Solution of word problems by Malaysian students: Insights from analysis of representations

Mohan Chinnappan
University of South Australia
<mohan.chinnappan@unisa.edu.au>

Munirah Ghazali
Science University of Malaysia
<munirah@usm.my>

Within the broad area of whole number operations, understanding and solving word problems continues to be an important area of inquiry. In the present study we draw on the framework of representational fluency to examine conceptual and procedural understandings that are exhibited by a group of Malaysian seven-year-olds as they attempted to solve 2-digit addition and subtraction word problems. Preliminary results show that the range of representations that were constructed by the participants as they searched the problem space was limited as was their ability to translate representations. Implications of these findings for further work about using representations are discussed.

Introduction

In its discussion about skills and mathematics competencies, NCTM Principle and Standards of School Mathematics (2000) has called for increased attention children's understanding of whole numbers and the use of this understanding to interpret and solve word problems. The solution of word or story problems is an important part of most primary and early-childhood mathematics curriculum. Despite recent instructional advances in practice, this area of mathematic learning continues to present considerable challenges to many students because multiple steps are involved in the solution process. Students have to read and understand the text. They will then have identify key parts of the text that are relevant to decoding the problem and developing the solution. In order to make these series of steps in the solution process explicit, we need a tool that is sufficiently context sensitive. In the present study, we draw on the framework of representation to track how students negotiate the word problem-solving environment.

The aim of our larger study is to document the range of representations that young children could construct and the level of fluency they exhibit in articulating the links among these representations. In this report we provide preliminary data about the representational range as a cohort of children attempted to solve two problems that involved 2-digit numbers.

Background to the study and research problem

Understanding whole numbers and operations are fundamental to children's number sense and their ability to apply those skills in making judgements and doing calculations. Studies of students' problem-solving competence with numbers and operations tend to identify a broad range of computational strategies that students could use. Counting up, Doubles and Bridging to 10 are examples of strategies that children could use when solving addition and subtraction problems. While the use of a particular strategy is indicative of students' competency with operations, such information is not useful for a) understanding

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why that strategy was preferred, b) why the strategy works and c) how to assist student when they apply the strategy incorrectly. We argue that the latter issues can be tackled head on by asking students to construct multiple representations of their strategies and reason the links between these representations – representational fluency. Analysis of strategy use from a representational fluency perspective can be expected to provide deeper insight in the lack of computational flexibility that has been reported as a continuing problems with children (Perry & Dockett, 2008).

In her study of Chinese and American teaching of elementary mathematics, Ma (1999:112) commented that

> 'being able to calculate in multiple ways means that one has transcended the formality of the algorithm and reached the essence of the numerical operations -- the underlying mathematical ideas and principles. The reason that one problem can be solved in multiple ways is that mathematics does not consist of isolated rules, but connected ideas. Being able to and tending to solve a problem in more than one way, therefore, reveals the ability and the predilection to make connections between and among mathematical areas and topics'.

Thus, the examination of alternative models of a solution or a solution strategy constitute a productive line of inquiry.

Over the past decade, considerable research ground has been covered in examining strategies used by students when performing the addition and subtraction, computations with non-contextualised problems (Blöte, Klein & Beishuizen, 2000; Russell, 2000; Verschaffel, Luwel, Torbeyns & Dooren, 2009). However, computations with contextualised problems introduce another layer of processing that involve interpreting and translating the text. In a more recent work on computational skills and problem solving, the added cognitive demands posed by the extra layer of linguistic information that has to be processed in word problem was acknowledged by (Fuchs, Fuchs, Hamlett, Lambert, Stuebing & Fletcher, 2008). According to Fuchs et al., (2008), while abstract computation problems are 'set up' for solution, word problem require students to analyse the text in order to identify missing elements and generate an equation that embodies the relationship between the given and missing elements. Thus, there is a need to unpack the relations between different elements in the problem environment of word problems.

Within the broad field of whole number concepts and operations, students' understanding and solution of addition and subtraction word problems have been the subject of sustained inquiry (Fuson, 1992; Verschaffel & De Corte, 1997). In an extensive review of literature, Verschaffel, Greer and De Corte (2007) commented the complexities underlying children's ability to solve arithmetic word problems must be examined both from a cognitive and socio-cultural perspective. The goal of our larger study is to gain insights into the effect of Malaysian culture in moulding students' conceptual understandings and problem-solving ability with arithmetic word problems. In so doing we aim to examine the range, flexibility and connectedness of representations that Malaysian children use in solving the above category of problems. In the report here we present selected data relevant about the type of representations and evidence of translation among representations.

Theoretical framework

Representations and fluency

The construct of representation has been in currency among researchers for a considerable time. Defined broadly, representation refers to the depiction or portrayal of a mathematical concept or entity. This definition suggests that a mathematical concepts can be given multiple representations. Indeed, the ability to construct multiple representations have been argued to provide a powerful window into children's depth of understanding (Schoenfeld, 1985; Barmby, Harries, Higgins, & Suggate, 2009). In the present study, we use the term *representation* to refer to 'external (and therefore observable) embodiments of students' internal conceptualizations (Lesh, Post & Behr, 1987: 34). In so doing we draw a distinction between internal and external representations. Internal representations reside in the long-term memory, and as such, are not observable. The manifestation of content and structure of internal representations in an observable mode constitutes external representation. *Representational fluency,* on the other hand, is indicative of translations between and within modes of representations. *Representational fluency* has been shown to be critical in building students' mathematical understanding (Goldin & Shteingold, 2001; Kaput, 1989).

Consistent with the above framework, the generation of representation is a precondition for demonstration of representational fluency. Thus, studies of representation fluency need to generate rich data about types of representations in the first instance before the question of fluency could be entertained. For example, a concept could be represented in different modes: tables, texts/verbal descriptions, graphs, symbols/notations and concrete/pictorial. For example, the addition of two whole numbers can be represented on a number line or as total of two groups of coins. Further, by relating the different components within and between representations, students could demonstrate the fluency there in. Skemp (1976) argued that connections among and within representations could provide insight into students' *relational* and *instrumental* understandings.

Research Questions

The following questions guided data generation and interpretation:

What is the range of representations that Malaysian children use in the solution of word problem involving addition?

What is the range of representations that Malaysian children use in the solution of word problem involving subtraction?

Methodology

Design

This was a descriptive study aimed at observing, recording and analysing participants' responses to a series of word problems.

Participants

Twenty six students from three regular intact Malaysian mathematics classrooms participated in the study. The age of children ranged from 7-8 years. All students were fluent in the Malaysian language – Bahasa Malaysia. The children had completed topics on addition and subtraction operations before the commencement of the study.

Tasks

In developing tasks that would assist us generate data relevant to the research questions, we were guided by two key design principles: a) the tasks were sensitive to the generation of multiple problem representations by the children and b) addition and subtraction problems had *part-part-whole* and *change* structures respectively (Carpenter, Fennema, Franke, Levi

& Empson, 1999). We chose problems with structures that were relatively easy as our focus was on representation and fluency. We selected 6 items from a pool of 2-digit addition and subtraction word problems from texts books that were used in Malaysian primary schools. The problems were anchored in Malaysian real-life contexts so that children could better relate to the meaning underlying the numbers. These problem were given to regular classroom teachers for their comment about the wording and authenticity of contexts. In this paper, we report students' responses to the following two problems (Table 1) which are English translations of the original problem which were in Bahasa Malaysia.

Table 1 – Focus Problems

Addition Problem	My brother went to a grocery store. He bought 8 apples and 7 oranges. How many pieces of fruit did my brother buy?
Subtraction Problem	Mr Smith has 13 computers in his store. He sold 5 computers. How many computers does Mr Smith have in his store now?
Subtraction Problem	Sally keeps 51 chicken. She sold 24 of the chickens at the market. Find the number of chicken that Sally has now?
Addition Problem	The Columbus Zoo has 36 seals and 47 penguins. Find the number of animals in the seal and penguin pools at the zoo.
Addition Problem	123 people visit the Butterfly exhibit on Monday and 98 people visit the Butterfly exhibit on Wednesday. How many people visit the Butterfly exhibit on both days?
Subtraction Problem	Alina bought a pair of shoes and a bag for 143 dollars. If Alina had 200 dollars to pay for these, how much change will she receive?

The researchers analysed problems in Table 1 with the view to generating representations that are based on students' learning experiences that were guided by the Malaysian national mathematics curriculum. This exercise yielded three broad categories of representations of word problems: verbal, visual, symbolic and/or algorithmic. These representations were given to the regular classroom teachers for comment before using it in our analysis.

Procedure

The primary source of data was one-to-one interviews that were conducted within the school environment during regular school hours. Prior to the interview, the teachers had briefed the students about purpose of the study and its value is helping them learn mathematics. The tasks were presented in Bahasa Malaysia, the medium of instruction in Malaysian national schools. Children were asked to solve the problem. Once they had produced a solution, they were asked to solve the problem in a different way. Following their second solution, children were encouraged to provide a third solution. When they commented that they could not think of other solutions, our final prompt was to get them to explain the relationships among the solutions that they had generated.

Concrete aid such as counters, graph papers, Unifix cubes and base-10 mats were provided for children to use. Our expectation was that the availability of multiple aids would assist them to formulate a range of solutions. The maximum duration of the interview was 30 minutes with most students completing the task within 20 minutes. All sessions were video-taped and transcribed for analysis. Our first level of data analysis involved the identification of frequency of three major categories of problem representations.

Results

Types of representation

Consistent with our conceptualisation of representational fluency, our first aim was to generate data about representation type before the question of fluency could be addressed. We report the percentage figures for representation types for addition subtraction problem (Table 2).

Table 2: Problem Representation

Representation		Addition Problem (%)	Subtraction Problem (%)
Verbal		8	0
Visual	Counters/Unifix cubes	25	21
	Number line	10	6
Symbolic/Algorithmic		46	62

Table 2 shows that in both problems, the Symbolic/Algorithmic representation dominated students' search in the problem space. The columns add to 89% because there were cases of students whose representations could not be coded as one the three above. A closer analysis of the algorithms indicate that children tended to favour the use of vertical algorithms which are commonly taught in their classroom. It is encouraging to note that a high percentage of representations that involve visuals was used in representing both the problems. As expected, a low percentage of the children could represent the problem verbally.

Representational fluency

Representational fluency, we have argued above, involved the construction of links among and between representation. Figure 1 shows an example of representation by one of the children, Ali (pseudonym) for the Addition problem (36+47) by using counters.

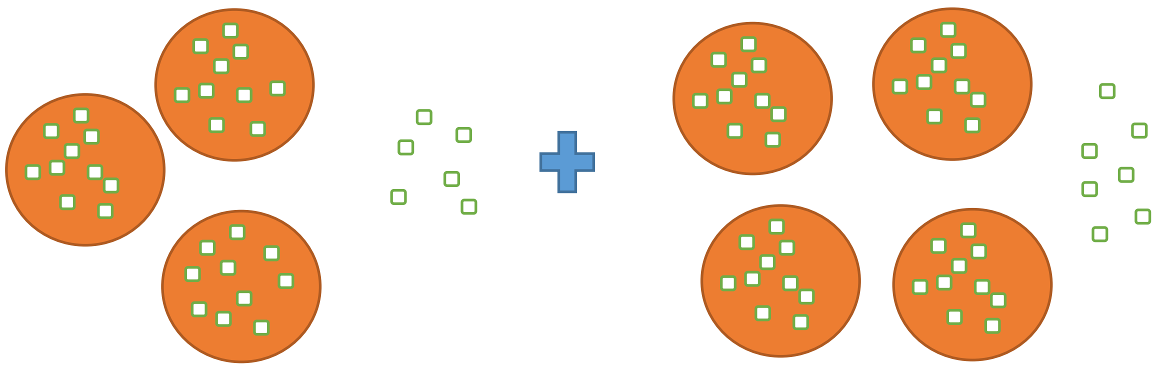

Figure 1

The above representation by Ali is indicative a number of key elements of his representation. Firstly, he was able code the problem as involving addition operation (use of addition symbol). The way the counters have been used suggests that Ali understands the base-10 system, where 36 is decomposed as 3 tens and 6 ones. He went on the work out the total as seven tens and 13 ones which was regrouped into eight tens and 3 ones (83). The above visual representation was then related to his vertical algorithm for adding 36 and 47. In being able to reason about the links between the components in Figure 1 with his vertical algorithm, he displayed representational fluency and translation between two representations. However, only 9% of the students could explicitly relate representations.

Discussion

The aim of the study was to analyse word problem-solving competencies by documenting preliminary evidence of problem representation and representational fluency among a cohort of Malaysian primary school children. Our assumption underlying this study was that a representationally-based analyses of students' problem-solving performance can be expected to provide insightful information about students' sense-making and understanding. In making progress with the given set of problems, children had to work with a number concepts within each representation and the shift they had to make between representations was not trivial. Representational fluency requires that children understand the conceptual underpinning of the problem and associated operations. Concepts such as place value and groupings are essential elements of such an understanding.

The predominance of use of Symbolic/Algorithmic representation was not totally unexpected as this mode is privileged in regular classroom instruction in Malaysia. The Malaysian mathematics curriculum is, by and large, examination driven. Success in these examinations require children's ability to produce correct answers rapidly. Algorithms as a form of representation are valuable tools but children need to understand the conceptual basis of such representations, a point that was highlighted analysis of procedural knowledge by Star (2005).

The results of this early study are encouraging in that we see evidence of children generating visual representations of solution of both the addition and subtraction problems. Counters were the main concrete aid that children used to depict ones and tens for the numbers. The processes of decomposition and regrouping of numbers with counters during the construction of representation provided windows into children's understanding of the decimal system. Malaysian culture, in the main, does not value or support talk in formal and informal contexts. We note, this aspect of their culture in the relatively low verbal representations.

In framing the research questions of the present study, our theoretical orientation was representations and fluency. The construct of representations provides a sensitive lens through which to better understand the interplay of socio-cognitive processes that underpin Malaysian students' understanding and solution attempts of word problems. As Kaput (1989) suggested, each representation of a problem could highlight or hinder an aspect of students' understanding. Thus, the opportunities for students to construct multiple representations of word problems will increase the likelihood of generating data that reveals not only the depth of students' understanding but also the interaction between socio-cultural and cognitive elements during the course of solution development. Toward that end, our future work will aim to develop word problems that are germane to the generation of multiple representations.

This is a preliminary report which forms part of our on-going larger research the aim of which is to understand Malaysian children's representations and associated flexibility or fluency. There are number of variables within a problem that could impact on the level and quality of representations including symbols, language and problem structure (Carpenter et al., 1999; Kintsch & Greeno, 1985). Our future work in this space will examine these factors more closely. Additionally, in the current report we have not provided data on the question of fluency which involves students reasoning among representations. As this is a pilot study, we are guarded in making general claims about representational fluency among Malaysian children that is based on limited data. However, the methodology and an initial data analytic procedures does provide directions for future debated and discussion for future work in this area.

The results of the study has implications to promote teachers' instructional agenda in their mathematics classrooms. It would seem that current priorities are mainly concerned with getting children to perform operations and produce correct answers to problems. There is a need to shift this algorithm-driven approach to engaging children that supports experimentation and experience with multiple problem representations (Lewis, 1989), and investigate why representations are related. Teaching for both skill development and conceptual understanding is crucial - these are learned together, not separately. But teaching in a way that helps students develop both understanding and efficient procedures is a complex and challenging task (Lawson & Chinnappan, 2015; Chinnappan & Forrester, 2014). Such an enterprise requires that teachers develop a sophisticated understanding of the basic mathematical ideas that underlie representational fluency, use tasks in which students develop these ideas, and recognize opportunities in students' work to focus on these ideas.

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