Zooming-in on Decimals

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This study investigated how six Year Four (9-10 years old) students interacted with a dynamic (zoomable) digital number line to demonstrate knowledge and understandings of decimal fractions. Results from a task-based interview indicated that the zoomable number line is able to assist students in developing conceptual understandings of decimal density, place value and relative size. The app proved to be a powerful mathematical representation of decimal fractions because of its dynamic affordances that allowed the students to 'see' concepts that would otherwise be 'unseeable' when using a traditional static number line.

Decimal fractions are considered to be of great significance in the primary mathematics curriculum due to their application and use in everyday life. However, decimal fractions and the related areas of ratio and proportion, are recognised as the most challenging and complex areas of mathematics for young children to learn (Lamon, 2001). Such complexities arise as students are exposed to fractions in numerous symbolic representations, including ratios, common fractions, decimals, and percentages, without deep understanding and knowledge of how use each notation correctly (Bobis, 2011). Decimal fractions notation can be difficult for children to comprehend, as part of the relationship, the denominator, is hidden (Steinle & Stacey, 2004). The relative size of numbers written in decimal notation is instead expressed through place value, and many young students struggle to extend their whole-number place-value knowledge to the comprehension of decimal fractions.

One of the key concepts required for understanding of decimal fractions is *decimal density*. Decimal density refers to the continuity property of rational numbers, whereby, between any two decimals there are an infinite number of other decimals (Widjaja, Stacey & Steinle, 2008). Many young children have extensive difficulties in recognising the density of decimals, as it is void in their whole-number knowledge (Widjaja et al., 2008). The discreteness feature of whole numbers is incongruous for understanding the density of decimals (Widjaja et al., 2008).

If students do not have a strong conceptual understanding of decimals they are more susceptible to developing a set of misconceptions associated with the topic area. Misconceptions are predictable, systematic mistakes that arise from an incorrect interpretation; broadly they are a mistaken way of thinking and understanding about a mathematical concept (Steinle, 2004). Young students' misconceptions about the meaning of decimal number notation has been well documented in the works of Steinle and Stacey (e.g. Steinle & Stacey, 2004; Steinle, 2004, Steinle & Stacey, 2001). However, less evidence has been accumulated on how to avoid and address these misconceptions. How then can teachers support children's comprehension of key concepts such as place value and decimal density when decimal fractions are introduced?

An important feature of mathematics education is working with appropriate mathematical representations to assist students in understanding the abstract concepts that are central to learning mathematics, yet there is uncertainty around the best representation for decimal fractions. Number lines have been suggested as a powerful representation of decimal fractions as they show where a decimal 'fits' between two-whole numbers

2018. In Hunter, J., Perger, P., & Darragh, L. (Eds.). Making waves, opening spaces *(Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia)* pp. 337-344. Auckland: MERGA.

(Thomson & Walker, 1996), which relates to decimal density. However, the mathematical meanings of the structure and multiple uses of number lines, can also be problematic for children (Teppo & van den Heuvel-Panhuizen, 2014; Way & Harding, 2017). The affordances of dynamic digital representations may offer a better learning alternative than traditional static number lines (Moyer-Packenham et. al., 2016; Tucker, Moyer-Packenham, Shumway& Gordon, 2016), but little research has been conducted about this particular digital tool. It is very difficult to display hundredths, and thousandths on a paper-based number line, however the sophisticated functions of the interactive web-app make this easily achievable. For example, users are able to stretch the number line and zoom-in to see how tenths 'fit' between two consecutive whole numbers and zoom-in again to see how hundredths are located between a pair of tenths.

The focus of this study was on exploring how children's interactions with the zoomable number line might support development their conceptual understanding of decimal fractions. The research question addressed by this paper asks; How do Year 4 students interact with dynamic digital representations, namely a 'zoomable number line', to demonstrate knowledge and understandings of decimal fractions?

Methodology

The theoretical position of this study is grounded in the work of Piaget (1978), who first introduced the task-based interview to uncover children's thinking about posed problems and believed that "true understanding takes place when the student makes discoveries for themselves" (Assad, 2015, p.17). A major tenet of constructivism suggests that learning occurs when students are actively involved in a process of meaning and knowledge construction, mediated through existing knowledge, as opposed to passively receiving information (Simon, 1995, p.116). This study recognised students' existing knowledge of decimal fractions and anticipated that individual students would respond to their explorations of the digital number line in different ways.

Task-Based Interviews

Gerald Goldin's (1993, 2003) research links constructivist theory with structured clinical interviews as a means of understanding conceptual knowledge, higher-level problem-solving processes, and children's internal constructions of mathematical meanings. Goldin (2003) outlines two key functions of task-based interviews, firstly, to observe the mathematical behaviour of children, often in an exploratory problem-solving context, and secondly, to draw inferences from observations to form conclusions about the problem solver's possible meanings, knowledge structures, cognitive processes, and whether these change through the course of the interview. Therefore, the tasks must be sufficiently open-ended to allow each student to respond in their own preferred way, and designed to maximise dialogue between the researcher and student (Hunting, 1997). The researcher uses probe questions to encourage further explanation, for example 'Can you tell me what you are thinking?', but remains neutral regarding 'correctness' of responses. The purpose of the interview is neither to teach nor to test knowledge, but rather to explore the student's ways of thinking and interacting with the mathematical representation.

Participants

The participants of the task-based interview were from a class of Year Four students (9-10 years old) in a suburban school in NSW. This year level was chosen because, according to the syllabus, in Year 3 decimal notation (tenths) has been introduced, and Year 4 students are working towards being able to "model, compare and represent decimals of up to two decimal places", and being able to "place decimals of up to two decimal places on a number line" (BOSTES, 2012, p.142). The class teacher was asked to recommend six children who would be comfortable in talking one-to-one with the researcher. Although the children were not intended to be representative of the class, the teacher was asked to ensure that the children spanned a range of general mathematical achievement levels, in case there were differences in the ways in which low and high achievers interacted with the digital number line. Six children were considered sufficient to reveal a variety of student responses, while not exceeding the imposed time constraints of an undergraduate Honours project.

Tasks

A set of five tasks were designed, with increasing levels of challenge, to encourage the students to interact conceptually with decimal notation and the positioning of decimals on a number line (see Table 1).

Table 1:
Interview tasks

Task 1 – Base knowledge Show the student card '4.2' What is this number? What is this? (interviewer gestures to decimal point) What does it mean?	Task 3 – Locating a decimal number Locate and describe its location on the digital number line. 0.3 0.12 0.9 1.55 Extension: *4.915 9.819* How did you locate the decimal fraction on the number line? How did you know what numbers the decimal was between?
Task 2 – Introduction to app The participant freely investigates the app by manipulating the toggles to zoom in and out, and scroll along the number line. What do those lines represent? What does that decimal number mean? What do you notice about the 'zoomable number line' app? Are there any features of the app that are confusing or difficult to use? What are some interesting or useful features of the app?	Task 4 – Comparing numbers Which is larger? (Predict then check) 0.4 or 4.5 0.86 or 1.3 0.3 or 0.4 1.85 or 1.84 3.71 or 3.76 Extension: 3.92 or 3.481 4.08 or 4.8 What strategy did you use to predict which decimal was larger? How do you know it is larger?
Task 5 – Contextualised problem solving *Five gymnasts are entered into a competition. Four of the gymnasts have performed their routines. Their scores, out of ten, were 9.8, 9.75, 9.79, and 9.76. What score must the last gymnast get in order to win the competition?*	

The interview began with a conversation about decimal fractions and an exploration of the functionality of the app. Following this, students located a sequence of one- and two-decimal-place numbers on the digital number line. They then compared a series of pairs of decimal numbers by identifying which was larger. The fifth task was presented as a contextualised word problem requiring the comparison of four decimals.

Data Collection and Analysis

Video-recording the interview was critical to collecting quality data as it was necessary to capture what was said and done by the child simultaneously, including gestures, expressions and emotional responses. In addition, a screen-recording tool from the application *QuickTime Player* was used to document manipulative actions that students made on the 'zoomable number line' web-app (Pierce, 2017), accessed on an iPad.

Data analysis involved repeated viewings of the video/audio recordings and screen-captures, to document the key features of each child's responses, with particular attention to manipulation of the digital number line, the mathematics concepts demonstrated, and the relationship between the two. These significant sayings and doings were then analysed for similarities and differences across the students' profiles, using a thematic analysis approach (Braun & Clarke, 2006).

Findings

The analysis revealed four major themes: decimal density, whole-number thinking, place value and relative size of decimals, and problem-solving. The four themes are presented in order of their predominance across the six students.

Decimal density

While completing the tasks, most students attended to the idea of decimal fractions existing between whole numbers, and smaller decimals existing between those decimal fractions, that is, the concept of decimal density. Through their actions and dialogue, they explained the zoom function of the digital number line as, "… a closer view to show you how far apart the numbers are and what decimals are in between" (Student 1).

For example, when locating the decimal 1.55, Student 5 explained "one-point-five-five is in the middle of one-point-five-six and one-point-five-four and also in the middle of one and two." By using the zoomable number line the student was able to demonstrate his developing understanding of number density as he recognised how wholes can be continually divided into smaller parts by describing one-point-five-five being in the middle of 1 and 2 as well as 1.56 and 1.54. The zoomable number line allowed Student 5 to exhibit his understanding of number density as he used the zoomed-in screen display (as seen in Figures 1 & 2) by gesturing to it when he named multiple locations of 1.55 on the number line.

Two students extended this idea as they commented on the continuity property of decimals whereby between any two decimals there are an infinite number of other decimals. Student 4 uncovered how, between any two distinct numbers, there are an infinite number of decimal fractions as he continually zooming in around the number 1. He was interested in discovering how many place values could be revealed in a decimal. The function of zooming-in to reveal increasing place values fascinated him. Similarly, when Student 6 was freely investigating the zoomable number line app he described his actions aloud: "It's zooming in… and now it's showing the decimals… and now it's showing the hundredths… and now it's showing the thousandths… and now it's showing the tens of thousandths."

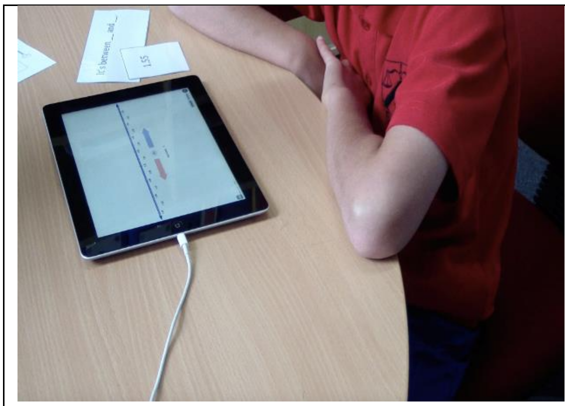

Figure 1. Screenshot of Student 5's interview showing zoomed in screen display of the web-app to complete a decimal location task.

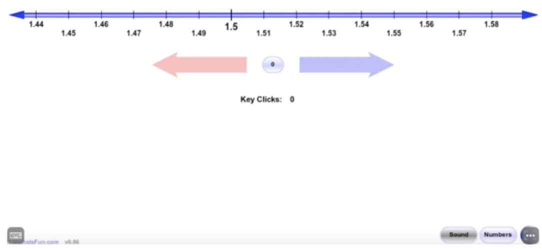

Figure 2. Screenshot of the 'Zoomable Number Line' web-app that was recorded using QuickTime Player whilst Student 5 was manipulating it.

Shifting 'whole number thinking'

Four out of the six students interviewed in this study displayed conceptual confusion based on their treatment of decimals as whole numbers. These students also displayed difficulty coordinating the number of parts and the size of the parts, as they had not developed the decimal-fraction link. The decimal-fraction link refers to the idea that decimals, like fractions, allow us to describe parts of a unit quantity (Steinle & Stacey, 2001). This was evident in the reading of decimals as whole numbers, for example, "one-point-fifty-five". However, during the interview, the students' conceptual understanding of decimals shifted from whole-number knowledge to being able to recognise and apply the decimal-fraction link through their interaction with the zoomable number line. For example, Student 3 was asked to locate the decimal 0.12 on the zoomable number line and describe its location by stating "it's between _ and _". Student 3 zoomed into 0.9 without revealing the hundredths and stated "zero-point-twelve is between zero-point-eleven and zero-point-thirteen". When prompted by the interviewer he zoomed into show hundredths and was then able to locate 0.12 by "scrolling back". The next question asked the student to locate 0.55 and describe its location on the number line. For this task, Student 3 instantly zoomed-in to reveal hundredths around the 1 and was then able to accurately locate 1.55.

When answering the same question, Student 1 went to locate 0.12 as a decimal after 0.9, after zooming in to reveal hundredths and seeing the numbers 0.88, 0.89, 0.9, and 0.91 on the number line he started scrolling left to locate the decimal after 0.1. The interviewer asked the student why he had self-corrected his working the student responded, "...because I read it wrong" the interviewer asked, "when did you realise that?" the student answered, "when I zoomed in". When asked to locate 0.12, Student 2 similarly to Student 1 went to locate it after 0.9, he realised the error when he zoomed in to reveal hundredths.

Place value and relative size of decimals

As described above, a common misconception held by children is thinking that the numerals to the right of the decimal point are another whole number. Student 3's persistent whole-number thinking led him to believe, in the context of comparing decimals, that "longer is larger". The scrolling function of the digital number line enabled the student to correctly compare the relative sizes of sets of decimals, which assisted him in discovering the concept of place value in decimals. For example, Student 3 was asked to predict whether 3.92 or 3.481 is larger, the student answered "three-point-four-hundred-and-eighty-one" reasoning that "four hundred and eighty-one is larger than ninety-two". The student was prompted to use the zoomable number line to check his answer. Student 3 accurately located 3.92 on the zoomable number line. He then zoomed in further to reveal thousandths and when probed he noted "I have to scroll back to find three-point-four-hundred-and-eighty-one". The interviewer asked the student again which of the two was larger and why, to which he responded "three-point-ninety-two because it's further up on the chart [number line]". At the end of the task-based interview the student was asked whether the app assisted him in answering a particular question, he replied "yes, the last question because I thought that the four-hundred-and-one decimal would be larger than the ninety-two decimal but it wasn't".

Problem-solving, explaining and justifying

Students with a stronger understanding of decimal notion still struggled with the abstractness of decimal notion, but the digital number line provided a tool for making predictions, testing them and then questioning their findings to 'see' the concept from various perspectives. In particular, Students 5 and 6 engaged with the dynamic nature of the zoomable number line to assist them in confirming and verbally justifying their solutions for the open-ended problem-solving task.

For example, after reading the *Task 5* problem-solving question aloud, Student 5 answered, "they must get an addition of zero-point-zero-six to win". When asked to explain his working the student reasoned, "because the lowest score is nine-point-seven-five and the highest is nine-point-eight and if you added zero-point-zero-six then it will be one over the next one and they will win." The student was asked to prove his answer using the zoomable number line he said, "the lowest score is at this point [gestures to 9.75] meanwhile the next one is one-point over [gesture to 9.76] and the next one is one-point under the highest [gestures to 9.79] and the highest is here [gestures of 9.8]." (See Figure 3)

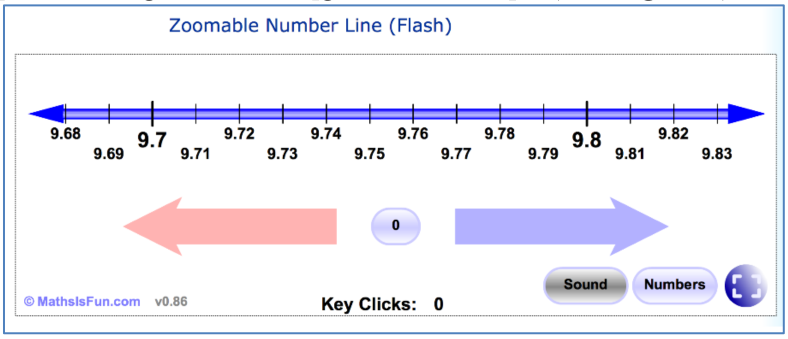

Figure 3. Screen shot of Student 5's number line during justification of problem solution.
(Pierce, 2017 at *Maths is Fun*)

The interviewer asked, "so what score could the competitor get to win?", student answered "6 more [gestures from 9.75 up 6-hundredths to 9.81] because then they will manage to get just one over the second-place at nine-point-eight-one." The interviewer then asked "did the zoomable number line help you answer the questions?" the student replied, "Yes, seeing the numbers on the number line helped me answer the question, I could see exactly where each number was and zoom in to see specifically where it was to make it easier."

Discussion and Conclusion

As anticipated, the ways in which each of the six students used the digital number line varied across the tasks, with a strong link between their approaches and existing understandings of base-10 numeration and decimal fractions. However, regardless of their prior knowledge, all children showed progress in their thinking during the 20-minute interview, particularly regarding decimal density, whole-number thinking and recognising the role of place value in communicating the relative size of decimal fractions. The students' interactions with the zoomable number line, at times, generated productive cognitive conflict, as the dynamic representation of decimal fractions challenged their existing understandings and misconceptions. Outside of the research-interview situation, these would have been ideal 'teaching moments'.

The number line is one of the few models that is functional for discussing the density of decimals. However, Steinle and Stacey (2004) have commented on the limitations of a static number line as they note that some students view it as a discrete representation of numbers at isolated points. The digital functions of the zoomable number line allowed the students to uncover decimal concepts that a static number line is not able to readily illustrate. The ability to move forwards and backwards along the number line enables students to see the continuous sequence of decimals, which assists them gaining a sense of relative magnitude and of addition and subtraction (Teppo & van den Heuvel- Panhuizen, 2014), and this was readily facilitated by the digital number line.

Pierce, Steinle, Stacey, & Widjaja (2008) argue for the need for fundamental reorganisation of whole-number prior knowledge in understanding the density of decimals. The dynamic affordances of the digital number line appeared to prompt the required reasoning in students, as they zoomed-in to reveal decimal fractions between two whole numbers, and zoomed-in further to see smaller and smaller parts appearing.

While keeping in mind the inferential limitations of a small-scale study, the findings clearly indicate the potential of dynamic, digital number lines for use as effective learning and teaching tools to develop conceptual understanding of decimal fractions, particularly regarding number density, comprehending place-value notation and the relative size of decimal fractions. All students, with minimal prompting from the researcher, engaged in self-directed exploration of decimals, and self-correction of reasoning as a direct result of their interactions with the zoomable number line. As the digital number line is capable of illustrating concepts that cannot be effectively represented on a traditional static number line, further research into how children perceive decimals with this digital tool could inform effective teaching practices for the prevention and correction of common misconceptions about decimal fractions.

References

Assad, D. (2015). Task-Based interviews in Mathematics: Understanding student strategies and representations through problem solving. *International Journal of Education and Social Science*, *2*(1), 17-26.

Bobis, J. (2011). Fractions: Best evidence and its implications for practice. In J.Way & J. Bobis (Eds.), Fractions: teaching for understanding, (pp. 11-20). Adelaide, SA: Australian Association of Mathematics Teachers.

Board of Studies NSW. (2012). *Mathematics K-10 syllabus for the Australian Curriculum.* Retrieved from https://syllabus.nesa.nsw.edu.au/download/

Braun, V. and Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3* (2), 77-101.

Goldin, G. (1993). Observing mathematical problem solving: perspectives on structured, task-based interviews. In B.Atweh, C.Kanes, M.Carss & G.Booker (Eds.), *Proceedings of Sixteenth Annual Conference of the Mathematics Education Research Group of Australasia.* (pp.303-309). Brisbane: MERGA.
Goldin, G. A. (2003). Representation in school mathematics: A unifying research perspective. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), A research companion to principles and standards for school mathematics (pp. 275–285). Reston, VA: The National Council of Teachers of Mathematics.
Hunting, R. (1997). Clinical interview methods in mathematics education research and practice. *The Journal of Mathematical Behavior*, *16*(2), 145-165.
Lamon, S. L. (2001). Presenting and representing: From fractions to rational numbers. In A. Cuoco & F. Curcio (Eds.), The role of representations in school mathematics—2001 yearbook (pp. 146–165). Reston, VA: NCTM.
Maths is fun. (2016). Zoomable Number Line. [online] Available at: https://www.mathsisfun.com/numbers/number-line-zoom.html.
Moyer-Packenham, P. S., Bullock, E. K., Shumway, J. F., Tucker, S. I., Watts, C. M., Westenskow, A., & Jordan, K. (2016). The role of affordances in children's learning performance and efficiency when using virtual manipulative mathematics touch-screen apps. *Mathematics Education Research Journal*, *28*(1), 79-105.
Piaget, J. (1978). Behavior and evolution (D. Nicholson-Smith, Trans.) New York: Random House. (Original work published 1976).
Pierce, Rod. (2017). Zoomable number line virtual manipulative. *Math Is Fun*. Retrieved 1 Mar 2018 from http://www.mathsisfun.com/flash.php?path=%2Fnumbers/images/number-line-zoom.swf&w=960&h=330&col=%23FFFFFF&title=Zoomable+Number+Line+(Flash)
Pierce, R., Steinle, V., Stacey, K., & Widjaja, W. (2008). Understanding decimal numbers: a foundation for correct calculations. *International Journal of Nursing Education Scholarship, 5(*1), 1-15.
Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective, *Journal for Research in Mathematics Education*, *26*(2), 114-145.
Stacey, K. & Steinle, V. (2003) Exploring the right, probing questions to uncover decimal misconceptions. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley (Eds), MERINO – Mathematics education research: Innovation, networking, opportunity (Proceedings of the 26th annual conference of the Mathematics Education Research Group of Australiasia, Vol. 2, pp. 634-641). Geelong: MERGA.
Steinle, V. (2004). Detection and remediation of decimal misconceptions. In B. Tadich, S. Tobias, C. Brew, B. Beatty & P. Sullivan (Eds), *Towards excellence in mathematics* (Proceedings of the annual conference of the mathematical Association of Victoria, pp. 460-478). Brunswick: MAV.
Steinle, V., & Stacey, K. (2001). Visible and invisible zeros: Sources of confusion in decimal notation. In J. Bobis, B. Perry & M. Mitchelmore (Eds.), *Numeracy and Beyond. Proceedings of the 24th Annual Conference of the Mathematics Education Research Group of Australasia* (Vol. 2, pp. 434-441). Sydney: MERGA.
Steinle, V. & Stacey, K. (2004). A longitudinal study of students' understanding of decimal notation: An overview and refined results. In I. Putt, R. Faragher, & M. Mclean (Eds), *Mathematics education for the third millennium, towards 2010* (Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Vol. 2, pp. 541-548). Sydney: MERGA.
Teppo, A., & van den Heuvel-Panhuizen, M. (2014). Visual representations as objects of analysis: the number line as an example. *ZDM—The International Journal on Mathematics Education*, *46*(1), 45-58.
Tucker, S., Moyer-Packenham, P., Shumway, J., & Gordon, K. (2016). Zooming in on children's thinking. Australian Primary Mathematics Classroom, 21(1), 23-28.
Watson, A., Jones, K., & Pratt, D. (2013). *Key Ideas in Teaching Mathematics: Research-based guidance for ages 9-19*. OUP Oxford.
Widjaja, W., Stacey, K., & Steinle, V. (2008). Misconceptions about density of decimals: Insights from Indonesian pre-service teachers' work. *Journal for Science and Mathematics Education in Southeast Asia*, *31*(2), 117-131.
Way, J. & Harding, C. (2017). Number lines: Observed behaviours and inferred cognitions of 8 year-olds. In Kaur, B., Ho, W.K., Toh, T.L., & Choy, B.H. (Eds.). *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 305-312. Singapore: PME.