Characteristics of Spatial Visualisation: Perspectives from Area of Composite Shapes

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This paper attempted to make explicit some of the underlying characteristics of spatial visualisation using the concept of area of composite shapes. By engaging students with metric-free tasks, we identify the type of perceptual and visual/spatial manoeuvres that they deploy in such situations. Interview data collected from three students in Grade 7, 8, and 9 are used to exemplify three key constituents of spatial visualisation: figure-ground perception, global and local perception, and gesturing. An observable discontinuity was discovered in coordinating different pieces of spatial information after disembedding the parts that constitute the whole. This paper concludes with pedagogical implications.

Imagine the task of finding the area of the region bounded by a square and an inscribed circle or the area of the region bounded by a circle containing a square. As you think about this experimental scenario, consider some of the things that may have come to your mind's eye as you visualized the actions without physically undertaking the task. Such situations may also arise in determining the nets of a cube or in finding the number of lines of symmetry of a figure. The specific spatial ability that is thought to underlie such processing is referred to as *spatial visualisation*. In the mathematics curriculum, a range of such spatial manoeuvres may be encountered especially as part of the geometry and measurement strand.

Spatial visualisation is an umbrella term that includes a range of visual/spatial manoeuvres (Carroll, 1993; Clements & Battista, 1992; Lowrie, Logan, & Ramful, 2017; McGee, 1979; Yakimanskaya, 1991) unlike mental rotation and spatial orientation, which have well-defined conceptual boundaries. Battista (2007) refers to spatial visualisation as "the ability to 'see', inspect, and reflect on spatial objects, images, relationships and transformations" (p. 843). Joining parts of a shape to construct its configuration and folding 2D nets to form 3D objects may constitutively involve different mental operations and both seem to fit the above definition of spatial visualisation. Similarly, imagining cross sections of given objects and anticipating the result of cutting a section of an object may also fit into this category. It appears that the etymology of the term 'spatial visualisation' as involving a visualisation and a spatial component tend to lead to an elusive interpretation of spatial visualisation as spatial reasoning itself. The important and open question then is: what is and what is not spatial visualisation? What may possibly provide partial answers to this question is the unpacking of the ways in which we operate with images in various tasks. Such a research endeavour may potentially elucidate the different kinds of manipulations that currently fall under the label of spatial visualisation.

This study is part of a spatial reasoning research programme that is attempting to unpack the ways in which spatial reasoning plays out in the Mathematics curriculum, more

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specifically in the Geometry and Measurement strand. This paper focuses on spatial visualisation as it occurs in finding the area of composite shapes. Much of the research on composing and decomposing shapes appear to emanate from early childhood and primary education (Clements, 2004; Sinclair & Bruce, 2015). This is not surprising given that such young children are at a stage where they are building the foundational concepts for geometric and spatial thinking. As students move into secondary school, understanding shape composition and decomposition becomes an important aspect of measurement of area. Students are required to have a sound understanding of the geometric aspect of shapes before they can apply the measurement concepts of area (Huang & Witz, 2013). In an attempt to understand how students generate, retain, and process visual/spatial images, and identify the characteristic of spatial visualisation, students engaged with tasks involving the segmentation of areas of composite shapes. Explicitly, the objective of this study is to characterise the type of perceptual and spatial manoeuvres that students deploy in finding the area of composite shapes, with the aim of tracking down the spatial manoeuvres that form part of spatial visualisation.

Analytical Framework

Visual Perception

Visual perception is associated with "the ability to see and interpret" (Hoffer, 1977, cited in, Gal & Linchevski, 2010) and has a strong conceptual link with spatial visualisation. Gal and Linchevski (2010) assert that "organization of perceptual data, recognition, and representation of objects in mind—all are a *sine qua non* base of visualisation" (p. 167). In this study, the composite area tasks required the participants to extract objects from the visual scene or to recognise shapes or objects (as entities). After the perceptual recognition and visual pattern recognition stages, the problem solver may represent this knowledge in terms of verbal/pictorial form or hierarchically in terms of mental images. Gal and Linchevski assert that: "[t]he mental picture that exists in the observer's mind consists of mental cuttings of the original physical (drawn) configuration in question. These mental cuttings separate the decomposed configuration into sub-units" (p. 177). Thus, the representation that we hold of a spatial task may consist of a complex configuration of mental images, parsed by our individual interpretation. It is also important to make the distinction between spatial and visual mental images as basic units of spatial visualisation. Spatial mental images contain information about the location, size, and orientation of entities while visual mental images represent shape information including other physical attributes such as colour and depth (Kosslyn, 1994, cited in, Sima, Schultheis, & Barkowsky, 2013).

Figure-Ground Perception

Visual perception includes, among others, figure-ground perception, perception of spatial relationships, and visual discrimination (Del Grande, 1990; Kovacs & Julesz, 1993). Figure-ground perception is emphasised given its salience in the current study. Figure-ground perception refers to the visual act of prioritising attention on a specific component/shape in a given configuration. Thus, a particular component/shape is foregrounded while others are left in the background. Another mental operation that may be involved in such a visual act is disembedding (Kovacs & Julesz, 1993). This mental operation allows an individual part from a partitioned whole to be lifted from its referent whole while keeping in mind its relation to the whole. Thus, both the part and the whole can be discerned as separate entities.

Global and Local Perception

Another concept that was relevant in this study is global and local perception (Enns & Kingstone, 1995). Global perception refers to perceiving the overall structure of the scene or image being processed, identifying the spatial relationship among elements and linking them together. Local perception focuses on boundaries of images, contrasts, and individual elements (Nayar, Franchak, Adolph, & Kiorpes, 2015). Nayar et al. (2015) suggested that adults often perceive images in a global form and will identify holistic shapes based on imagined edges "rather than a collection of…local elements" (p. 39). Nayar et al. found that children as young as 10 years of age can move between local and global perception and processing.

Gestures

Gestures give evidence of the spatial representations that individuals hold while solving tasks. Hostetter and Alibali (2008) assert that co-gestures are commonly used by students in working with spatial tasks. Logan, Lowrie, and Diezmann (2014) found that primary school-aged children utilised gesture as a support mechanism for their cognitive processing when engaging with spatial tasks. This tangible form of image making acted as an insight into their spatial thinking and strategy use, providing observable details of some cognitive and conceptual aspects of their learning (Alibali, 2005).

The Context of the Study

This study is situated within a Government Partnerships for Development (GPFD) project funded by the Department of Foreign Affairs and Trade (DFAT) involving University of Canberra mathematics educators working with Indonesian mathematics teachers in a disadvantaged community. Specifically, this study was part of a teacher professional development programme, where the Indonesian mathematics teachers were engaged in classroom action research related to spatial reasoning.

Participants

The three students who are the subject of this paper were high-ability students in their classroom context and were from Grade 7 (S1-female), 8 (S2-female), and 9 (S3-male). Given the preliminary nature of this study, the focus was on high-ability students to have a proxy of the accessibility of the chosen spatial tasks. Similarly, one student from each of the three grade levels was chosen for exploratory purposes. The three students were given a pre-test prior to the interviews to gauge their prerequisite knowledge of area. All of them were proficient in measuring the area of basic shapes such as rectangle, square, circle, and various triangles.

Task Design

The composite area tasks were designed to engage students in coordinating spatial information as they occur in the interpretation of the area of 2D shapes. The intent was to create a context where students would manipulate visual/spatial images which are regarded as the basic units of spatial visualisation. The tasks were metric-free (see Figure 1) and therefore required students to describe the procedure rather than work with measurements such as the area formula. The tasks were organised into three levels: finding area by segmenting polygonal shapes (Level 1 - e.g., Tasks 1a & 1b); finding area by working with the difference among polygonal shapes (Level 2 - e.g., Tasks 2a & 2b); and finding area by working with differences involving circles (Level 3 - e.g., Tasks 3a-d). Figure 1 exemplifies sample tasks from each of the three levels.

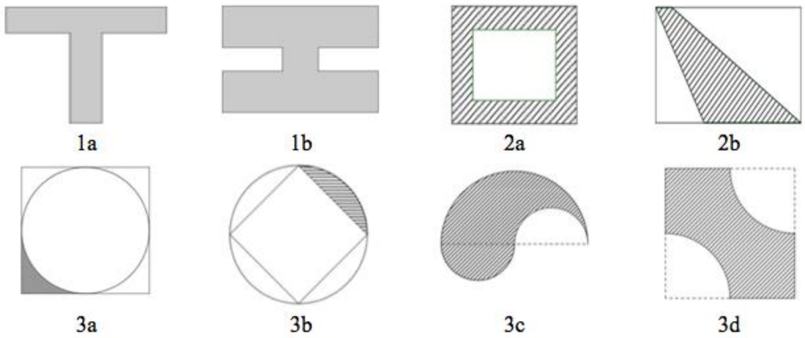

Figure 1. Sample tasks at Levels 1, 2, and 3.

Data Collection and Analysis

The interviews took place at a school in a remote area in West Nusa Tenggara, Indonesia. The interviews were conducted after school hours, and they were anonymously video recorded (faces were not captured). Interview guidelines were developed by the researchers in collaboration with the two teachers working with the participants. After the teachers conducted the interviews, the video recordings were uploaded onto a password-protected shared Google drive. Retrospective analysis of video data focused on the identification of the spatial manoeuvres that the students deployed along with the gestures they made. The figural processing of the tasks was scrutinized, following the written inscriptions that participants made on the paper provided. The interviewers' strategy of occasionally asking students to explain twice how they solved the tasks was beneficial.

Results and Discussion

Three distinct manoeuvres underlying spatial visualisation were identified: (i) figure-ground perception; (ii) global and local perception, and (iii) gestures. The data is presented along the three themes to display their incidence in the students' attempts to find the area of the composite shapes.

Figure-Ground Perception

Figure-ground perception as a visual act was clearly apparent through the explanations and gestures that the students made with their fingers as they foregrounded particular shapes and put others in the background. Figures 2(a) and 2(b) illustrate how students focused on particular shapes in their attempts to find the shaded area in Task 2b. The students used their fingers to show the shapes they considered (which are indicated by the bold outline in Figure 2).

S3: Method 1 is to find the area of the trapezium [trace the outline 2(a)], then subtract with the area of this triangle [trace the outline 2(b)]. The second method, find the area of this trapezium [trace the outline 2(c)], then subtract the area of this triangle [trace the outline 2(d)].

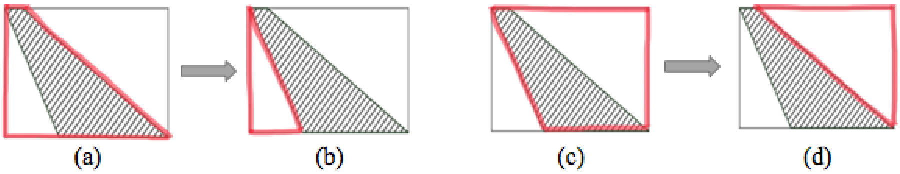

Figure 2. Foregrounding and backgrounding particular shapes by S3 (Methods 1 and 2).

Manipulation of spatial/visual images. An example is provided to illustrate how student S1 handled spatial/visual images. In her first approach to Task 2b, S1 coordinated the three segments as follows (see Figure 3):

S1: First, we find the area of the rectangle [trace the outline 3(a)], then find the area of triangle A [trace the outline 3(b)], then triangle B [trace the outline 3(c)]. Add the area of triangle A and the area of triangle B. So, the area of this (shaded part) is the area of the rectangle subtract the area of A and B.

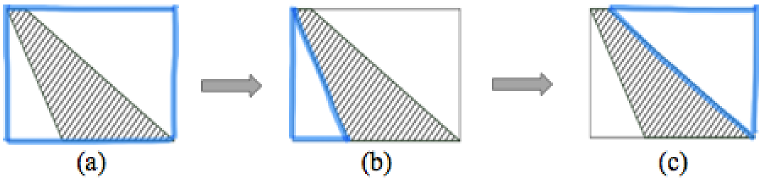

Figure 3. Segmentation of space by S1 in Task 2b using triangles.

When the teacher asked S1 if she had another method, she used two trapeziums within the picture to provide a highly spatial explanation (see Figure 4):

S1: Find the area of trapezium A [trace the outline 4(a)], then find the area of trapezium B [trace the outline 4(b)], and find the area of the rectangle [trace the outline 4(c)]. To find the area of the shaded region is the area of trapezium A, add the area of trapezium B. The result of this addition subtracts with the area of the rectangle.

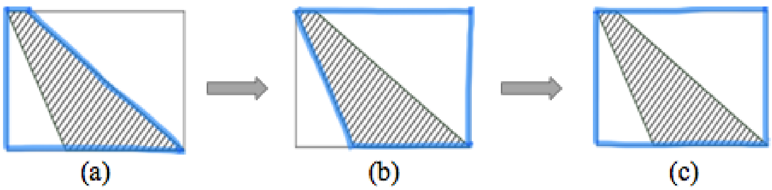

Figure 4. Segmentation of space by S1 in Task 2b using trapeziums.

In this example, the student may have visually superimposed Figures 4(a) and 4(b) and realised that she filled the whole space in the rectangle but counted the shaded region twice. Thus, she subtracted the area of the rectangle to find the area of the shaded region.

Global and Local Perception

Two major patterns were identified from the students' strategy of finding the area of the shaded region of composites shapes, namely, global and local approaches. The local approach focused on visualising and decomposing smaller parts of the task and particular segments of the shapes. For instance, in Figure 5(b), S1 first split the circle into four parts and focused on one quarter of the circle and a smaller square to explain: "the area of the shaded region can be found from the area of a small square subtract the area of a quarter of the circle." Similarly, in Figure 5(d), she attempted to split the shaded region internally, drawing a line in the semicircle region to show a smaller circle.

The global approach was identified when students interpreted the task holistically and visualised or added lines or shapes to enclose the graphic. For example, in Figure 5(a), S3 drew lines outside the T shape in Task 1a to construct a bigger rectangle. In Figure 5(c), he drew a curved line to show an enclosing circle and then drew a vertical line to split the circle into four parts. In summary, it appears that S1 tended to approach the tasks locally while S3 tended to use both local and global strategies.

Figure 5. Sample global and local approaches to find area.

(a) Global approach-S3 (b) Local approach-S1 (c) Global approach-S3 (d) Local approach-S1

Gestures

Two types of gestures were evident from the video records: (i) rotation of the composite shapes, and (ii) movement of fingers on the boundary of the shapes (the students would occasionally use their pen to trace such movement). The given shape was rotated to position it in such a way that it allowed them to identify and disembed known shapes or to perform horizontal or vertical segmentation. The rotating action was also apparent when the students were attempting to identify shapes within a shape as illustrated in Figure 6.

(a) (b) (c) (d)

Figure 6. Illustration of rotating the shapes to identify shapes within shapes.

Student S2 first rotated the shape to position it horizontally to highlight: "a right triangle" (Figure 6(a)). Then she rotated the shape three more times in Figures 6(b)-(d) and mentioned: "there are three triangles and one rectangle".

The gestures indicate that the students were gathering perceptual information from the drawn objects by their sensory system (Gal & Linchevski, 2010). The trace of the fingers (with or without a pen) made explicit the focus of their attention. On many occasions, students S1 and S3 were observed stopping momentarily to focus on the composite shape while rotating it in different directions, particularly when prompted to find a second method to obtain the area of the shaded region. Among the three students, S1 tended to use more gestures.

Disembedding and Assembling a Composite Shape as a Part-Part-Whole Structure

All three students could readily disembed the different shapes from the given composite. However, the apparent discontinuity that one of them (S2) experienced was in assembling the spatial information together as a part-part-whole structure after disembedding the parts. Two examples are provided (from Tasks 2b and 3d) to show this distinct aspect of spatial visualisation in coordinating different pieces of spatial information. In Task 2b, although S2 produced three distinct spatial/visual images, she could not relate them in a part-part-whole structure.

> S2: We find the area of this triangle [trace the outline 7(a)], then find the area of this triangle [trace the outline 7(b)]. Then we find the area of the rectangle [trace the outline 7(c)]. After that, we add those three shapes. Then divided by two because this (pointing to the shaded area) is about a half of the rectangle.

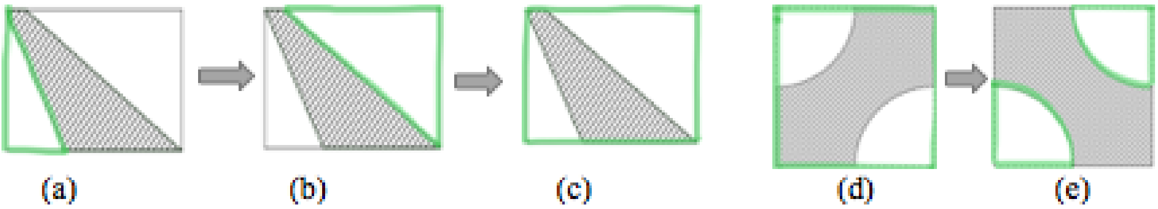

Figure 7. Approach used by S2.

Similarly, in Task 3d (see Figure 1), she could not coordinate the different parts as reflected in the following script:

> T: What shapes can you see?
>
> S2: A square [point 7(d)] and quarters of circle [point 7(e)].
>
> T: If I ask you to find the area of the shaded region, how would you find it?
>
> S2: First, find the area of both the quarter circle [point 7(e)]. Then we find the area of the square, this outside part [point 7(d)], then add them.

Conclusion and Implications

This study provides illustrative examples of the characteristic spatial manoeuvres that underlie spatial visualisation in working with areas of composite shapes. The measurement aspect of area was purposively put in the background to be able to identify the ways in which students manipulate spatial/visual images. The paper attempted to capture at a fine-grained level of detail the perceptual processes that may be in operation in holding spatial/visual images. In particular, Gal and Linchevski's (2010) articulation of perception was found to be pertinent in focussing on students' actions in spatial tasks. Additionally, this study prompted the authors to bring in the concept of local and global perception from Enns and Kingstone (1995). Different students see different things in a particular spatial task. While some operate at the local level, others tend to be more global in approach.

Although the processing of spatial information is different from that of numerical information, there are some parallel strategies that may exist between the two. This study showed instances of the part-part-whole structure as students handled different pieces of spatial/visual information. The part-part-whole structure is well established in the domain of numbers (Baroody, 1999). Although limited to three students, the findings of this study provide much motivation to continue to unpack the salience of spatial reasoning in its different forms in the mathematics curriculum.

Reflecting on the ways in which the students interacted with the metric-free tasks, the following suggestion is provided that can potentially enhance students' spatial visualisation experiences. Students should be given non-metric experiences to interpret area as the amount of covering in composite shapes and articulate part-part-whole relationships. Textbooks can be helpful in promoting non-metric area tasks. Premature introduction to the quantitative approach to area may lead to an overreliance on numbers. Spatial experiences with area in the form of global and local interpretation may not be naturally occurring and hence may need instructional prompts.

References

Alibali, M. W. (2005). Gestures in spatial cognition: Expressing, communicating and thinking about spatial information. *Spatial Cognition and Computation, 5*(4), 307–331.

Baroody, A. J. (1999). Children's relational knowledge of addition and subtraction. *Cognition and Instruction, 17*(2), 137–175.

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 843–908). Charlotte: Information Age Publishing.

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytic studies*. Cambridge University Press.

Clements, D. H. (2004). Geometric and spatial thinking in early childhood education. In D. H. Clements, J. Sarama, & A. M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 267–297). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. *Handbook of research on mathematics teaching and learning*, 420–464.

Del Grande, J. (1990). Spatial sense. *The Arithmetic Teacher, 37*(6), 14–20.

Enns, J. T., & Kingstone, A. (1995). Access to global and local properties in visual search for compound stimuli. *Psychological Science*, *6*(5), 283–291.

Gal, H., & Linchevski, L. (2010). To see or not to see: Analyzing difficulties in geometry from the perspective of visual perception. *Educational Studies in Mathematics, 74*(2), 163–183.

Hostetter, A. B., & Alibali, M. W. (2008). Visible embodiment: Gestures as simulated action. *Psychonomic bulletin & review*, *15*(3), 495–514.

Huang, H.-M. E., & Witz, K. G. (2013). Children's conceptions of area measurement and their strategies for solving area measurement problems. *Journal of Curriculum and Teaching, 2*(1), 10–26.

Kovacs, I., & Julesz, B. (1993). A closed curve is much more than an incomplete one: Effect of closure in figure-ground segmentation. *Proceedings of the National Academy of Sciences, 90*(16), 7495–7497.

Logan, T., Lowrie, T., & Diezmann, C. M. (2014). Co-thought gestures: Supporting students to successfully navigate map tasks. *Educational Studies in Mathematics, 87*(1), 87–102.

Lowrie, T., Logan, T., & Ramful, A. (2017). Visuospatial training improves elementary students' mathematics performance. *British Journal of Educational Psychology*, *87*(2), 170-186.

McGee, M. G. (1979). Human spatial abilities: Psychometric studies and environmental, genetic, hormonal, and neurological influences. *Psychological Bulletin, 86*(5), 889.

Nayar, K., Franchak, J., Adolph, K., & Kiorpes, L. (2015). From local to global processing: The development of illusory contour perception. *Journal of Experimental Child Psychology*, *131*, 38-\–55.

Sima, J. F., Schultheis, H., & Barkowsky, T. (2013). Differences between spatial and visual mental representations. *Frontiers in Psychology, 4*(240), 1–15.

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM, 47*(3), 319–329.

Yakimanskaya, I. S. (1991). *The Development of Spatial Thinking in Schoolchildren. Soviet Studies in Mathematics Education.* (Vol. 3). Reston: National Council of Mathematics.