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Determination of Metaphors of Middle School Pre-service Mathematics Teachers about the Concept of Mathematical Thinking*

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Summary

This research aims to determine the perceptions of middle school pre-service mathematics teachers regarding the concept of "mathematical thinking" through the usage of metaphors. The phenomenology method, which is one of the qualitative research methods, is adopted in this study. Data of the research was obtained by the pre-service mathematics teachers filling the sentences such as "Mathematical thinking is like ... because …". The data was analysed using a content analysis method. The statements of pre-service teachers starting with "because" were coded. Then relevant codes were united together, and eight categories were developed. Some codes were taken as they were, whilst some were combined and thus, 54 features were formed. At the end of the research, it has been determined that the pre-service teachers have produced 156 different metaphors about mathematical thinking. In addition, it was determined that the metaphors, categories, and features concerning mathematical thinking had a positive meaning in general. Besides, it was determined that the perceptions of grades 1st-4th pre-service mathematics teachers centred upon four different categories. Finally, it was concluded that mathematical thinking was considered to be a process-oriented concept by females and a result-oriented concept by males. Metaphors, categories, and features revealed within the scope of the study may guide researchers in their studies on mathematical thinking.

Keywords: *Middle school pre-service mathematics teachers, Mathematical thinking, Metaphor, Perception, Phenomenology*

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Introduction

The individual learns operations and acquires a way of thinking through the process of mathematic learning (Tall, 2006). This way of thinking is known as "Mathematical Thinking" (MT) (Baltaci, 2016). MT can be defined as using mathematical techniques, concepts, and methods directly or indirectly during the problem-solving process (Henderson et al., 2003). MT is also a process which makes it easier to understand complex structures by bringing ideas together (Mason, Burton, and Stacey, 2010).

People voluntarily or involuntarily use MT to analyze their encountered events and phenomena (Arslan and Yildiz, 2010). It means that individuals need MT in every moment of their lives (Alkan and Bukova-Guzel, 2005). They need MT to "understand the world and environment in which they live" (Mason et al., 2010) and to "give purposeful, systematic, accurate, and precise meanings to the incidents they encounter" (Sevgen, 2002). As a natural tool for solving real-life problems, MT poses a vital importance for mathematics learning and teaching (Stacey, 2006). This matter is also expressed in the principles and standards developed for school mathematics by the National Council of Teachers of Mathematics [NCTM], which was established in the United States of America (NCTM, 2000). Also, the importance of MT has been emphasized in primary and middle school mathematics teaching programs that were published in different years in Turkey (Ministry of National Education [MoNE], 2013, 2018).

With the updated mathematics teaching programs, teachers are expected to acquire the necessary skills and refresh themselves to cover and improve the MT of students (Vui, 2007). Therefore, it is required to lay an emphasis on teacher training studies concerning MT (Hughes, 2006). In this context, it may be a suitable strategy to bring these conditions in teachers before the service in order to provide teachers with training in MT skills. It is possible to state that pre-service teachers have important duties and responsibilities in raising the future generations, conducting effective mathematics teaching and bringing out MT skills in students. It is necessary to observe the MT styles of pre-service teachers who have these duties and responsibilities. This situation is important from four perspectives. Firstly, it is required for mathematics educators to know the perceptions of pre-service teachers concerning MT (Tataroglu-Tasdan, Celik, and Erduran, 2013). This is because as long as educators know the MT of pre-service teachers, they will be able to question their own mathematics knowledge and teaching. In addition, analyzing the perceptions of pre-service teachers concerning MT may help to teach mathematics at faculties of education more effectively and to increase the number of individuals with higher MT skills. Secondly, it is also important for pre-service teachers to know their own perceptions concerning MT. Because as long as pre-service teachers are aware of their own MT processes, they will be able to develop their teaching within the frame of these thoughts when they

actually become teachers. In addition, the knowledge of pre-service teachers regarding their own perceptions concerning MT may help them make appropriate decisions in their professional life. Furthermore, considering the perceptions of pre-service teachers in the context of supporting their MT will enable them to understand, learn, internalize, and materialize their mathematical knowledge (Gozen, 2001). Thirdly, the perceptions of pre-service teachers concerning MT may provide researchers with important feedback related to the possible opportunities or problems of mathematics teaching. Finally, determining to what extent the pre-service teachers, who are the candidate implementers of programs, have MT skills which they aim to bring in their students will light the way for future teaching programs and environments (Mumcu and Akturk, 2017). Due to these four matters, the research is aiming to reveal the perceptions of pre-service teachers concerning MT have been in the center of mathematics education studies during recent years (Ciftci, 2015).

Pre-service teachers will create new perceptions concerning MT "using their experiences from their student life, teachers they encounter with and observations they make" (Tortop, 2013). Metaphors are often used to detect these perceptions (Deringol and Gulten, 2016). Metaphors serve to look at different points and to develop different perspectives (Sanchez, Jose, and Victor, 2000). Moreover, metaphors also contribute to revealing the way these concepts are perceived (Rizvanoglu, 2007). Many abstract concepts that are used as the products of daily life or gained experience will be reconstructed to concrete concepts with metaphors in an easy, meaningful and understandable way (Lakoff and Johnson, 1980). Metaphors also reflect the thoughts of individuals (Bozlk, 2002). For this reason, metaphors can be considered to be important tools which reflect the past, present, and future perceptions of pre-service teachers regarding MT. Thus, the study aimed to determine the perceptions of middle school pre-service mathematics teachers regarding the concept of MT and according to that purpose, metaphors were used. It is possible to state that MT and problem solving are similar processes (Nunokawa, 2005). In this context, it is believed that using metaphors in determining the perceptions of pre-service mathematics teachers using MT as a problem-solving process in their classes concerning MT will further strengthen the present study.

Method

The study was conducted by adopting the phenomenology method. This method deals with the interpretation of personal views and events of individuals (Cekmez, Yildiz, and Butuner, 2012). This type of study uncovers the examples, explanations, and experiences that will help us to better understand a concept or phenomenon (Yildirim and Simsek, 2006).

Study Group

The study group of the research consists of a total of 218 pre-service teachers, of which 165 of them are females and 53 are males, who study the Elementary School Mathematics Teacher Program during the spring semester of 2016-2017 academic years. The pre-service teachers were included in the study group who participated in a MT-related conference or took a course at the faculty of education.

Development and Application of Data Collection Tool

A two-stage form was used as a data collection tool in the study. In the first step of the form, there were two questions which determined the genders and grade levels of pre-service teachers. In the second step of the form, there were two sentences concerning the concept of MT. The data of the study was gathered into two sentences; "MT is like ... because ...". In these sentences, there were three elements as expressed by Forceville (2002) namely "the subject of metaphor", "the source of metaphor", and "the features attributed from the source to the subject of metaphor". The word "MT" in the sentence of "MT is like the ocean" constitutes the subject of a metaphor. The concept of "ocean" in the statement of "MT is like the ocean" indicates the source of a metaphor. The expression of "it is endless" in the sentence of "MT is like the ocean because it is endless" emphasizes the feature attributed from the source to the subject of the metaphor.

In the data collection form, the sentences were formed through the examination of the studies (Culha-Ozbas and Aktekin, 2013; Erickson and Pinnegar, 2017) where metaphors were used as data collection tools. Following the discussion of the structure and the quality of the semi-structured sentences in the form with the two field experts, the form was made ready for a pilot study. Following this, the pilot study was applied to review the data collection tool and determine the time to be given to the study group with five of the mathematics pre-service teachers from each grade who are studying in the 1st, 2nd, 3rd, and 4th grades. In the main implementation, the data collection forms were given to the pre-service teachers and they were asked to produce a metaphor for the MT and explain the metaphor within 15 minutes. These forms that were filled out by the pre-service teachers were used as the data collection tool in the study.

Data Analysis

The data analysis was completed in five steps (Saban, 2008).

Naming: At this stage, all the statements related to the MT concept by the participants were transferred to the Excel table. Then, unrecognized expressions were corrected by maintaining the coherence and the number of different metaphors were identified. Each participant produced a metaphor, and a list of 156 metaphors was prepared.

Elimination: In order to understand whether the pre-service teachers expressed their metaphors in a clear way or not, the participant expressions were reviewed in terms of "subject", "source", and "feature attributed from the source to the subject". In this context, the data was eliminated if it was not appropriate for the purpose of the study. Thus, six forms were excluded from the analysis due to the irrelevant association and a decision was made to analyse 212 forms.

Code, category, and feature creation: At this stage, data analysis was performed using the content analysis method. Firstly, the statements on MT of the pre-service teachers were gathered under the titles of "source" and "feature attributed from the source to the subject" within the scope of the analysis. Then, the features of MT that were used by pre-service teachers to produce metaphors were examined and the participant expressions starting with "because" were coded. When the coding was completed, the word or group of words that best reflect the explanations of the mathematics pre-service teachers identified and a total of 74 codes developed. After the coding process was performed, the related codes were assembled, and eight categories were created. Finally, 54 features for MT were developed by taking some codes as they are or combining some of them. The feature can be defined as "the attribution that allows something to be separated from similarities or other things" (Turkish Language Association [TLA], 2011, p.1867). It was thought to be appropriate to name as feature the codes formed through considering this definition and the sentences composed through combining the related codes with each other. Since some of the pre-service teacher explanations may be included in many categories, these metaphors are listed in more than one category.

Providing validity and reliability: The concept of MT, identification of participants, and the process of collecting and analysing data were extensively explained to provide validity. Direct quotations were related to the features were also given. The codes generated by the first researcher were given to the second researcher to ensure the reliability of the study and the codes generated by the two coders were checked for compatibility. The numbers of consensuses and divergences were determined to calculate the consistency of the codes. The coding reliability was found to be 93.0% through the implementation of the [Consensus / (Consensus + Divergence) x 100] (Miles and Huberman, 1994) formula. Following this, the codes were pre-classified and 14 categories were created. Developed categories were presented for the examination of two mathematics educators and two Turkish language teachers. As a result of the exchange of opinions with these persons, some categories were united. In this way, the number of categories was reduced to eight. The reliability scores between the researchers were calculated as 88.0% for the codes in the first category, 90.0% for the codes in the second category, and 91.0% for the codes in the third category. For the codes in the other categories, the percentage of consistency varies between 93.0% and 97.0%. The researchers met and discussed the

codes that did not have a consensus on them and an agreement on the codes and the categorizations were ensured through the exchanging ideas.

Calculation of frequency and percentage values: The frequencies and percentages of the metaphors and categories and the frequency values of the features were calculated. The frequency and percentage values of the metaphors and categories are given in tables, whereas the frequencies of the features are given within parentheses in the findings. The data was organized according to the metaphors expressed by the middle school pre-service mathematics teachers and then the categories and tables were created. The features are given in straight sentences.

Results

156 metaphors expressed by the pre-service teachers on MT are given in Table 1:

Table 1. *All metaphors formed by the pre-service teachers concerning the concept of MT*

Metaphors	*f*	%	Metaphors	*f*	%	Metaphors	*f*	%
1.Life	10	4.7	53.Sailor knot	1	0.5	105.Scales	1	0.5
2.Tree	6	2.8	54.Love	1	0.5	106.Space shuttle	1	0.5
3.Game	6	2.8	55.Mixed feelings	1	0.5	107.Mathematical abilities	1	0.5
4.Puzzle	6	2.8	56.Disease	1	0.5	108.Pi number	1	0.5
5.Playing chess	4	1.9	57.Solving a complex node	1	0.5	109.Three dimensional objects	1	0.5
6.Ocean	3	1.4	58.Field training courses	1	0.5	110.Creating statistical data	1	0.5
7.Intelligence games	3	1.4	59.Climbing stairs	1	0.5	111.Abacus	1	0.5
8.World	3	1.4	60.Raising a child	1	0.5	112.Ripped socks	1	0.5
9.A busy machine	3	1.4	61.Knit	1	0.5	113.Merchant's thought	1	0.5
10.Calculator	3	1.4	62.Building wall	1	0.5	114.Grocery credit book	1	0.5
11.Eating chocolate	2	0.9	63.Understanding girls	1	0.5	115.Turkey's mines	1	0.5
12.Varnish	2	0.9	64.Looking for a needle in a haystack	1	0.5	116.Peacock feather	1	0.5
13.GeoGebra software	2	0.9	65.Theorem	1	0.5	117.Gold	1	0.5
14.Basis of problem-solving	2	0.9	66.Making events simpler	1	0.5	118.Pearl	1	0.5
15.Computer	2	0.9	67.Overcoming the challenges of life	1	0.5	119.All drugs in pharmacy	1	0.5
16.Food processor	2	0.9	68.Detective	1	0.5	120.A perspective painting	1	0.5
17.Rubik's cube	2	0.9	69.Trying to understand the universe	1	0.5	121.Egyptian pyramids	1	0.5
18.Philosophy	2	0.9	70.Creative thinking	1	0.5	122.Water cycle	1	0.5
19.Labyrinth	2	0.9	71.War	1	0.5	123.Mother	1	0.5
20.Imagination	2	0.9	72.War preparation	1	0.5	124.Folk poet Neset Ertas	1	0.5
21.Building a home	2	0.9	73.Crossword	1	0.5	125.Listening to music	1	0.5
22.Sudoku	2	0.9	74.Ferris wheel	1	0.5	126.Song	1	0.5
23.A long way	2	0.9	75.Puppet	1	0.5	127.Watching a movie	1	0.5
24.Sun	2	0.9	76.Play dough	1	0.5	128.Football match	1	0.5

25.Water	2	0.9	77.Hide and seek	1	0.5	129.Photography	1	0.5
26.Sea	2	0.9	78.Lamp	1	0.5	130.Dancing	1	0.5
27.Raindrops	2	0.9	79.Lantern	1	0.5	131.Ecological niche	1	0.5
28.Sands	2	0.9	80.Light	1	0.5	132.Playing an instrument	1	0.5
29.Time	1	0.5	81.Binoculars	1	0.5	133.Intelligence of a fox	1	0.5
30.Space	1	0.5	82.Mirror	1	0.5	134.Gymnastics	1	0.5
31.Universe	1	0.5	83.Lens	1	0.5	135.Running contest	1	0.5
32.Stars in the space system	1	0.5	84.Artist's thoughts	1	0.5	136.Cooking	1	0.5
33.Sky	1	0.5	85.Bird's-eye look	1	0.5	137.Breathing	1	0.5
34.Iceberg	1	0.5	86.Artwork	1	0.5	138.Unravelling of knitting	1	0.5
35.Cloud	1	0.5	87.Coach	1	0.5	139.Freedom	1	0.5
36.Polar star	1	0.5	88.Poet	1	0.5	140.Eating	1	0.5
37.Snowflakes	1	0.5	89.Apple falls on Newton's head	1	0.5	141.Sleeping	1	0.5
38.Where the sun rises	1	0.5	90.Getting on ranger	1	0.5	142.Therapy	1	0.5
39.Cactus	1	0.5	91.Storybook	1	0.5	143.Make-up	1	0.5
40.Flower	1	0.5	92.Writing a book	1	0.5	144.Gossip	1	0.5
41.Couch grass	1	0.5	93.Computer system	1	0.5	145.Food	1	0.5
42.Horse	1	0.5	94.Honeycomb	1	0.5	146.Clock	1	0.5
43.Cow	1	0.5	95.Eating a walnut	1	0.5	147.Cider vinegar	1	0.5
44.Cook's flour, salt, and oil	1	0.5	96.Computer keyboard and mouse	1	0.5	148.Field courses	1	0.5
45.Dragon	1	0.5	97.Internet	1	0.5	149.Bottle	1	0.5
46.Bee	1	0.5	98.Wi-Fi	1	0.5	150.Key holder	1	0.5
47.Black hole	1	0.5	99.Elevator	1	0.5	151.Knife	1	0.5
48.Children's intelligence	1	0.5	100.Car engine	1	0.5	152.Reel of sewing machine	1	0.5
49.Privileged person	1	0.5	101.Reaching from the general to the specific	1	0.5	153.A rotating wheel	1	0.5
50.Poor person	1	0.5	102.Living the mathematics	1	0.5	154.Treadmill	1	0.5
51.A building under construction	1	0.5	103.Washing machine	1	0.5	155.Habit	1	0.5
52.Winding a lapped rope into a ball	1	0.5	104.Lie detector	1	0.5	156.Planning	1	0.5

When Table 1 is examined, it is understood that the first four MT-related words which come to the minds of the mathematics pre-service teachers are "life", "tree", "game", and "puzzle".

Table 2 shows the categories comprising the answers of pre-service teachers concerning the concept of MT:

Table 2. *Categories developed for MT*

Categories	*f*	%
1.MT as a cognitive concept	53	25.0
2.MT as a developing concept that produces and delivers solutions	37	17.5
3.MT as a versatile and endless concept	31	14.6
4.MT as a sequential, combining/complementary, and cumulative concept	30	14.2
5.MT as a concept that is vital and adds meaning to life and is integrated with mathematics	30	14.2
6.MT as a concept that illuminates/guides and that requires patience, hard work, time, and care	29	13.7
7.MT as a concept that emphasizes individual characteristics, gives happiness, making addiction, and cherishing losing or winning feeling	23	10.8
8.MT as a mixed/complex and mysterious concept	12	5.7

In Table 2, it is understood that the category "MT as a cognitive concept" is at the forefront. It was seen that 25.0% of the pre-service teachers produced 45 metaphors that emphasized the cognitive dimension of MT.

Table 3. *Metaphors under the category of "MT as a cognitive concept"*

Metaphors	*f*	Metaphors	*f*
1.Tree	4	24.Raindrops	1
2.Computer	2	25.Honeycomb	1
3.Game	2	26.Couch grass	1
4.Intelligence games	2	27.Listening to music	1
5.Varnish	2	28.Habit	1
6.Playing chess	2	29.Understanding girls	1
7.Imagination	1	30.Building wall	1
8.Space	1	31.Egyptian pyramids	1
9.Treadmill	1	32.Puppet	1
10.Lantern	1	33.Washing machine	1
11.Calculator	1	34.Mirror	1
12.Gymnastics	1	35.Pearl	1
13.Horse	1	36.Life	1
14.Internet	1	37.GeoGebra software	1
15.Writing a book	1	38.Cooking	1
16.Key holder	1	39.Wi-Fi	1
17.Detective	1	40.Artwork	1
18.Eating	1	41.Cider vinegar	1
19.Labyrinth	1	42.Ocean	1
20.Trying to understand the universe	1	43.Sky	1
21.All drugs in pharmacy	1	44.Gold	1
22.Food	1	45.Storybook	1
23.Freedom	1		

When Table 3 is examined, it is seen that pre-service teachers seemed MT mostly to "tree" metaphor in this category. The 14 features of the metaphors that make up the MT category as a cognitive concept and the quotations of these features are presented below:

1. MT puts brain to work (f: 3) and improves it (f: 10).

MT is like intelligence games because ... we think while playing an intelligence game... So, we're running our brain.

MT is like gymnastics because when it is done regularly, gymnastics develops the body. MT also develops the brain.

2. MT makes it possible to learn new information (f: 8) and makes it easier to remember the learned information (f: 1).

MT is like a puppet because every time we play the puppet, it makes different moves. MT also does the same. Every time you think, new information emerges.

MT is like a computer because it makes it easier for people to keep the information they learn...

3. MT develops practical intelligence (f: 4) and thinking (f: 4).

MT is like imagination because sometimes we imagine impossible events ... As the questions become difficult, we may need to consider the impossible possibilities. Thinking in this way contributes to practical intelligence...

MT is like a lantern because it improves thinking by illuminating the rooms of our mind.

4. MT requires to think logically (f: 4) and to contemplate (f: 3).

MT is like writing a book because it requires thinking reasonably.

MT is like intelligence games because it requires thinking.

5. MT allows creating new ideas (f: 6).

MT is like a game because playing a game ... allows free evaluation of ideas just like MT.

6. MT allows people expand their horizons (f: 4) and freely use the information (f: 1).

MT is like a tree because a tree greens the environment. MT greens our minds, expands our horizons.

MT is like freedom because both are against restriction. MT allows us to use the information freely for any problem.

7. MT is based on imagination (f: 2) and cause effect relationship (f: 1).

MT is like GeoGebra software because both MT and GeoGebra are based on imagination.

MT is like trying to understand the universe because the results are all interconnected that we found during the studies about the harmony in the environment. A nature event is the result of another event, and it is also the cause of another event. MT is like this...

8. MT is a reasoning activity (f: 2).

MT is like a labyrinth because MT is the reasoning. We think about what strategy we will go on and what path we will go through.

9. Engaging brain with new knowledge develops MT (f: 1).

MT is like a tree because MT requires time to improve and requires making effort. As a new planted tree needs regular irrigation, we need to regularly irrigate our brains to improve MT power. The water of our brain is new information …

10. MT helps people to build their own math worlds (f: 1).

MT is like a treadmill because the ability of a treadmill to reduce weight of an individual is important. Nowadays, students ... see mathematics as just a means to get a better score from the exam ... they only take care of the part that will work ... However, someone who knows the essence of the process knows that you get into form on that treadmill. So, someone who uses the MT consciously builds his own mathematical world...

11. It reflects the accumulation of knowledge about human MT (f: 1).

MT is like a mirror because people may reflect their knowledge ... as much as their knowledge and ability to interpret...

12. People can think mathematically at every age (f: 1).

MT is like habit because if you can think mathematically in seventeen, you can think mathematically in seventy.

13. Sometimes people can think mathematically and sometimes not (f: 1).

MT is like Wi-Fi because sometimes it works and sometimes not. Our mind can sometimes think mathematically, sometimes not.

14. MT is a process in which creativity is revealed (f: 1).

MT is like a storybook because the writer aims to put his creativity while writing a storybook. People also demonstrate their creativity in this process.

It was seen that 17.5% of the pre-service teachers produced 33 metaphors on MT that emphasized the dimensions of producing solution, result orientation, and development.

Table 4. *Metaphors under the category of "MT as a developing concept that produces and delivers solutions"*

Metaphors	*f*	Metaphors	*f*
1.Life	4	18.Playing chess	1
2.Basis of problem-solving	2	19.Sun	1
3.Food processor	1	20.Labyrinth	1
4.Philosophy	1	21.Key holder	1
5.Light	1	22.Ocean	1
6.Overcoming the challenges of life	1	23.Intelligence of a fox	1
7.Gold	1	24.War	1
8.Computer keyboard and mouse	1	25.Reel of sewing machine	1
9.Internet	1	26.Running contest	1
10.Turkey's mines	1	27.Tree	1
11.Merchant's thought	1	28.GeoGebra software	1
12.Solving a complex node	1	29.Space	1
13.Calculator	1	30.Field courses	1
14.Imagination	1	31.Time	1
15.Mother	1	32.Universe	1
16.Three dimensional objects	1	33.Children's intelligence	1
17.Breathing	1		

When Table 4 is examined, it is understood that the metaphor of "life" comes to the forefront in this category. The four features of the metaphors formed this category and the quotations of these features are presented below:

1. MT offers different solutions to people (f: 7) and helps solve everyday problems (f: 19).

MT is like life because in life it is not clear what to expect. Also, we cannot predict what we will face with during the MT process. MT offers us different solutions.

MT is like philosophy because MT tries to find solutions to the problems that people may encounter in their lives like philosophy.

2. New information is learned everyday on MT (f: 1) which is in constant development (f: 4).

MT is like time because time keeps moving. MT is also in constant progress and development.

MT is like space because ... there are always new things in space. MT is a field like space where new information is learned every day.

3. MT provides to reach a goal in a short time and in the right way (*f*: 4).

MT is like a calculator because if a person responds to mathematical questions in a short period of time with correct answers, he acts as a calculator.

4. Field courses (f: 1), research (f: 1), and GeoGebra software (f: 1) develop MT.

MT is like field courses because the MTs of the pre-service teachers increase over time with the field lessons.
MT is like a tree because it develops from a small nucleus; it grows and gives fruit ... Like MT because you begin to think mathematically on a subject and do research to develop and enlarge MT...
MT is like GeoGebra software because the GeoGebra software helps to improve MT...

It was seen that 14.6% of the pre-service teachers produced 29 metaphors that emphasized the versatile and endless structure of MT.

Table 5. *Metaphors under the category of "MT as a versatile and endless concept"*

Metaphors	*f*	Metaphors	*f*
1.Sea	2	16.Folk poet Neset Ertas	1
2.A busy machine	2	17.Gossip	1
3.Creative thinking	1	18.Apple falls on Newton's head	1
4.Bird's-eye look	1	19.Ocean	1
5.Photography	1	20.A rotating wheel	1
6.Computer	1	21.Sudoku	1
7.Food processor	1	22.Sands	1
8.Scales	1	23.Stars in the space system	1
9.A long way	1	24.Sky	1
10.Snowflakes	1	25.Sleeping	1
11.Game	1	26.Pi number	1
12.Water	1	27.Ripped socks	1
13.Water cycle	1	28.Bottle	1
14.Cow	1	29.Cactus	1
15.Mathematical abilities	1		

When Table 5 is examined, it is seen that the most frequently repeated metaphors are "sea" and "a busy machine". The four features of the metaphors that make up this category and the quotations of these features are presented below:

1. MT is an endless (f: 13) cyclical (f: 1) process.

MT is like an ocean because ocean has no end. MT is like this as well. There is no end.
MT is like a busy machine because it is a constant state of continuation.
MT is like a rotating wheel because it is a cyclical process.

2. MT requires thinking through different perspectives (f: 11).

MT is like photography because it requires different perspectives. MT also requires thinking about different angles.

3. MT is in every area of life (f: 5).

MT is like mathematical abilities because it is in every area of life.

4. MT contains everything inside (f: 4).

MT is like folk poet Neset Ertas because he is pronounced as joy-trouble-love. So, he contains everything in himself. Just like MT...

It was observed that 14.2% of the pre-service teachers had produced 23 metaphors that emphasized sequential, combining/complementary, and cumulative dimensions of MT.

Table 6. *Metaphors under the category of "MT as a sequential, combining/complementary, and cumulative concept"*

Metaphors	*f*	Metaphors	*f*
1.Puzzle	6	13.Unravelling of knitting	1
2.Game	2	14.World	1
3.Food processor	2	15.Playing chess	1
4.Egyptian pyramids	1	16.Life	1
5.Car engine	1	17.Winding a lapped rope into a ball	1
6.Computer system	1	18.Bee	1
7.Sailor knot	1	19.Mixed feelings	1
8.Peacock feather	1	20.Rubik's cube	1
9.Creating statistical data	1	21.Reel of sewing machine	1
10.Clock	1	22.Raindrops	1
11.Making events simpler	1	23.Knit	1
12.War preparation	1		

When Table 6 is examined, it can be seen that the most frequently repeated metaphor is "puzzle". The eight features of the metaphors that make up this category and the quotations of these features are as follows:

1. MT has a systematic structure (*f*: 15).

MT is like unravelling of knitting because when you follow the right steps in a systematic manner, you get the result you want.

2. MT moves from part to whole or from whole to part (*f*: 7).

MT is like a puzzle because it allows you to go from part to whole or from whole to part.

3. MT collects ideas and ties them to a conclusion (*f*: 3).

MT is like a food processor because the food processor is used to combine materials when preparing food. MT also combines ideas about a subject and blends them into the result.

4. MT depends on experiences (*f*: 2).

MT is like Egyptian pyramids because it depends on the experience of the craftsmen to make the pyramids. Likewise, experience in MT is crucial and MT depends on experience.

5. MT requires proceeding by thinking future steps (*f*: 1).

MT is like playing chess because we always should think a few steps ahead and move correctly.

6. After solving the problems, MT takes the final shape (*f*: 1).

MT is like winding a lapped rope into a ball of wool because first you need to solve the rope, then you need to give shape. Just like MT, it solves the problem and then takes the final shape.

7. Once the rule is learned, MT becomes a habit (*f*: 1).

MT is like a bee because even though the bee is not wise, it has a habit. MT becomes a habit after learning the rule.

8. The consequences cannot be achieved without knowing what MT is (*f*: 1).

MT is like mixed feelings because if you do not understand it, you will not get it...

It was observed that 14.2% of the pre-service teachers produced 25 metaphors that emphasized the dimensions which are vital and that add meaning to life and are integrated with mathematics.

Table 7. *Metaphors under the category of "MT as a concept that is vital and adds meaning to life and is integrated with mathematics"*

Metaphors	*f*	Metaphors	*f*
1.Life	3	14.Ecological niche	1
2.Water	2	15.A building under construction	1
3.Calculator	2	16.Poor person	1
4.Building a home	2	17.Poet	1
5.Theorem	1	18.Playing chess	1
6.Living the mathematics	1	19.Abacus	1
7.Love	1	20.Cook's flour, salt, and oil	1
8.Song	1	21.Sands	1
9.Mother	1	22.Grocery credit book	1
10.Disease	1	23.Playing an instrument	1
11.Sea	1	24.Where the sun rises	1
12.World	1	25.Three dimensional objects	1
13.Space shuttle	1		

When Table 7 is examined, it is understood that pre-service teachers in this category assumed MT mostly to the "life" metaphor. The six features of the metaphors that make up this category and the quotations to those features are as follows:

1. MT is integrated with everyday life (f: 2) and adds meaning to daily life (f: 5).

MT is like the sea because there are many living things in the sea. MT also includes offers for solutions to many everyday problems. For this reason, MT is integrated with life.

MT is like life because MT of people develops as life goes on and people try to make sense of life by putting every moment of life into MT.

2. MT has a great importance (*f*: 6).

MT is like the world because there will be no life if MT does not exist, as the world.

3. MT is always needed everywhere (*f*: 5).

MT is like water because it is always necessary in everywhere.

4. MT is related to numbers and operations (*f*: 5).

MT is like abacus because it is related to numbers and operations.

5. MT is integrated with mathematics (*f*: 5).

MT is like playing chess because we use MT in a chess game ... motions of the stones form geometric shapes. So, the outcome of every MT is mathematics.

6. In MT, mathematical symbols, signs (f: 1) or formulas (f: 2) are used.

MT is like building a home because if we want to build a house first it is necessary to plan and it is important to use symbols and signs related to mathematics in MT.

MT is like theorem because mathematical formulas are used in the theorem proofs as they are in MT.

It was seen that 13.7% of the pre-service teachers produced 27 metaphors that emphasized the dimensions of illumination/guidance, patience, hard work, time, and care.

Table 8. *Metaphors under the category of "MT as a concept that illuminates/guides and that requires patience, hard work, time, and care"*

Metaphors	*f*	Metaphors	*f*
1.Tree	3	15.A busy machine	1
2.Artwork	1	16.Planning	1
3.Pearl	1	17.Polar star	1
4.Looking for a needle in a haystack	1	18.Coach	1
5.Climbing stairs	1	19.Lamp	1
6.Dragon	1	20.Sun	1
7.Raising a child	1	21.Binoculars	1
8.Sailor knot	1	22.A perspective painting	1
9.Make-up	1	23.A long way	1
10.Detective	1	24.Dancing	1
11.Reaching from the general to the specific	1	25.Knife	1
12.Philosophy	1	26.Lie detector	1
13.Eating	1	27.Lens	1
14.Elevator	1		

When Table 8 is examined, it is understood that the most repeated metaphor is "tree". The five features of the metaphors that make up this category and the quotations for those features are presented below:

1. MT leads the people (f: 5) and reveals the truth (f: 3).

MT is like a coach because a coach guides player as well as MT guides people.
MT is like a lie detector because it allows us to distinguish between right and wrong.

2. MT requires a certain amount of time and effort (*f*: 8).

MT is like a tree because if you spend time and effort, and fulfil all its care, it will give you beautiful fruits. MT is similar.

3. Careful attention to detail is necessary in MT (*f*: 6).

MT is like a detective because all the events are related to each other. So, it is necessary to pay attention to any smallest detail to reveal the current situation. A detective should also pay attention to the tiny details to solve an event.

4. MT is difficult (f: 4) and requires patience (f: 1).

MT is like climbing stairs because as you go up the stairs it becomes harder.
MT is like artwork because ... creating artwork requires patience. MT process also requires patience.

5. MT provides a more comprehensive and clear view of details (f: 2).

MT is like binoculars because it allows us to see the details more clearly and comprehensively.

It was observed that 10.8% of the pre-service teachers produced 22 metaphors that emphasized MT as a concept which emphasizes individual characteristics, provides happiness, making addictions, and cherishing losing or winning feelings.

Table 9. *Metaphors under the category of "MT as a concept that emphasizes individual characteristics, gives happiness, making addiction, and cherishing losing or winning feeling"*

Metaphors	*f*	Metaphors	*f*
1.Eating chocolate	2	12.Imagination	1
2.Artwork	1	13.Mother	1
3.Getting on ranger	1	14.Privileged person	1
4.Playing an instrument	1	15.Dancing	1
5.Crossword	1	16.Football match	1
6.Hide and seek	1	17.Intelligence games	1
7.Looking for a needle in a haystack	1	18.Field training courses	1
8.Watching a movie	1	19.Play dough	1
9.Therapy	1	20.Eating a walnut	1
10.Game	1	21.Artist's thoughts	1
11.Flower	1	22.A busy machine	1

Table 9 shows that the metaphor of "eating chocolate" is at the forefront in this category. The eight features of the metaphors that make up this category and the quotations of these features are as follows:

1. MT is fun (f: 8) and addictive (f: 2).

MT is like watching a movie because I think MT is fun as watching movies.
MT is like eating chocolate because you cannot give up after you start. Just like MT.

2. MT gives people a sense of losing or winning a game (*f*: 4).

MT is like intelligence games because if every move we make in the intelligence games is correct, we win the game, and if we make a wrong move, we lose.

3. MT occurs in a harmony (f: 2) and relaxes people (f: 2).

MT is like a busy machine because ... it is in harmony.
MT is like therapy because it relaxes people.

4. MT is unique (*f*: 3).

MT is like imagination because MT is unique as much as imagination.
MT is like artwork because as in the work of art, there are many beauties in MT...

5. MT creates a sense of privilege in people (f: 1) and makes people feel valuable (f: 1).

MT is like a privileged person because if a person realizes that he thinks mathematically and develops himself, he feels himself privileged...
MT is like a mother because ... MT makes you feel worthy like a mother.

6. People give shape to MT (*f*: 1).

MT is like play dough because MT is a process that we create with certain materials. We shape it...

7. It is necessary to actively participate in the MT process (*f*: 1).

MT is like dancing because if you need to be active to dance, you should get into the process for MT...

8. The more original MT is the more valuable and effective it becomes (*f*: 1).

MT is like thoughts of an artist because the more unique your artist's thought is, the more valuable and influential it is. Similarly, MT becomes valuable and effective with uniqueness.

It was seen that 5.7% of the pre-service teachers produced 12 metaphors that emphasized the concept as mixed/complex and mysterious.

Table 10. *Metaphors under the category of "MT as a mixed/complex and mysterious concept"*

Metaphors	*f*	Metaphors	*f*
1.Rubik's cube	1	7.Life	1
2.Cow	1	8.Cloud	1
3.World	1	9.Space	1
4.Black hole	1	10.Egyptian pyramids	1
5.Philosophy	1	11.Iceberg	1
6.Ferris wheel	1	12.Sudoku	1

When Table 10 is examined, it is seen that the pre-service teachers did not agree on a certain concept in this category; and therefore, they created different concepts. The five features of the metaphors that make up this category and the quotations of these features are as follows:

1. MT is filled with games, traps (f: 1), and mysteries (f: 3).

MT is like life because life is full of games and traps.
MT is like space because there are many unknown and unresolved problems in mathematics. MT is also an area that has a lot of mystery like space...

2. MT has a mixed/complex (f: 2) structure with up and downs (f: 1).

MT is like a Rubik's cube because every time I see Rubik's cube, its complicated structure comes to mind.
MT is like Ferris wheel because sometimes you go down and sometimes you go up.

3. There are undiscovered aspects of MT.

MT is like a black hole because ... there are things in it we have not discovered.

4. MT requires seeing what is not given.

MT is like a sudoku because it is necessary to see the numbers that are not given in the sudoku. MT is not only doing the operations that are in sight but also seeing you not being given...

5. MT cannot explain everything.

MT is like philosophy because we find everything by examining until to the finest detail, but still we cannot give an explanation to some of them.

The comparison of the developed categories of the MT concept in terms of genders and grade levels is given in Table 11:

Table 11. *Comparing the categories of MT according to the genders and grade levels*

Categories	Female		Male		1st Grade		2nd Grade		3rd Grade		4th Grade	
	f	%	*f*	%	*f*	%	*f*	%	*f*	%	*f*	%
1.MT as a cognitive concept	43	81.1	10	18.9	13	24.5	14	26.4	15	28.3	11	20.8
2.MT as a developing concept that produces and delivers solutions	24	64.9	13	35.1	11	29.7	8	21.6	10	27.0	8	21.6
3.MT as a versatile and endless concept	23	74.2	8	25.8	6	19.4	14	45.2	4	12.9	7	22.6
4.MT as a sequential, combining/complementary, and cumulative concept	27	90.0	3	10.0	15	50.0	6	20.0	6	20.0	3	10.0
5.MT as a concept that is vital and adds meaning to life and is integrated with mathematics	20	66.7	10	33.3	11	36.7	7	23.3	9	30.0	3	10.0
6.MT as a concept that illuminates/guides and that requires patience, hard work, time, and care	22	75.9	7	24.1	7	24.1	7	24.1	9	31.0	6	20.7
7.MT as a concept that emphasizes individual characteristics, gives happiness, making addiction, and cherishing losing or winning feeling	17	73.9	6	26.1	6	26.1	6	26.1	8	34.8	3	13.0
8.MT as a mixed/complex and mysterious concept	9	75.0	3	25.0	5	41.6	3	25.0	1	8.3	3	25.0

According to Table 11, it can be seen that the female pre-service mathematics teachers adopt "MT as a sequential, combining/complementary, and cumulative" concept; and that males are more likely to adopt "MT as a developing concept that produces and delivers solutions". Moreover, the first graders adopt MT as "a sequential, combining/complementary, and cumulative"; the second graders adopt as "versatile, endless"; third graders adopt as "a concept that emphasizes individual characteristics, gives happiness, making addiction, and cherishing losing or winning feeling"; and the fourth graders adopt MT as "mixed/complex and mysterious".

Discussions, Conclusions, and Suggestions

The large number of metaphors was needed to explain the MT concept in a holistic way. The need for numerous metaphors to explain an abstract concept such MT is thought to be a normal situation. In addition, the metaphors expressed by mathematics pre-service teachers shows that there is also awareness of MT. It is a positive situation for pre-service teachers who have not yet started their professional lives to have a conscious view of MT. It can be said that presenting lectures about the MT to the pre-service teachers or taking the MT-related courses is proven to be effective in increasing this awareness. Metaphors related to the MT concept of pre-service teachers who have not taken selective course or have not participated in MT related conference will be determined, and the results will be compared with the metaphors of this study.

It has been determined that the most produced metaphors on the MT concept are "life", "tree", "game", and "puzzle" by mathematics pre-service teachers. Examining the explanations of

these metaphors, it is understood that the pre-service teachers have generally positive opinions about the concept of MT. Therefore, it should not be forgotten that the usage of positive metaphors by mathematics pre-service teachers regarding mixed or abstract concepts of mathematics can positively affect their perceptions of the concepts that they are going to teach in their professional life. For this reason, positive metaphors on MT can be used to educate generations who appreciate and love mathematics.

It has been found that a large number of categories was needed to explain the MT concept. It can be suggested that perceptions of pre-service teachers include differences and are multifaceted for the concept of MT. Alkan and Tasdan (2011) also found that mathematics pre-service teachers can reveal the multidimensional structure of MT and make different definitions for MT. It is thought that the reason for different perceptions of pre-service teachers related to MT is that the concept of MT has a wide and complex structure. In addition, when all the categories are examined, it is concluded that pre-service teachers have a generally positive view of the MT concept. These categories can be used to form items and to name factors in scales to be developed for MT.

When the categorization related to the MT concept is examined, it is revealed that the most metaphor included category is "MT as a cognitive concept". As a result, it can be said that pre-service mathematics teachers primarily perceive the MT as a mental process. When the codes related to MT are examined, it is seen that the participants emphasize the cognitive direction of MT more. When a problem is encountered, MT is used to handle the problem in various ways (Ferri, 2003). The MT skills of individuals develop in the mental process of problem-solving (Ersoy, 2012). For this reason, it is recommended that education faculties should attach importance to problem-solving activities in order to improve the MT skills of pre-service teachers.

The category that produces the second most metaphors is the category of "MT as a developing concept that produces and delivers solutions". This category shows that some pre-service teachers explain the perceptions of MT depending upon the solution process. Possessing the knowledge and skills related to thinking provides positive contributions in using high-level thinking processes and solving problems in everyday life (Teong, 2002). For this reason, it may be useful to inform pre-service teachers about how to use the MT in their daily life and in solving mathematical problems. Some pre-service teachers also made statements that new information on MT may be produced over time. This suggests that pre-service teachers know that MT is dynamic rather than static. If the rapid development of science is considered, pre-service teachers should be encouraged to read resources related to MT and participate in activities such as courses or seminars to continually renew and refresh their knowledge.

Another category emerging from the findings of the study is the "MT as a versatile and endless concept". It is understood from this category that the participants regarded the MT as a multidimensional and unlimited concept and thus important. Given that MT is a process that will last from the cradle to the grave (Bulut, 2009), it is understood that how important it is to educate teachers of high MT skills in their education faculties. For this reason, teacher training programs may need to be renewed and updated according to the needs of the time. At this point, it can be as a significant development that the Council of Higher Education put in place the updated teacher training programs in 2018.

According to research findings, another category where metaphors produced is "MT as a sequential, combining/complementary, and cumulative concept". It is understood that the pre-service teachers emphasize systematic structure and need for experience in this category. There are a lot of studies in the literature that state that MT is a process and this process is progressive, and phases must be systematically monitored (Liu, 2003; Yildiz, 2016). It is also stated that the MT experiences of pre-service teachers are not sufficient (Alkan and Tasdan, 2011; Tataroglu-Tasdan et al., 2013). Pre-service teachers need to know the systematic structure of problem-solving and should be able to use its stages effectively to have high MT skills. It is thought that it would be beneficial to give more information about the systematic structure, phases of MT and the problem-solving and about process/concept knowledge in the lessons for pre-service teachers with inadequate knowledge or training.

It was determined that mathematics pre-service teachers see MT as a mathematical concept and think it is important for life. This situation shows itself in the category of "MT as a concept that is vital and adds meaning to life and is integrated with mathematics". This category shows that the pre-service teachers have knowledge regarding the importance of MT. MT is of a vital importance for individuals (Mason et al., 2010). It can be seen as rejoicing situation that pre-service teachers produced metaphors that emphasize the importance of MT for life. As a result, it can be said that pre-service teachers with a positive perception on the importance of MT can help them to develop MT skills of students in the future. It has also been determined that some participants indicated the MT is related to numbers and operations and the MT uses mathematical symbols, signs, and formulas. This may indicate that some pre-service teachers perceive the MT as a thinking way unique to mathematics. It should not be forgotten that MT is not only used in mathematics, but also in all problem-solving situations that people encounter in their daily lives. In this context, academicians have great duties to prevent the misapprehension of pre-service teachers about "benefiting from MT only in mathematics". Academicians may prevent this situation by providing examples of how the MT works in solving a problem in daily life and in mathematics.

Another category related to MT is in the form of "MT as a concept that illuminates/guides and that requires patience, hard work, time, and care". In this category, it is understood that the pre-service teachers have created metaphors that MT is leading them. Therefore, it can be said that pre-service teachers consider MT as a functional thinking system. A non-functional thinking system also creates a sense of inhibition in the individual addition to loss of time and energy (Celuch and Slama, 1999). For this reason, it is very important for the pre-service teachers to have detailed information and see examples of how the MT can lead them. This category also includes metaphors that MT requires patience, effort, time, and attention. This may be related in reason of mathematics and the abstract structure of MT. As a matter of fact, Schinck, Neale, Pugalee, and Cifarelli (2008) determined that students at secondary and undergraduate levels describe mathematics as a structure that is difficult and requires being active. In order to have a clearer view of the discrete structure of MT, it is suggested to open selective courses intended for MT in mathematics teaching programs of faculties of education.

MT can also be perceived; as an activity that takes care of individual characteristics and cherishes happiness in addition to the activity mode that makes an addiction and competitive environment. The category of "MT as a concept that emphasizes individual characteristics, gives happiness, making addiction, and cherishing losing or winning feeling" supports this situation. This category points out the importance of its usage in MT-related worksheets that allow individuals to be active and successful by considering the individual characteristics of the pre-service teachers. In addition, since intelligence games plays a positive role in MT skills (Ott and Pozzi, 2012), it is recommended to use intelligence games in lessons and open courses on mental games in undergraduate programs, to entertain pre-service teachers and create a competitive environment (Yildiz and Hacisalihoglu-Karadeniz, 2018).

"MT as a mixed/complex and mysterious concept" is the last category that emerges from the research findings. This category can be indicated that some pre-service teachers perceive MT as a complex and difficult concept and thus they have negative thoughts about MT. The reason for this perception towards MT may be due to the nature of mathematics. This is because mathematics, which needs effort to be learned and taught, is generally regarded as a difficult and boring subject (Sengul, Katranci, and Gerez-Cantimer, 2014). It is thought that giving more detailed information about the learning and teaching of mathematics to the pre-service teachers during the university years may reduce the negative perceptions determined in this study.

The MT perceptions of pre-service teachers are found to be in two different orientations. It has been determined that females consider MT as a process-oriented and males see MT as a result-oriented concept. Aydin and Ubuz (2014) also found that undergraduates have a difference towards

MT in terms of genders. The reason for the differentiation of the views of the participants towards MT is the natural difference between females and males. In this context, research on whether the perceptions of MT of primary school, middle school, and secondary school students, mathematics teachers, and academics are changing according to the genders can add useful information to the literature.

In the analysis based on grade levels, the MT perceptions of mathematics pre-service teachers having their first, second, third, and fourth grades were found to be in four different orientations. It can be said that pre-service mathematics teachers at different grade levels have different perceptions on the MT concept. Aydin and Ubuz (2014) also determined that the MT of undergraduate students differs according to their grade levels. MT is directly related to individual development and education (Alkan and Bukova-Guzel, 2005). Perceptions of pre-service teachers on MT may change over time. It is believed that undergraduate lessons and problem solving/posing processes in these lessons have been effective in this change on the perceptions of pre-service teachers. The fact that the structure of MT can be constantly developed by cognitive and social learnings may indicate that as the class level increases, MT will also increase (Tataroglu-Tasdan, 2014). Therefore, it is suggested to examine whether the perceptions of the students in different grade levels and the pre-service teachers who are studying in different programs changes in terms of their grade levels or not.

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