The Development of Addition and Subtractions Strategies for Children in Kindergarten to Grade 6: Insights and Implications

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This paper provides insight about the development of addition and subtraction strategies for nearly 22,000 Australian primary school children in 2016. The children were each assessed by their teacher using a task-based assessment interview that identified the strategies they used to mentally perform addition and subtraction, and matched these to a growth point framework. The findings highlight the broad distribution of strategies used by children in each grade level and suggest that few children, including those in Grade 6, reach Growth Point 6 that involves the mental calculation of two-digit and three-digit numbers. These findings have important implications for classroom teaching and professional learning.

Mathematics education provides knowledge that ultimately empowers people to access further education, employment and active citizenship. *Numeracy Now* is a system-wide strategy launched by a Catholic diocese in New South Wales to transform the learning and teaching of mathematics so that all students might thrive mathematically.

Key approaches for the initiative were: developing the mathematics instruction leadership of primary school principals and curriculum leaders; classroom teachers using the Mathematics Assessment Interview (MAI; Gervasoni et al., 2011) and an associated framework of growth points (Clarke et al., 2002), using this data to guide school-based reform supported by system consultants and using the Extending Mathematical Understanding intervention program (Gervasoni, 2015) in the second year of schooling and beyond to provide intensive specialised instruction for children who were mathematically vulnerable.

This paper reports on one aspect of this initiative in order to highlight issues associated with children's development of addition and subtraction strategies for calculating. This aspect was the use of the assessment interview and an associated framework of growth points to determine the Addition and Subtraction Strategies growth points reached by all children across the system in 2016. The insights gained and the implications for classroom instruction and teacher professional learning will be discussed with the view to recommending approaches for enhancing mathematics learning for all.

Using Assessment Interviews to Identify Children's Number Knowledge

Clinical assessment interviews are widely used by teachers in Australia and New Zealand as a means of assessing children's mathematical knowledge. This follows three large scale projects that informed assessment and curriculum policy formation in Victoria, NSW and New Zealand: *Count Me In Too* (Gould, 2000) in NSW, the Victorian *Early Numeracy Research Project* (Clarke et al., 2002), and the *Numeracy Development Project* (Higgins, Parsons, & Hyland, 2003) in New Zealand.

A common feature of each of these projects was the use of a one-to-one assessment interview and an associated research-based framework to describe progressions in mathematics learning (Bobis et al., 2005). Teachers participating in each project indicated that the benefits of the assessment interview, though time-consuming and expensive, were

(2017). In A. Downton, S. Livy, & J. Hall (Eds.), *40 years on: We are still learning! Proceedings of the 40th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 269-276). Melbourne: MERGA.

considerable in terms of creating an understanding of what children know and can do, and for planning instruction. Indeed, an important feature of clinical interviews is that they enable the teacher to observe children as they solve problems to determine the strategies they used and any misconceptions (Gervasoni & Sullivan, 2007). They also enable teachers to probe children's mathematical understanding through thoughtful questioning (Wright, Martland, & Stafford, 2000) and observational listening (Mitchell & Horne, 2011).

The insights gained through these assessments inform teachers about the particular instructional needs of each student more powerfully than scores from traditional pencil and paper tests, the disadvantages of which are well established (e.g., Clements & Ellerton, 1995). Bobis et al. (2005) concluded that one-to-one assessment interviews and associated frameworks assisted to move the focus of professional learning in mathematics from the notion of children carefully reproducing taught procedures to an emphasis on children's thinking. It is broadly accepted that the traditional focus on taught procedures for calculating can negatively impact on children's number sense (Clarke et al., 2006) and may impede children's development of powerful mental reasoning strategies for calculating (Narode, Board, & Davenport, 1993). Thus, because of the deep insight about children's mathematical knowledge gained through the use of one-to-one assessment interviews, the Mathematics Assessment Interview was chosen as the assessment tool for the *Numeracy Now* initiative. It was anticipated that the data obtained would provide important insights for teachers and that the cohort data would provide a rich snapshot of children's addition and subtraction strategies for principals and system leaders.

The Mathematics Assessment Interview and Growth Points

One aspect of the *Numeracy Now* initiative is the annual assessment of nearly 22000 Kindergarten (five-year-old) to Grade 6 (11-year-old) children's whole number knowledge at the beginning of each year using the Mathematics Assessment Interview (MAI). This assessment tool is a refinement of the original Early Numeracy Interview (Department of Education Employment and Training, 2001) developed as part of the *Early Numeracy Research Project* (ENRP; Clarke et al., 2002).

The Mathematics Assessment Interview is a clinical interview with an associated research-based framework of growth points that describe key stages in the learning of nine mathematics domains. The principles underlying the construction of the growth points were to: describe the development of mathematical knowledge and understanding in the first three years of school in a form and language that was useful for teachers; reflect the findings of relevant international and local research in mathematics (e.g., Fuson, 1992; Gould, 2000; Mulligan, 1998; Steffe, von Glasersfeld, Richards, & Cobb, 1983; Wright, Martland, & Stafford, 2000); reflect, where possible, the structure of mathematics; allow the mathematical knowledge of individuals and groups to be described; and enable a consideration of children who may be mathematically vulnerable.

The growth points form a framework for describing development in nine domains, including four whole number domains: Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies. The processes for validating the growth points, the interview items and the comparative achievement of children in project and reference schools are described in full in Clarke et al. (2002).

To illustrate the nature of the growth points, the following are the growth points for Addition and Subtraction Strategies. These describe the strategies children use to calculate.

1. Counts all to find the total of two collections.

2. Counts on from one number to find the total of two collections.
3. Given subtraction situations, chooses appropriately from strategies including count back, count-down to and count up from.
4. Uses basic strategies for solving addition and subtraction problems (doubles, commutativity, adding 10, tens facts, other known facts).
5. Uses derived strategies for solving addition and subtraction problems (near doubles, adding nine, build to next 10, fact families, intuitive strategies).
6. Extending and applying. Given a range of tasks (including multi-digit numbers), can use basic, derived and intuitive strategies as appropriate.

Each growth point represents substantial expansion in knowledge along paths to mathematical understanding (Clarke, 2013). They enable teachers to: identify the zone of proximal development for each child in each domain so instruction may be customised and precise; identify any children who may be vulnerable in a given domain; and highlight the diversity of mathematical knowledge in a class. The whole number tasks in the MAI take between 15-25 minutes for each student and are administered by the classroom teacher. There are about 40 tasks in total, and given success with a task, the teacher continues with the next tasks in a domain (e.g., Addition and Subtraction Strategies) for as long as the child is successful. If the child cannot perform a particular task correctly, the interviewer moves on to the next domain or moves into a detour, designed to elaborate more clearly any difficulty a child might be having in a particular area. To illustrate, Figure 1 shows three questions from the GP6 Addition and Subtraction Strategies section of the interview. Words in italics are instructions for the interviewer, and the symbols and arrows indicate which question to ask next. Teachers report that the assessment interview provided them with insights about children's mathematical knowledge that might otherwise remain hidden (Clarke, 2013). This was an important reason for using this assessment instrument as part of the *Numeracy Now* strategy.

Assessing Children's Whole Number Knowledge

The data examined in this paper were collected from all 56 Catholic primary school in one of the three Catholic Dioceses in Sydney. Participants included 21,884 Kindergarten (five-year-old) to Grade 6 (11-year-old) children who were assessed at the beginning of 2016 by their classroom teachers using the *Mathematics Assessment Interview* (MAI).

The teachers followed a detailed script to present each task to children and recorded their responses and strategies on an interview record sheet. These responses were analysed and coded by the teachers using a scoring rubric to determine the growth points children reached in the domains of Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies. The use of a detailed script, record sheet and scoring rubric were important for increasing the validity and reliability of the assessment data. Further, all school leaders and teachers had participated in professional learning focused on understanding and implementing the MAI and growth point framework. This also increased the trustworthiness of the assessment data across schools.

25) Multi-Digit Strategies
I am going to show you some questions. Tell me the answer**.**
Show the white cards for the following questions ***[at any stage]:*** ☺ → **Section D Q29a**
a) 68 + 32 **b)** 25 + 99 **c)** 100 – 68

For the next two questions (d) and (e) read the questions (no cards provided).

d) half of 30 **e)** double 26

26) How Many Digits?

a) *Show the blue card with 134 + 689.*

Please read the card to me. Is the answer to this more than 1000 or less than 1000?

Please explain. ☺ → **Section D Q29a**

b) *Show the blue card with 1246 – 358.*

Please read the card to me. Is the answer to this more than 1000 or less than 1000?

Please explain.☺ → **Section D Q29a**

27) **Estimating and Calculating Addition**

a) *Show the yellow card with 347 + 589.*
Please read this to me. Please estimate the answer to this.
(*If necessary, prompt:* "What would the answer be 'round about'?")

No estimate, or one outside the range 800-1000 → **Section D Q29a**

b) Can you work out the exact answer to this in your head? *(936)*

If "yes", encourage the child to try to do so. If not successful (or if the response to the question in part c) is "No"), make the following request:

c) Please use the pencil and paper to work it out <u>any way you like</u>. ☺ → **Section D Q29a**

Figure 1. Three MAI tasks for Addition and Subtraction Strategies Growth Point 6.

The growth points determined for each student were checked, entered into an Excel spreadsheet in each school and collated by system leaders. These data were next analysed by the research team, according to the ethical guidelines approved for the research, to determine the percentage of children on each growth point in each whole number domain and grade level. Only the growth point data for the Addition and Subtraction Strategies domain are reported in this paper.

Insights about Children's Addition and Subtraction Strategies

Examination of grade level growth point distributions for the Addition and Subtraction Strategies domain across the first seven years of schooling provides a rich picture of the development of children's knowledge and strategies, and enables associated insights and issues to be identified. Figure 2 shows the growth points for 21,884 children from Kindergarten to Grade 6 in 2016. This includes all children attending primary schools in the Diocese.

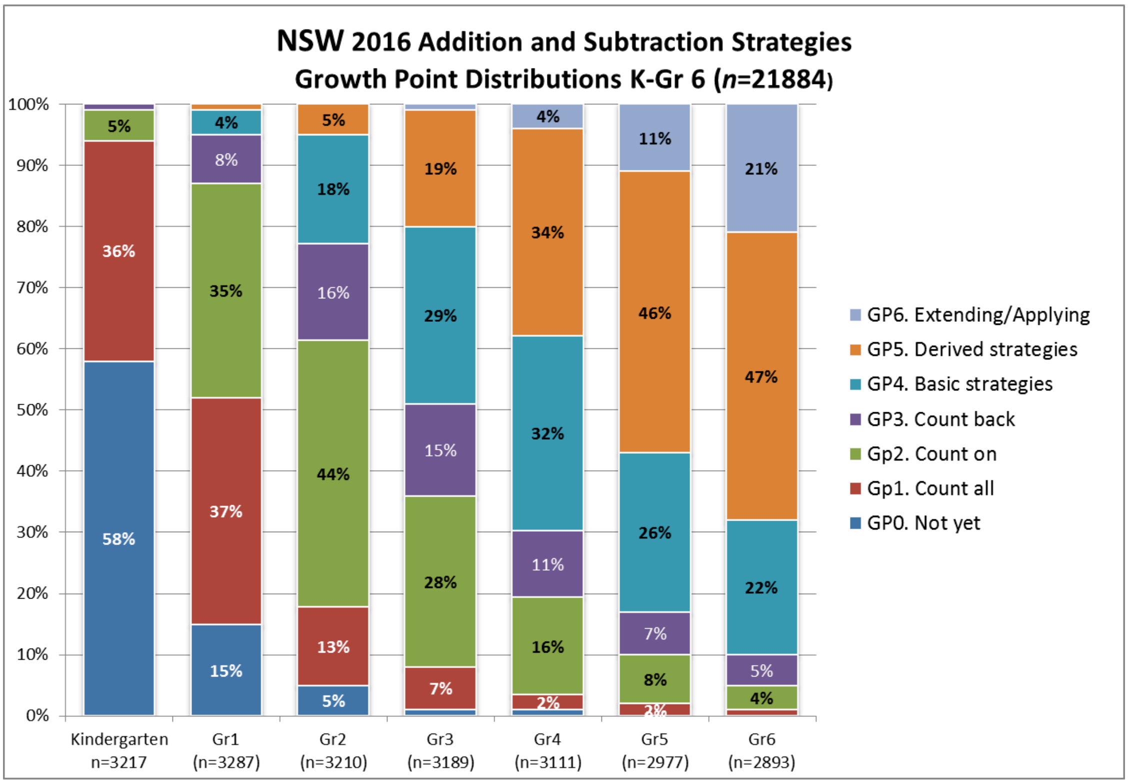

Figure 2. Addition and Subtraction growth point distributions for children in Grades K-6.

Four key insights concerning children's development emerge from analysing the growth point (GP) data:

1. Progress from one grade level to the next is evident with the median growth point increasing from Kindergarten to Grade 5 by one growth point each year, at which point the median growth point increase ceases;
2. There is a wide spread of growth points in each grade level, particularly from Grade 1 to Grade 6;
3. The 80th percentile growth point (GP5-derived strategies) does not increase from Grade 3 to Grade 5; and
4. The 20th percentile growth point does not increase from Grade 2 to Grade 3 (GP2-count-on) and from Grade 5 to Grade 6 (GP4-basic strategies).

A key focus for learning and teaching in addition and subtraction is children's development and use of basic and derived strategies for calculating as opposed to counting-based strategies. Analysis of the systems data highlights three issues associated with children's development of these strategies:

1. Large groups of children from Grade 3 to Grade 6 use counting-based strategies to calculate (GP 1, GP2 and GP3), including 51% of beginning Grade 3 students;

2. The majority of Grade 4 to Grade 6 children successfully use basic or derived strategies for calculating with one-digit numbers, such as for doubles, tens facts and near doubles; and
3. Very few Grade 4 to Grade 6 children reach GP6, which involves mental calculations with two- and three-digit numbers, such as 24 + 99 or 100 – 68.

Discussion

The findings presented in the previous section highlight several important issues related to the instructional needs of children in the Addition and Subtraction Strategies domain. First is the broad distribution of growth points in each grade level, and the wide distance between the lowest and highest growth points in each grade level. This shows that children in each grade have diverse learning needs, and underscores the complexity of classroom teaching and of teachers responding to children's diverse knowledge and experiences. It is clear that there is no teaching "formula" that will meet all children's needs. Thus, it is essential that classroom teachers are (1) able to identify children's current knowledge in order to plan suitable learning opportunities for them, and (2) have the ability to differentiate learning opportunities for individuals according to the range of knowledge represented in a teaching group.

The second point is that children's progress through the growth points from one grade level to the next is clearly obvious in most grade levels. However, it is also clear that progress from one growth point to the next is more challenging at some points for certain groups of students. For example, progress from GP5-GP6 appears to be challenging for many children. Knowledge about these challenging points in children's learning and about how to assist children to reach them is necessary to enable teachers to be most effective, and may be a useful focus for professional learning programs.

A third issue highlighted by analysis of the growth point distributions is the significant number of children using counting-based strategies beyond Grade 2. Counting strategies add considerably to the cognitive load in children's working memory and may lead to these children having difficulty accessing many of the concepts explored in their classrooms when the ability to mentally calculate is assumed.

A fourth point arising from the analysis is that few children reached the highest growth point (GP6), even by Grade 6. On one hand, this provides confidence that the MAI is a useful assessment tool throughout the primary school. However, this also emphasises the importance of teachers creating learning environments that enable all to thrive and reach the higher growth points. It appears that this is not currently so for many children.

Finally, the analyses suggest that several growth points represent challenging aspects of learning in addition and subtraction. These include using basic and derived strategies as opposed to counting-based strategies in addition and subtraction, and mentally calculating with two-digit and three-digit numbers. It is recommended that professional learning opportunities for teachers focus on these challenging growth points and associated powerful pedagogical actions and tools that will assist all children to learn these strategies.

Conclusion

The findings presented in this paper suggest that meeting the diverse learning needs of children is a challenge, and requires teachers to be knowledgeable about how to identify each child's current mathematical knowledge and to customise instruction accordingly. This calls for rich assessment tools capable of revealing the extent of children's knowledge and strategies, and an associated framework of growth points capable of guiding teachers' curriculum and pedagogical actions. Growth points help teachers to identify children's zone of proximal development in order to design effective learning opportunities for children, and in order to adjust activities to increase engagement and remove features that create barriers to learning. Thus, reference to a framework of growth points helps to ensure that instruction for children is closely aligned to their initial and on-going assessment, and is at the "cutting edge" of each child's knowledge (Wright et al., 2000).

A particular issue highlighted through examining the system date is that few children reach GP6 which requires them to mentally calculate with two-digit and three-digit numbers. Only 21% of beginning Grade 6 students reached GP6. This contrasts to a longitudinal study by Clarke et al. (2006) who used the same assessment interview and growth point framework to measure children's progress following the *Early Numeracy Research Program*. In their study, 22% of children reached GP6 at the end of Grade 4 and 59% at the end of Grade 6.

Researchers have long since noted that, despite curriculum advice to the contrary, many teachers introduce children to formal written methods for adding and subtracting two-digit and three-digit numbers in Grade 2 and Grade 3 (e.g., Narode, Board, & Davenport, 1993). The evidence presented in this paper suggests that this is before most children mentally calculate with numbers in this range. We hypothesise that this leads many children to approach calculations in a procedural way and may curtail the development of their number sense and flexible mental strategies for calculating with larger numbers. We suggest that teachers introduce children to formal written methods after they reach GP6 and can flexibly and creatively calculate mentally with two-digit and three-digit numbers. This aligns with NSW syllabus guidelines.

> From Early Stage 1 there is an emphasis on the development of number sense, and confidence and competence in using concrete materials and mental, written and calculator techniques for solving appropriate problems. Algorithms should be introduced after students have developed mental strategies for computing with two- and three- digit numbers. (NSW Board of Studies, p. 34)

We suggest that formal written methods should be reserved for calculations that are too complex to perform mentally. A later introduction of written methods may increase the conceptual understanding that children can apply to solving more complex calculations in efficient and flexible ways. We also recommend that teachers focus more intensely on children's development of basic and derived strategies in the first three years of primary school as a means of reducing many children's persistent use of counting-based strategies. The initial use of ten-frames, structured bead-strings and arithmetic racks to model, describe and simulate and discuss these basic and derived strategies may be a useful approach.

Assisting children to learn mathematics is complex, but teachers who are equipped with the pedagogical knowledge and actions necessary for responding to the diverse needs of individuals are able to provide children with the learning opportunities and experiences that enable them to thrive. With respect to children learning efficient, flexible strategies for mentally calculating with two-digit and three-digit numbers, the findings presented in this paper call for new pedagogical approaches in the schools included in this study.

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