Transitions in Mathematics Education

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The Merriam-Webster dictionary defines *transition* as: (a) the passage from one state, stage, subject, or place to another: change; or (b) a movement, development, or evolution from one form, stage, or style to another. The word transition can refer to an active shift of the person in space and time or status, for example; it can also refer to developments taking place within the person. Transitions may be anticipated by those involved, and hence planned for, or they may result from unexpected changes in people's lives. Transitions can occur at various points throughout a person's educational trajectory, and here we include student development across primary, secondary, and tertiary sectors; also, the transitions made between cultural contexts in schools. Beach (1999) noted that (consequential) transitions consist of "changing relations between persons and their social activities represented in signs, symbols, texts, and technologies" (p. 119). In this symposium, we will be considering transitions in mathematics education affecting both students and teachers, specifically in relation to representations in the first two papers and to values in the third.

In *Transitions in Language Use in Primary School Online Mathematical Problem Solving*, Duncan Symons and Robyn Pierce adopt a Bakhtinian lens to examine upper primary school students' use of informal and formal language registers in CSCL mathematical problem solving. They argue that online discussion assists in the development of mathematical language as demonstrated by students' use of a *transitional mathematical register* combining new mathematical words with their own natural language.

In *Mathematical Writing and Writing Mathematics: The Transition from Secondary to University Mathematics*, Caroline Bardini and Robyn Pierce present a framework based on their research on students' use and understanding of mathematical symbols, recognised as crucial in students' successful transition from school to university mathematics. In particular, the framework supports a fine-grained analysis allowing better appreciation and understanding of the subtle differences in students' experiences with symbolic expressions.

In *The Valuing of Deep Learning Strategies in Mathematics by Immigrant, First-generation, and Australia-born Students: Transitions Between Cultural Worlds*, Abi Brooker, Marian Mahat, and Wee Tiong Seah draw on an ecological systems model of students' learning experiences to take an intercultural approach towards transitions in mathematics education. Their focus is on the many school students in Australia who move between cultures on a daily basis, particularly those who achieve well in international assessments. They consider that the multicultural nature of many Australian classrooms provides an opportunity for students to learn from different values and perspectives to enhance their learning. Identifying the values that students have for deep learning (and their preferred strategies for learning) might offer valuable insights into how students' engagement with and abilities in mathematics can be better supported on a wider scale.

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Transitions in Language Use in Primary School Online Mathematical Problem Solving

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Online text discussion highlights "teachable moments". A Bakhtinian lens is used to examine upper primary school students' use of informal and formal language registers in mathematical problem solving in a Computer Supported Collaborative Learning (CSCL) environment. The authors argue that online discussion assists in the development of mathematical language as the data shows evidence of students' use of a 'transitional mathematical register' combining new words discussed in class with their own words that the "community of practice" would not think of as part of mathematical vocabulary.

Transitional Mathematical Register

Students in the upper primary years of schooling are in a state of flux or transition (Attard, 2010; Downs, 2003). At this age (11 to 12 years), students are preparing for the transition from primary school to secondary school as well as the social and physiological transition from childhood to adolescence. Both Attard (2010) and Downs (2003) note that the transition to secondary school requires students to negotiate new social, organisational and academic structures. Academic expectations also change at this point. In mathematics, this includes the use of mathematical language.

Barwell (2012) describes this as a transition from an informal to a formal mathematical register where "register" is understood in the sense that it is used by Halliday (1978, p. 195) as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings." For students working mathematically at this level, the informal register encompasses everyday words such as "going" or "pointy," while "multiply", "equation," and "median" lie in the formal register.

In this paper, we report on an investigation into the notion that a *Transitional Mathematical Register* (TMR) exists between the informal and formal registers. We demonstrate the interplay, and movement between registers in students' use of language by analysing two short excerpts of online discussion between students tasked with solving problems by using mathematics.

Theoretical Framework

This study draws on Barwell's (2012) application of the Bakhtinian (1981) dialogic perspective as a means to expose the tensions that exist between informal and formal mathematical language. He demonstrates that the formal mathematical register is privileged throughout international curricula by pointing to a tendency of these documents to require simple, informal mathematical language to describe mathematical ideas in the earlier years of schooling, whilst working towards usage of the "formal mathematical register." He argues that informal and formal registers are always required and are always in tension.

Barwell (2012) suggests that privileging the formal mathematical register within the curriculum is not ideal, because it places greater importance on the *correct* use of

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mathematical language at the potential expense of meaning making. Bakhtin's (1981) view of language was that it is situated, dynamic, and dialogic. He sees languages as being either unified (unitary) or, to use his term, in a state of *heteroglossia*. The theoretically complete formal mathematical register can be seen as a unified language. The tensions and centripetal forces that exist within curricula, our schools and educational institutions, and society, expect a single, agreed upon language and register for the teaching and learning of mathematics. Valuing only this unitary language may inhibit students' experimentation and trialling of new and unfamiliar language. As a theoretical construct, there is a place for the formal mathematical register, however in our lived reality it is unlikely to truly exist.

Context of the Research

Students in the present study worked in a Computer Supported Collaborative Learning (CSCL) environment. The work reported here took place in a larger project conducted in an Australian primary school. Thirty-eight percent of students attending the school are from a language background other than English. This has implications for working in a CSCL environment which places demands on students' abilities in the area of literacy. Participants were 54 Year 5 students (10 to 12 years old) allocated to ten online, mixed ability groups. Over the ten weeks in which the unit was delivered, students collaboratively solved and/or investigated nine mathematical problems incorporating an aspect of each content strand of the Australian Curriculum: Number & Algebra, Statistics & Probability, and Measurement & Geometry (ACARA, 2014).

Students were expected to engage in iterative asynchronous online discourse where they would build on each other's ideas. No online adult facilitator took part in the CSCL. This decision was taken in order to avoid discussion between students being heavily influenced by an "expert" other. Each week for the first seven weeks, prior to students commencing work online, an hour of standard classroom discussion was facilitated by the first author of this paper. This time was spent with the class performing three tasks. First discussing expectations of behaviour, and appropriate approaches to collaboration within the online space. Second reviewing the previous week's solutions and discussing students' perceived challenges and successes then finally reading through and discussing the following week's problem.

The first piece of data presented (on *Wallpaper Symmetry*) was facilitated using this approach, however the second piece of discussion data (on geese "V patterns"), was from the ninth week of the intervention. At this stage, instead of providing the students with classroom support, they were expected to use this hour to work in the online space. They received no specific advice from the facilitator. They were only allowed to communicate in the online environment (they were not permitted to speak to each other). We were interested in gaining an understanding about whether the students would be able to apply the language and problem-solving strategies without any teacher support.

Findings

Attempts at conveying mathematical ideas or concepts have been bolded, highlighting attempted use of formal or informal mathematical registers. In *Wallpaper Symmetry* students were asked to represent line/ mirror symmetry, rotational symmetry and translational symmetry using Microsoft Word and making use of the shapes provided. They uploaded their file and then described how symmetries were used in their wallpaper.

Olivia: i think to **work this out** we would need to **choose a shape** with the **pointy sides** (dont really know how to say it) so it would be easier with for us to do it does anyone agree with me?
Chris: What **shape** is everyone deciding on. i was thinking of a **hecsigon**
Olivia: i **changed it** i have **done a triangle** i **created something like a fan** so when it **spins** you could see **the pattern** and also **it would never changes** i have uploaded mine to edmodo.
Zander: **Mirror/Line Symmetry- line symmetry** means when you have a **shape** or anything, and you **cut it in half, it looks exactly the same size and lining on every single thing as the other side.**
Rotation Symmetry- rotational symetry means, depending on **how many pointy sides they have**, say for example, i had a **plus sign +,** it has **4 pointy sides**. So then, after you **move it 4 times**, it **goes back to the same spot**.
Reflection Symmetry- reflection symmetry means if you have a **picture of your face, you keep drawing that, making look the same height, the same lengh**, and etc.

Figure 1. Discussion of Wallpaper Symmetry Problem.

In Figure 1 we see examples of students struggling to express their mathematical thinking. Olivia's opening statement suggests a tension caused by her desire to use the formal mathematical register while lacking the words to do so. Her language is disjointed but when read carefully we see that Olivia is developing an understanding that rotational symmetry means a shape will look identical when rotated on its axis the number of degrees corresponding to its order of rotation. She does not have the words to describe this accurately but is able to make her meaning known through her use of the TMR.

It is interesting to note that Olivia's choice of words to represent mathematical thinking evolves and becomes a shared language through dialogue with other students in her group. Zander adopts her TMR phrase to progress the discussion. This shared language to create shared meaning exhibits Barwell's (2012) interpretation of Bakhtinian dialogism.

In Figure 2 (Modelling Middle School Mathematics, 2014) students explored the *V pattern* made by flocks of geese as they fly. The discussion below, between Indigo and Maddie was typical of discussion that occurred during this investigation.

Indigo: The **rule** is it is **going up by twos** as an **odd number** so instead of the **simple 2 4 6 8** it is **1 3 5 7 9** etc

Indigo: and my **formula** I am still **working out**

Indigo: OK I have **found** a **formula**! What I did was (say the **square** was **b2** and the **number** in it was **three**) I did **=b2+2** because **3 plus 2 is five** which is the **next equation** in the **pattern**. It **goes up by two every time** so that would be the **formula**.

Maddie: Do you know how to **drag down the numbers** so you can **go to 100**?

Maddie: That is very good Indigo. I liked how you explained the **formation**.

Indigo: thank you Maddie

Figure 2. Discussion of Geese Problem.

In Figure 2, we see Indigo experimenting with the identification of growing patterns. She is able to establish that each successive term changes by the same amount as the preceding term. Her mathematical vocabulary is not yet developed enough to express herself using formal vocabulary. However, she is able to establish meaning through her utterances. In the study, we saw many examples of students using the phrase *going up by*, which, while not part of the formal mathematical register, is helpful in the meaning making process. A tension exists between Indigo's desire to communicate her ideas and her lack of approved formal words. Indigo's emergent language provides another example of a TMR.

Discussion and Implications

Upper primary school is one bridging point in student mathematical language development. No longer are students only required to use the language of basic place value and four operations, they must begin to develop language for more sophisticated concepts; such as algebraic and relational thinking. While the goal is their use of formal mathematical language, students make sense of these new concepts through appropriating familiar language in combination with the new formal vocabulary. This hybrid language, or *TMR*, allows students to reason and communicate their emerging understandings.

The dialogic nature of language is also evident in this data. Students who have been exposed to new terms in the classroom use a variation of this formal mathematical vocabulary within the CSCL environment. The language is used with various degrees of precision. This data supports Barwell's (2012) contention that insisting on use of the formal mathematical register rather than acknowledging this transition phase could be counter-productive. We see students' use of the TMR as evidence of "teachable moments" when students' correct ideas should be validated but formal language modelled without any suggestion that the student is in some way 'wrong' so that the dialogic cycle may continue to have impact.

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Mathematical Writing and Writing Mathematics: The Transition from Secondary to University Mathematics

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This paper reports findings from research on students' use and understanding of mathematical symbols, which has been recognised as playing a major role in students' success in mathematics. One goal is to identify potential trouble spots for the usage of symbols as students travel the path between school and university mathematics. We present a framework that supports a fine-grained analysis allowing us to better apprehend the subtle differences in students' experiences with symbolic expressions. This is illustrated by considering some first-year mathematics students' written responses to questions and insights from interviews with senior secondary teachers and university lecturers and tutors.

Students' use and understanding of mathematical symbols, what we call 'symbolic literacy' (Bardini, Pierce& Vincent, 2015) plays a major role in students' success in mathematics. Gaining fluency with symbols is especially important at university, when not only does mathematics become much more symbolic, but its writing is more subtle and requires increased 'flexibility' from the reader. This paper reports findings from an on-going three-year project on students' symbolic literacy. One of the project's goals is to identify and investigate potential trouble spots for the usage of symbols as students travel the path between school mathematics and mathematical sciences subjects at university. We present a theoretical framework that has been shown to support a fine-grained analysis allowing us to better apprehend the subtle differences in students' experiences with symbolic expressions between their previous encounter in school and their new journey at university. This is illustrated with three examples and finally some implications for teaching are suggested.

Literature Review

The transition from school to tertiary mathematics has been studied from various perspectives. Thomas (2008) summaries some of these in the introduction to a special edition of the Mathematics Education Research Journal. He notes that learning in mathematics progresses along a spiral path where students revisit concepts from new perspectives. This seems to be challenging for many students: de Guzmán et al. (1998) observed that to make the transition from school to tertiary mathematics students need to organise their knowledge to allow a global perspective supporting making connections, modifying views and adapting to new domains. Reporting on a recent study comparing mathematics in the secondary and tertiary contexts, Corriveau (2017) found that "even if the same concepts are used at both levels e.g. concept of domain, they are conceptualised differently" (p. 156). This study seeks to identify some of the obstacles students encounter in revising or expanding their thinking.

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This Study: Methodology and Theoretical Framework

Methodology

In this study, data were collected from 279 first-year mathematics students at three Victorian universities. All students were enrolled in a subject with the prerequisite of Year 12 Mathematics Methods or equivalent. Participants responded to fortnightly surveys, during tutorials, posing a probing mathematical question designed to gauge their "symbolic literacy" according to the topic they were studying. Students' responses have been categorized and responses analysed for likely links to students' past mathematical experience.

We also interviewed experienced senior secondary school teachers, university lecturers and tutors (from four Victorian universities) asking them about their students' difficulties writing and understanding symbolic mathematics. Transcripts of these interviews have been analysed by the research team in order to identify themes in their responses.

Framework

The framework that we have applied to our analysis of the symbols of concern to students and their teachers is based on the work of Serfati (2005), who provides us with an epistemological approach to mathematical notations that takes into account both the syntactical properties of a symbol and the mathematical concept(s) conveyed.

Serfati's work advocates that we consider three distinguishing features of any symbolic expression. In our simplified version of his approach we describe these components as:

- the *materiality*. The materiality of a symbol focuses on its 'physical' attributes (what it looks like). A classic example is the = sign. Materiality includes the category the symbol belongs to (letter, numeral, specific shape, conjunction etc.).
- the *syntax*. The syntax of a symbol relates to the rules it must obey in symbolic writing. This includes the number of operands for symbols standing for operators but also the appropriateness of placing certain symbols adjacent to one another.
- the *meaning*. The meaning of the symbol is the concept being conveyed, for example the representation of an unknown or of a given operation. Meaning for Serfati is that commonly agreed by the community of mathematicians and it does not refer to a person's individual understanding.

Results and Discussion

First, we saw evidence of students strongly attached to fixed materiality so that if the symbol, letter, used is changed then the student does not recognise the syntax or the meaning of the expression. In statistics staff commented for example that students do not recognise linear functions expressed using letters or the arrangement other than y=mx+c for example $y = \hat{a} + \hat{b}x$ or not using y and x.

Tutor MB001: …straight line-all the schools say now that's x, y… they say y=mx+c… but in the university level, or the mathematical convention is y=a+bx. We put the constant first…that's the same. A lot of students couldn't see that's equivalent.

In the example shown in Figure 1 students were required to "realise" the denominator. While students concerned knew the technique of multiplying by the complex conjugate, a new skill, they did not recognise the difference of two squares formed in the denominator.

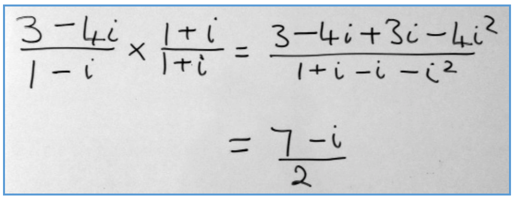

Figure 1. Student solution not recognising difference of two squares.

In later discussion, the tutor commented that many students only recognised (a-b)(a+b) = a^2–b^2. The change in materiality impeded recognition of a helpful pattern.

Second, we saw evidence of students unthinkingly applying syntax templates (Bardini et al., 2015) that they have met before but without consideration of the context or the domain in which they are working. Asked to explain the meaning of -1 for in $\sin^{-1}(x)$ students responded in the variety of ways shown in Table 1.

Table 1

Students' Meanings for -1 in $\sin^{-1}x$ *(n = 204)*

Student response	n	%
Inverse	78	
Inverse (arcsin)	15	54.4%
arcsin	17	
Flipped in y = x axis	1	
1/sin(x)	50	
Reciprocal of sin(x)	15	37.7%
cosec(x)	12	
1/sin(x) or arcsin	3	
cosec(x) as it is the inverse	3	
If capital S, arcsin (x) otherwise cosec(x)	1	7.9%
Inverse of sin. However, could also be written as 1/sin(x)	1	
Other responses	8	

Finally, we see evidence of students working without consideration of whether the symbols they have chosen create expressions that would make sense for another person reading this work. University staff consistently commented (see examples below) on their concern that students write series of symbols that do not make sense. They also comment that students often seem hesitant to use a mix of words and symbols.

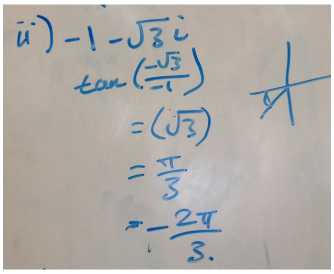

Figure 2. Student solution.

Tutor (CB0304a) …please consider the logical progression…these lines don't follow from each other

Tutor (CB304b) I think that they can understand the answer but they're failing is to express them step by step, so I say that they are lacking communication.

Teachers spoke of marking in Year 12 examinations where multiple choice and 1 or 2 mark questions reward the final answer rather than meaningful communication.

Conclusions and Implications

Analysis of the student data and staff interviews suggests that the weaknesses in the bridge between school mathematics and university mathematics began to develop in the junior secondary years where students learned to recognise patterns of symbols relying on the cue of fixed symbols. Relying on unthinking recognition continues to cause difficulties as students encounter familiar syntax templates in new contexts and domains (vectors, complex numbers). These weaknesses appear to be papered over rather than remedied at the senior secondary level as students focus on the skills required to maximise their marks on examination questions. An emphasis on speed and correct final answers values pattern recognition over mathematical thinking and communication. University staff speak of the need for words and symbols for meaningful communication in mathematical solutions.

In order to overcome these potential weaknesses and prepare school students to use mathematics in life and at university we need to explicitly model and value variety in the letters and domains for our examples and model and value clear, correct mathematical communication. In doing this it may be helpful to ask students questions or give instructions such as: Read this maths out aloud please. How else could we write this? What does this mathematical sentence mean? Could someone else, who had not been in this class, follow your working and use it to solve a new problem? Show me another way to represent and solve this problem.

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The Valuing of Deep Learning Strategies in Mathematics by Immigrant, First-Generation, and Australia-Born Students: Transitions Between Cultural Worlds

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Although foreign-born and first-generation students are in constant transitions between their home and host pedagogical cultures, they have performed better than their locally born peers in Australia. In this paper, we draw on Urie Bronfenbrenner's ecological systems approach to child development, focusing on the child's perspective to understand how cultural transitions (e.g., Gorgorió, Planas, & Vilella, 2002) interact with learning. This framework not only allows us to understand how these daily transitions facilitate different ways in which the various deep learning competencies are valued, but also assist to identify implications for even more effective mathematics learning by all students.

Introduction

The latest TIMSS 2015 (Thomson, Wernert, O'Grady, & Rodrigues, 2016) and PISA 2015 (Thomson, de Bortoli, & Underwood, 2016) results show that Australian students' performance in assessments for mathematical knowledge, its understanding, and its applications have not improved over the last 20 years or so. At the same time, mathematics education reforms in many other countries have led to noticeable improvements in their students' mathematics ability, resulting in these students surpassing their Australian peers in assessable mathematical knowledge. In fact, the latest PISA assessment results indicate that whilst Australia's ranking has dropped relative to other countries, its ranking would have been worse if it had not been supported by the performance of students from immigrant families. In PISA 2015, mathematical literacy scores of students from Australian-born families were significantly lower than those attained by first-generation and foreign-born students, as noted by Thomson et al. (2016):

> 10% of Australian-born students were high performers compared to 14% of first-generation students and 14% of foreign-born students. At the lower end of the mathematical literacy proficiency scale, the proportions of low performers for Australian-born and foreign-born students were similar (22%), while the proportion of first-generation students was 18%. (p. 65)

The mathematics learning experiences of these foreign-born and first-generation students are transition processes (Gorgorió, Planas, & Vilella, 2002). These students attend the same school as their Australian-born peers, have the same teachers, have the same in-class opportunities, and are generally treated equally in the (mathematics) education systems. Yet, there seems to be a cultural pattern in which students who have grown up moving between the Australian culture (in the mainstream classroom) and their respective home cultures (in the family) outperform those who do not make such transitions (Australian-born).

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We are interested in investigating why this pattern has emerged, and in what educators can do to improve the mathematics performance of students from Australian-born families. Drawing on developmental science (e.g., ecological models of child development), educational science (e.g., deep learning competencies and curriculum) and multiculturalism studies, we suggest that students' approaches to mathematics education are at least partially informed by their family cultural environments; and that encouraging similar approaches to learning in other students might help to promote a positive shift in Australian students' mathematics abilities.

An Ecological Systems Model of Students' Learning Experiences

The ecological systems model of child development (Bronfenbrenner, 1992) has been found useful to describe the learning experiences of immigrant children (e.g. Jensen, 2007; Paat, 2013). It describes children's development as nested within a series of interrelated systems – *microsystems* are systems with which the child interacts directly, such as family and school; *mesosystems* are the relationships between those microsystems; *exosystems* are elements of the society that affect the child's life but are typically out of the child's reach, such as mass media, industry developments, market structures, and local government; *macrosystems* are the broader ideologies and values of the culture; and *chronosystems* refer to the specific point in history in which all of these systems exist.

Although all of these systems are likely to shape students' mathematics education to some degree, the relationship between two microsystems (school and family) is particularly pertinent for this paper. We take an alternative position to this model by considering it from the perspective of the active child. As the child interacts with each microsystem, s/he takes on the values, meets relevant expectations, and masters the resources of the system.

How well the child navigates between these systems has strong implications for his or her development. On the one hand, stronger mesosystems (connections between family and school) can have positive consequences for a child's learning, as parental involvement in school-based activities reinforces important messages about the value of education for the child (Bishop, 2006; Lee & Bowen, 2006). Internationally, research consistently demonstrates this relationship between parents' and children's attitudes and behaviours towards education (e.g., Chiu, Pong, Mori, & Chow, 2012; Gibbs, Shar, Downey, & Jarvis, 2016; Li, 2016; Wang & Eccles, 2013). On the other hand, there is increasing evidence of an "immigrant paradox", in which children who successfully navigate their multiple cultural worlds are able to draw strengths from these cultures and experience more positive outcomes (in academic, mental health, and social domains) than their peers (e.g., Marks, Ejesi, & Garcia Coll, 2014). In other words, for the first-generation and foreign-born students, both their home and the Australian cultures are represented in the different micro-, meso-, exo-, macro- and chronosystems. As they are often required to practice cultural switching, they develop stronger deep learning skills than their mono-cultural or mono-linguistic peers. In a school context where deep learning is valued, this should result in higher academic performance.

Australian Culture: Shaping Children's Learning Experiences and Valuing Deep Learning Processes

The *school* context in Australia, however, is largely mono-linguistic, where the medium of instruction in almost all schools is the English language. This is despite the fact that Australia is a culturally diverse nation, in which 27% of its population were born overseas and a further 20% are first-generation (born in Australia with at least one immigrant parent; *Australian Bureau of Statistics*, 2013). This is also despite the fact that migrant and indigenous groups speak 500 other birth languages. The mono-linguistic culture of Australian classroom thus positions cultural (and linguistic) diversity as a challenge, despite its usefulness as a resource and learning opportunity for students (Scarino, 2014).

We are concerned with how deep learning strategies are represented and taught to students (of mathematics). In particular, we ask: how much do foreign-born and first-generation students value deep learning strategies? And are some deep learning strategies more useful for these students than others?

We acknowledge that Australian schools consider deep learning as a useful study approach within formal education. Australian educators' interest in deep learning is not unique: there is currently a strong international interest in the ways in which school curriculum promotes deep learning (e.g., Fullan & Langworthy, 2013). The *New Pedagogies for Deep Learning* global partnership (between Australia, Canada, Finland, the Netherlands, New Zealand, Uruguay, and USA), identifies six competencies that reflect deep learning processes essential for learning: character, citizenship, collaboration, communication, creativity, and critical thinking (DEECD, 2015). Many of these are embodied in the Australian Curriculum as capabilities that students are expected to develop. In particular, the four proficiencies of understanding, fluency, problem solving, and reasoning are reflected in the Victorian Curriculum for Mathematics. The *New Pedagogies* website (Victoria Department of Education and Training, 2014) hosts a range of case studies that demonstrate the varied ways in which students learn these competencies, including school culture (modelling good practice, school rules, and school meetings) and structured learning activities (e.g., projects, class discussions, competitions). Yet, the actions that reflect the valuing of particular attributes need not be the same in different cultures (Seah & Andersson, 2015). We further ask: If the six competencies identified by DEECD (2015) constitute deep learning strategies, what do they look like for the foreign-born and first-generation students as they move in and out of different cultures daily across systems in their mathematics learning? How similar or different are these students' deep learning strategies compared to their Australia-born peers in the same class?

What Does This Mean for Students in the Australian Context?

In our attempt to understand why foreign-born and first-generation immigrant students perform better in mathematics as they transition to Australian schools, we have found Bronfenbrenner's (1992) ecological systems model of child development useful. It has reminded us that as we assess the mathematics learning experiences of foreign-born and first-generation immigrant students, the different cultural forces and influences they experience on a daily basis has the potential to enrich the ways in which they internalise the deep learning competencies. To the extent that the home and Australian cultures do not clash, the immigrant students appear to have a richer repertoire of cultural knowledge to draw upon, with which to value attributes of deep learning of mathematics (and other subjects).

In the meantime, new questions are raised. These include: how much do foreign-born and first-generation students value deep learning strategies in mathematics? What do these strategies look like for the immigrant students? Are some deep learning strategies more useful for these students than others? We would recommend that these questions be subjected to future inquiries. Indeed, the values of students with better academic outcomes might offer insights into areas of needed support for those who are struggling. At the same time, foreign-born and first-generation immigrant students' approaches to learning are strongly interconnected with their transitions between cultures, and offer useful insights for improving all students' engagement and performance within the Australian mathematics education system.

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