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The views of high school students about proof and their levels of proof (The case of Trabzon)

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Abstract

This study was conducted to determine the views of high school students about proof and these students' levels of proof. Case study method was used in this descriptive study. The data of the study were obtained by conducting a questionnaire which consists of eight open-ended questions to total 125 10th grade students studying in two different secondary schools in Trabzon during 2006-2007 school year. The views of the students regarding proof were coded and their levels of proof were analyzed based on the classification of Miyazaki related to proof. At the end of the study, high school students' competencies of doing proof were found below the desired level, as well as their use of different proof types. Based on these results, it's recommended to give room in the classes to activities that would improve students' mathematical thinking skills and increase their levels of proof.

Keywords: Mathematics education; mathematical proof; matrhematical reasoning; proof levels; student views.

1. Introduction

Since mathematics is logical, not phenomenological in terms of its method and results; the mathematician aims at making a perceived relationship certain, not explaining it (MNE, 2005). Since the procedure followed in the production of mathematical knowledge is unique to mathematics (Alkan & Altun, 1998), judging to the trueness or falseness of an argument, theorem or expression in mathematics comes after a process called proof. Proof is fundamental to doing and knowing mathematics. It is the basis of mathematical understanding and essential for developing, establishing, and communicating mathematical knowledge (Stylianides, 2007) and also it is an essential part of improving thinking. However many mathematics educators believe that focusing exclusively on the logical nature of proof can be harmful to students' development. Such a narrow view leads students to focus on logical manipulations rather than on forming and understanding convincing explanations for why a statement is true (Alibert & Thomas, 1991). Researches have suggested various roles that proof plays

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in mathematics (De Villiers, 1990; Knuth, 2002a): a) to verify that a statement is true, b) to explain why statement is true, c) to communicate mathematical knowledge, d) to discover or create new mathematics, e) to systematize statements into an axiomatic system.

One of the most important tools in learning mathematics is considered to be proof (Knuth, 2002b). Proof is made up of some universally accepted methods. Proofs can be made mainly either by induction or deduction. Deduction type of proof involves several methods such as direct proof, proof by contra positive, and proof by contradiction (Baki, 2008; Moralı, Uğurel, Türnüklü & Yeşildere, 2006). The aim of a mathematical proof can be stated as proving the trueness or falseness of an argument for every case and condition (Baki, 2008), as well as demonstrating the interrelations of the justifications (Lee, 2002). In other words, showing the trueness of an expression is only one of the reasons of proof. Moreover, proof has several other purposes such as *explanation, systemization, communication, discovery of new results, justification of a definition, developing intuition, providing autonomy* (Weber, 2003).

A mathematician analyses the problem or expression first. Then, the mathematician inquires about whether an expression is true or false by looking at a previous proof and examines how this expression may be derived by utilizing the proven theorems. This process ends with the implementation of proof or by showing that the expression is false. However, a proof needs to be accepted by the mathematics society in order to be regarded as a real proof (Lee, 2002).

The development of a proof depends on individuals' gaining of various logical thinking ways. Logical thinking begins to appear at an elementary school child. During the secondary education period, the individual possesses the ability for abstract thinking and has already been covered a long distance (Altıparmak & Öziş, 2005). At the end of this period, students are expected to attain the following skills (NCTM, 2000): a) recognize reasoning and proof as fundamental aspects of mathematics, b) make and investigate mathematical conjectures, c) develop and evaluate mathematical arguments and proofs and d) select and use various types of reasoning and methods of proof. Proof has an important role in the attainment of these skills. With the development of mathematical proof skills, the mathematical understanding skill of the individual will also improve.

With the increasing importance attached to proof in mathematics, the thinking processes and development of students from diverse age groups have become the subject of many studies (Knuth, 2002b; Stylianides, 2007). However, doing a proof is considered to be a challenging, fearsome, and unlovely process by many students at all levels including the elementary, secondary and higher education (Almeida, 2003; De Villiers, 1990; Jones, 2000; Raman, 2003). In a study by Moralı et al. (2006), most pre-service teachers were found to have either no or insufficient views about doing proofs. In another work by Özer and Arıkan (2002), it was found that high school students could not use proof methods and techniques sufficiently and could not do proof at an expected level. Similar results were reached, in another study by Almedia (2001). When the role of doing proofs in mathematics education is taken into account, it's obvious that the number of studies is scarce in this field in Turkey. For this reason, the main purpose of this study is to determine the views of high school students about proof and their levels of proof.

2. Method

This is a descriptive study using case study method. Although case studies are widely used in both qualitative and quantitative inquiries, in the case of qualitative research they enable in depth investigation of a single or a number of cases, phenomena or events with a limited sampling (Çepni, 2007).

2.1. Participants

The study group consists of total 125 randomly selected 10^{th} grade students studying in two different high schools in Trabzon during the spring term of 2006-2007 school year.

2.2. Data collection tools

The data of this study were collected using a questionnaire consisting of eight open-ended questions. The first two of these questions consist of the views of students about proof and the necessity of proof, and the last six questions consist of questions developed by Özer and Arıkan (2002) to determine students' levels of proof. The items used survey are as follows:

Item 1: What's mathematical proof according to you? Can you define it briefly?, *Item 2:* Is there a need for mathematical proof? Why?, *Item 3:* Show the trueness of the expression "the sum of two consecutive numbers are three times the middle number.", *Item 4:* Show the trueness of the expression "the sum of five consecutive numbers is five times the middle number", *Item 5:* Show that "the sum of two odd numbers is an even number.", *Item 6:* Show that "if a is an even number and b is an odd number, then $a^2 + b^2$ is an odd number.", *Item 7:* **S**how that "if b and c are divisible by a, then (b+c) is divisible by a.", *Item 8:* Show that " for every m, n €N, $(a^m)^n = (a)^{mn}$".

2.3. Data analysis

Item 1 and Item 2 were thematically classified in regard to student responses' similarities and differences (Merriam, 1988; Yin, 1994). Two of the authors evaluated students' responses separately and all disagreement points were solved by negotiation. Students' responses to last six questions were analyzed based on the classification of Miyazaki (2000) regarding mathematical proof and their levels of proof were presented with one example for each. The levels of proof of Miyazaki are as in Table 1.

Table 1. Proof levels of Miyazaki

	Contents	
Representation	**Inductive reasoning**	**Deductive reasoning**
Functional language of demonstration	Proof D	Proof A
Other language, drawings and/or manipulable objects	Proof C	Proof B

Miyazaki classified proof into four groups as Proof A, Proof B, Proof C and Proof D. According to Miyazaki, Proof A is the type of proof in which deductive reasoning is involved and a functional language is used in the course of doing a proof. Proof B is the type of proof in which deductive reasoning is involved and other language, drawings and movable objects are used in the course of doing a proof. Proof C is the type of proof in which inductive reasoning is involved and other language, drawings and movable objects are used. Proof D is the type of proof in which inductive reasoning is involved and a functional language is used. Miyazaki evaluated Proof A as the most advantageous and Proof C as the least advantageous level in school mathematics. On the other hand, Miyazaki states that Proof B and Proof D are middle level proofs in between Proof A and Proof C.

3. Results and Discussion

In this chapter, the data obtained from the surveys were analyzed and discussed and the results were presented in tables.

The views of the students related to the definition of proof were coded and the percentages and frequencies of these codes with an example student answer were given in Table 2.

Table 2. Student answers and the percentages and frequencies of the codes generated for Item 1

Codes	f	%	Student Responses
Showing the correctness of a result	21	16.8	A method used in mathematics to prove the trueness of the implemented operations.
Demonstrating the way mathematical operations are carried out in detail	21	16.8	Showing the way any mathematical operation is done in depth.
Showing the result numerically	24	19.2	Making use of numerical data to prove something.
Showing the correctness of a proposition, formula or theorem	25	20.0	Showing the validity of some rules which are accepted in mathematics society. In other words proving its trueness.
Giving a logical explanation	3	2.4	Basing an expression to logic instead of rules.
Showing the result in various ways	4	3.2	Finding the result of an operation by using different means.
Showing that an equation is justified for all values	1	0.8	Showing that a case satisfies a rule for all numbers.
Representing a given expression mathematically	12	9.6	Finding out the construction of an expression with mathematical concepts.
Finding the unknowns based on the knowns	5	4.0	Generating a formula based on other formulae and information.
No answer	9	7.2	

The students' answers regarding the definition of proof were found to gather under nine different codes. Among these codes, the codes of "*Showing the correctness of a result", "Demonstrating the way mathematical operations are carried out in detail", "Showing the result numerically", and "Showing the correctness of a hypothesis, formula or theorem"* were found to outweigh the others. Furthermore, it was determined that the students' definitions of proof were similar to those reported in the literature (De Villiers, 1990; Lucast, 2003; Stylianides, 2007; Weber, 2003). Furthermore, the definitions related to proof being gathered under nine different codes indicates that the students have many different views concerning proof. The results of the studies by Knuth (2002a), Knuth (2002c), Reid (2002) and Varghese (2009) support this result.

The views of the students related to the necessity of doing proof were coded and the percentages and frequencies of these codes with an example student answer were given in Table 3.

Table 3. Student answers and the percentages and frequencies of the codes generated for Item 2

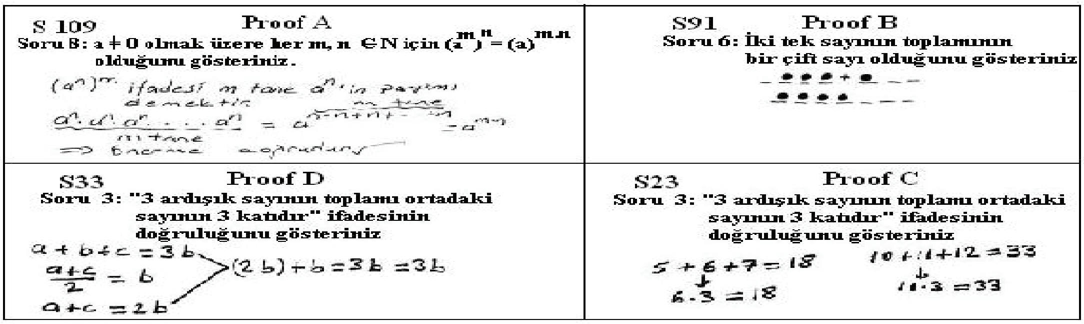
	Codes	f	%	Student Responses
Necessary	Facilitating comprehension	38	30.4	There's a need. Because mathematics deals with perceiving rather than memorizing and proofs facilitate the comprehension of the subject.
	Providing permanent learning	28	22.4	Proofs are needed for the better comprehension of the subject and permanent learning
	Enabling the realization of right and wrongs	36	28.8	There's a need. If no proof is used, we can not know whether our operations are correct or false.
	Improving the view of mathematics	3	2.4	Proof is necessary. Because doing proof improves one's perspective and facilitates life.
	Making mathematics meaningful	2	1.6	Necessary. Since illogical, merely memorized mathematics is meaningless and will be easily forgotten. In other words, mathematics has no meaning without proof.
	Increasing persuasiveness	1	0.8	There's a need. Because anything which we don't know its reason are not permanent and persuasive.
	Enabling the construction of new knowledge	1	0.8	Necessary. Because we can deliver new knowledge and formula to mathematics.
Not necessary	Proofs are not asked in examinations	2	1.6	Not necessary. We have an absolute confidence in the formula whose logic is understandable. Asking proofs in examinations is unnecessary. Since it's not included in Student Selection Examination for Higher Education .
	Trust in mathematical generalizations	4	3.2	In my opinion, it's not necessary. Indeed, these formulae have already been proven during their construction.
Blank		10	8	

The students' answers regarding the necessity of proof were found to gather under nine different codes. From these codes, the codes of *"Facilitating comprehension", "Enabling the realization of right and wrongs", and "Providing permanency"* were found the outweigh the other codes. These findings are in line with the results of the works by De Villiers (1990) and Weber (2002), although slightly. Furthermore, some students who are unaware of the cognitive benefits proofs would provide were found to justify their views by the fact that proof is not used in central examinations such as Student Selection Examination for Higher Education and some stated that they have confidence in mathematical generalizations since these formulae and generalizations are established by doing proof.

Students' responses to the questions asked to determine the students levels of proofs were analyzed based on the classification of Miyazaki regarding mathematical proof and their proof levels were presented in Table 4.

Table 4. The frequency and percentages of the proof levels of the students as regards the questions and a sample student response

Items	Proof A		Proof B		Proof C		Proof D	
	f	%	f	%	f	%	f	%
Item 3	75	60	0	0	49	39.2	1	0.8
Item 4	75	60	0	0	48	38.4	2	1.6
Item 5	54	43.2	1	0.8	70	56	0	0
Item 6	51	40.8	0	0	72	57.6	2	1.6
Item 7	61	48.8	0	0	61	48.8	3	2.4
Item 8	34	27.2	0	0	84	67.2	7	5.6
Total	350	46.7	1	0.1	384	51.2	15	2.0



When we look at the proof done in order to show the trueness of the 3rd, 4th, 5th, 6th, 7th and 8th questions, it can be seen that 46.7% of the students did proofs in type Proof A which requires the use of variables to show the trueness of the expression, whereas 51.2% of the students did proofs of type Proof C which requires the use of induction method and giving numerical values to show the trueness of an expression. This finding doesn't align with the results of the study by Özer and Arıkan (2002). Because in this study, almost half of the students showed the trueness of the given expression by using algebraic or functional language, and almost all of the remaining students tried to show the trueness of the expression by giving numerical values.

4. Conclusion and Recommendation

This study was conducted to determine the views of high school students about proof and their levels of doing proofs. Based on the results of this study, the conclusions and recommendations were presented as follows:

Furthermore, it was determined that the students' definitions of proof were similar to those reported in the literature. This shows that the students participated in this study have sufficient level of views about the definition of proof. However, teachers are recommended to take into account the potential doing proof has in the learning and teaching of mathematics and organize the activities in their classes accordingly since some students do not have a sufficient level of view about proof.

When we examine the views of the students related to the necessity of doing proof, it can be said that the majority of the students believe in the necessity of doing proof but they live some difficulties in practice. For persuading the students who think that doing proof is not necessary about the necessity of it and minimizing the relevant problems, the following recommendations were made: The task of informing students about how doing proof will contribute to them in improving their mathematical and logical thinking skills, and understanding the relationships between concepts certainly belongs to teachers. Teachers may perform this duty by providing the students with opportunities to express their standpoints on the subjects, discuss and by encouraging them to do proof. Students want to learn the trueness of a proposition with its reasons. If we teach mathematics with descriptive proofs, we may enable them to understand and enjoy mathematics better. If the processes related to the instruction of the subject are implemented properly, we may bring about individuals who learn the concepts with underlying reasons think creatively and may provide different solutions to the problems (Altıparmak & Öziş, 2005).

According to the findings derived from the questions for determining the levels of proof of students, nearly half of the students were found to do proofs at the level of Proof A which is described as the most advantageous level of proof in Miyazaki classification and the other half were found to attempt to justify the expression by giving numerical values and do proofs at the level of Proof C which is described as the least advantageous level of proof in Miyazaki classification. This demonstrates that the participating students could not use proof methods and techniques sufficiently and could not do proof at an expected level. Since the students can not use proof methods and techniques sufficiently and have difficulty in using mathematical language, it's recommended to develop and use in classes, activities that will improve students' mathematical thinking skills and increase their levels of doing proofs

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