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Comparing the old and new 6th-8th grades mathematics curricula in terms of Van Hiele understanding levels for geometry

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Abstract

This study was conducted with the aim of comparing the behaviors and attainments related to the plane geometry in the old and new 6th-8th grades mathematics curricula (MC) in terms of Van Hiele geometry understanding levels. With this purpose document analysis method was used. In the study, the levels of behaviors and attainments in the old and new 6th-8th grades MC were determined according to Van Hiele theory. As a result of the study, it was found that although the number of attainments in the old mathematics curriculum related to plane geometry decreased in the new one, the percentage of levels of some of the topics was increased. Moreover it was found that both the old and new MC involve higher order behaviors and attainments for 6th grade such as deductive reasoning.

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1. Introduction

Geometry is the branch of mathematics, describing the point, line, plane, plane and space shapes, the relationships between these shapes, the measures of geometrical shapes such as length, angle, area and volume (Dursun & Çoban, 2006). The aim of the geometry is learning the properties of the geometrical shapes in plane and space, finding the relations between them, describing geometrical position, explaining transformation and proving geometrical arguments. Students start to see, know and understand the physical world around them from small ages and in the following years they continue their education with higher levels of geometrical thinking that is supposed to develop inductively and deductively in later years. Moreover they can analyze problems, solve them and build a relation between mathematics and life. In fact, the solutions to many everyday problems the people face require basic geometry skills (Hızarcı, 2003). For this reason, geometry education occupies a prominent place in primary

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education. Geometry may be a hard to understand subject because it is constructed on abstract structures. Since these abstract structures do not address students' lives directly, this brings learning difficulties (Mullis et al., 2000; cited by Durmuş, Toluk & Olkun, 2000). In order to minimize these difficulties, geometry lessons in primary and secondary education should be presented compatible with the understanding levels of the students.

The researches conducted to understand geometry were usually built upon Van Hiele geometry understanding levels (Burger & Shaughnessy, 1986; Fuys, 1985). These levels show the approaches and understanding levels for geometry (Baki, 2006). Because transition from one level to other depends on the quality of the education and the subject matter, an educational approach driving students to discovery, critical thinking and discussion will promote the development of students in these levels and will enable the rapid transition to higher levels. If Van Hiele geometry understanding test is conducted to teachers before instruction and their geometry understanding levels are determined, teachers may consider these levels as well as the attainments of the curriculum during their instruction and will be successful in leading the students to higher levels (Yılmaz, Turgut & Kabakçı, 2008). Besides, knowing these levels will help teachers organize their instructions (Altun, 2006).

Van Hiele geometry understanding levels

Van Hiele theory was developed by Pier M. Van Hiele and Dina Van Hiele-Gelfod in Utrecht University during doctorate studies (Olkun & Toluk, 2003). According to Van Hiele theory there are five phases of geometrical understanding (Güven, 2006). These are;

Level 0: Visualization level

A student in this level deals only with the image of the shape given. Cannot distinguish geometrical properties of the shape and perceives the shapes as a whole. The students define, name and compare the shapes with their appearances. The characteristics of the visualization Fuys, Geddes and Tiskler (1988) described are as follows (To gain more detailed knowledge about this and the other levels, the Ph.D. thesis of Güven (2006) can be referred):

- Can identify a shape as a whole and define it verbally according to its appearance.
- Can construct, draw and copy a shape.
- Can name geometrical shapes with standard or non standard names.
- Can solve problems that do not highlight the properties of the shape.
- Knows the parts of a shape but cannot analyze the shape according to these parts.

Level 1: Analysis level

A student in this level distinguishes the properties of the shape but properties are perceived independently. A student may list the properties of a geometrical shape but cannot relate these properties with each other. Some of the properties of analysis level determined by Fuys, Geddes and Tiskler (1988) are as follows:

- Recognize and can test the relations between the parts of the shape.
- Can determine the properties of the shapes experimentally and can generalize the properties in a shape class.
- Can solve geometry problems by using the known properties of a shape.
- Uses the properties of shapes and formulizes them.
- Cannot formulate and use formal definitions.

Level 2: Informal deduction level

In this level, the student starts to relate the properties with each other. Definitions, axioms, postulates are meaningful for the student but deductions are not understood yet. In this level, students may follow a proof step by step by they cannot do it themselves. Some of the properties of informal deduction level determined by Fuys, Geddes and Tiskler (1988) are as follows:

- Can determine the minimum number of properties to define a geometrical shape.
- May follow a proof and can give recommendations about the steps.

- May give multiple explanations for a proof and tries to confirm these using diagrams.
- Cannot distinguish the difference between a statement and its inverse.

Level 3: Deduction level

A student in this level can order the relations. Moreover, he can use theorems, axioms and definitions in making geometrical proofs. He can determine if and only if conditions and can use these in a proof. In this level different theorems can be proven by using previously proven theorems and axioms. Some of the properties of deduction level determined by Fuys, Geddes and Tiskler (1988) are as follows:

- Understands the necessity of undefined terms, definitions and postulates.
- Can find the relation between a theorem and its inverse and prove both.
- Can compare different proofs of a theorem and describe the differences.
- Can determine if and only if conditions of a formal definition or can describe the equivalent of a definition.

Level 4: Rigor

A student in this level can interpret and apply the axioms, theorems and definitions of Euclidean geometry in non-Euclidean geometries. He can recognize the similarities and differences of different axiomatic systems. Some of the properties of rigor level determined by Fuys, Geddes and Tiskler (1988) are as follows:

- Can compare axiomatic systems. (Like Euclidean and non-Euclidean geometries)
- Can understand the independence of an axiom, and equality with another axiom.
- Searches an area where a mathematical theorem or principle can be used.
- Can produce theorems in different axiomatic systems.

Considering the cognitive development levels of Piaget, Olkun and Toluk-Uçar (2006) stated that the grades 1, 2 and 3 are in visualization level, the grades 4, 5 and 6 are in analysis level, the grades 7, 8 and 9 are in informal deduction level and grades 10, 11 and 12 are in deduction level in the development of geometrical thinking. The levels of some grades can be shown differently in literature. Since the researchers find the classification of Olkun and Toluk-Uçar more appropriate, it is more common. From this classification, it is expected that the behaviors and attainments in 6th grade in the old and new MC should concentrate on analysis level and the behaviors and attainments in the 7th and 8th grades should concentrate on informal deduction level. In this context, determining the levels on which the behaviors and attainments of the 6th-8th grades MC and comparing these levels with Van Hiele geometrical understanding levels are important.

1.1. The aim of the study

This study aims to compare the behaviors and attainments related with plane geometry in the old and new 6th-8th grades MC in terms of Van Hiele geometry understanding levels.

2. Method

A document analysis method was used in this study. Document analysis is an examination in which related records and documents are gathered and are coded according to a norm and system (Çepni, 2007). In this study the behaviors and attainments in the old and new 6th-8th grades MC were examined and these behaviors and attainments were classified considering the properties stated Fuys, Geddes and Tiskler (1988). For example in the old 6th grade mathematics curriculum, the behavior of "Naming a line segment and reading and writing with symbols" was grouped in the visualization level since there's a property such as "Can name the objects with standard and non standard names" property. The same processes were followed for the other behaviors and attainments. Later the levels determined by two researchers for the behaviors and attainments were compared. The reliability was calculated with the formula $\text{Reliability} = \frac{\text{Consensus}}{\text{Consensus} + \text{Dissidence}}$ and reliability was found a high value as .89 (Miles & Huberman, 1994). Later the behaviors and attainments placed in different levels by the researchers were discussed and these behaviors and attainments were placed in more suitable levels by taking expert support.

3. Findings and interpretation

3.1. Findings related to the behaviors and attainments in 6th grade MC

In Table 1 the behaviors and attainments related with plane geometry in the old and new 6th grade MC were compared in terms of Van Hiele geometry understanding levels.

Table 1. The Van Hiele geometry understanding levels of behaviors and attainments in the old and new 6th grade MC

Van Hiele Geometry Understanding Levels	Old 6 th Grade MC		New 6 th Grade MC	
	Number of Behavior	Percentage (%)	Number of Attainment	Percentage (%)
Visualization Level	16	24	2	9
Analysis Level	37	56	11	50
Informal Deduction Level	13	20	9	41
Deduction Level	0	0	0	0
Rigor Level	0	0	0	0
Total	66	100	22	100

When we look at Table 1, we can conclude that the number of behaviors related with plane geometry in visualization, analysis and informal deduction level in the old 6th grade mathematics curriculum is double the number of the attainments in the new curriculum and in both MC there are no behaviors or attainments in the deduction and rigor levels. Furthermore, it is shown in Table 1 that the percentages of the visualization and analysis levels decreased in the new mathematics curriculum, though the percentage of deduction level increased. This situation was interpreted by the researchers as a student in 6th grade is in analysis level and in transition to 7th grade his level shifts to informal deduction level so the visualization level is decreased and the informal deduction level is increased in the transition from 6th to 7th grade to enable the students to think more complexly.

3.2. Findings related to the behaviors and attainments in 7th grade MC

In Table 2 the behaviors and attainments related with plane geometry in the old and new 7th grade MC were compared in terms of Van Hiele geometry understanding levels.

Table 2. The Van Hiele geometry understanding levels of behaviors and attainments in the old and new 7th grade MC

Van Hiele Geometry Understanding Levels	Old 7 th Grade MC		New 7 th Grade MC	
	Number of Behavior	Percentage (%)	Number of Attainment	Percentage (%)
Visualization Level	10	8	0	0
Analysis Level	48	38	17	46
Informal Deduction Level	68	54	20	54
Deduction Level	0	0	0	0
Rigor Level	0	0	0	0
Total	126	100	37	100

When we look at Table 2, we can conclude that the number of behaviors related with plane geometry in visualization, analysis and informal deduction level in the old 7th grade mathematics curriculum is more than the number of the attainments in the new curriculum and in both MC there are no behaviors or attainments in the deduction and rigor levels. Furthermore, it is shown in Table 2 that the percentage of the visualization level decreased in the new mathematics curriculum, though the percentage of analysis level increased and the percentage of the informal deduction level did not change. Because the researchers regard a student in 7th grade as in informal deduction level, the decrease in the percentage of visualization level is found positive.

3.3. Findings related to the behaviors and attainments in 8th grade MC

In Table 3 the behaviors and attainments related with plane geometry in the old and new 8th grade MC were compared in terms of Van Hiele geometry understanding levels.

Table 3. The Van Hiele geometry understanding levels of behaviors and attainments in the old and new 8th grade MC

Van Hiele Geometry Understanding Levels	Old 8 th Grade MC		New 8 th Grade MC	
	Number of Behavior	Percentage (%)	Number of Attainment	Percentage (%)
Visualization Level	0	0	0	0
Analysis Level	11	30	4	29
Informal Deduction Level	26	70	10	71
Deduction Level	0	0	0	0
Rigor Level	0	0	0	0
Total	37	100	14	100

When we look at Table 3, we can conclude that the number of behaviors related with plane geometry in analysis and informal deduction level in the old 8th grade mathematics curriculum is more than the number of the attainments in the new curriculum and in both MC there are no behaviors or attainments in the visualization, deduction and rigor levels. Furthermore, it is shown in Table 3 that the percentage of the visualization level decreased in the new mathematics curriculum, though the percentage of analysis level increased and the percentage of the informal deduction level did not change. Since the researchers assume that a student in 8th grade is in informal deduction level, the uniformity in the percentage of visualization level and the increase in the analysis and informal deduction level in the new mathematics curriculum are perceived as positive.

4. Results and recommendation

The following are the findings of the study:

1) Considering that a student in 6th grade is in analysis level, it was found that both the old and new 6th grade MC involve too high attainments and behaviors such as deduction.

2) It was found that the percentage of informal deduction in the new 7th grade mathematics curriculum did not change.

3) The percentages of the analysis and informal deduction levels did not change in the new 8th grade mathematics curriculum.

4) When we consider that the students in 7th and 8th grades are in informal deduction level, it can be interpreted that both the old and new MC in these levels do not involve too high behaviors and attainments.

Regarding these findings the following are recommended:

1) When we consider a student in the 6th grade as in analysis level, we can recommend an increase in the number of attainments in analysis level in 6th grade.

2) Considering the students in 7th and 8th grades as in informal deduction level, a recommendation may be to increase the attainments in the deductive level in the new 7th and 8th grades MC.

3) To prepare the 8th graders to 9th grade and the 7th graders to 8th grade, attainments in deductive level may be added to the new 7th and 8th grades MC.

4) Similar studies may be performed for the behaviors and attainment in the 1st-5th grades and 9th-12th grades.

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