

## Improving Elementary 8<sup>th</sup> Class Students' Thoughts about Nature of Mathematical Knowledge

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**Abstract:** In this study, improving elementary 8<sup>th</sup> class students' thoughts about nature of mathematical knowledge was aimed. Case study research method was used in the study. 25 students at 8<sup>th</sup> grade at a public elementary school in Trabzon were participated in the study in 2010-2011 school year. Four worksheets have been prepared on the nature of mathematical knowledge. The semi structured interviews were made with six students just before the application and one month later. Collected data were interpreted with content analysis. In the study, it is determined that students learnt the word meaning of mathematics and the place where it appeared first. Also it is seen that students noticed mathematics came in a changing and developing state from past to now, mathematical symbols and figures used in different ways. That's why it is advised to develop and implement different activities which can develop thoughts of students about nature of mathematical knowledge.

*Keywords:* Mathematics education, history of mathematics, students' views, nature of mathematical knowledge.

Students have opinions and beliefs which are shaped by their school years (Aydın, 2010). An individual's beliefs are understood as his subjective, experience based, often implicit knowledge and emotions on some matter or state of art (Pehkonen & Pietila, 2010). Toluk Uçar, Pişkin, Akkaş and Taşçı (2010) stated that many negative views, such as mathematics is consist of only calculations and it can only be done by intelligent people, are shared among people and children are affected by this kind of negative views. This kind of negative views leads to a set of prejudices in students. The greatest obstacle of students in learning mathematics can be said as bias and looking at it as a notional, difficult and meaningless science (Taşkın, Yıldız & Arslan, 2010). Duatepe Paksu (2008) describes mathematical beliefs as a person's view of the world of mathematics, that is perception of approach to mathematics and mathematical studies. According to Schommer (1993), personal beliefs about nature of knowledge and learning are called as epistemological beliefs.

Epistemological beliefs deal with the nature of knowledge and the processes of knowing and have been seen as a significant factor in college students' academic performance (Hofer, 1999; Schoenfeld, 1992; Whitemire, 2004). Aydın (2010) describes epistemological beliefs as more basic assumptions about borders, definitiveness and criteria of knowing and he indicated that meanings such as 'opinions, views, subjective beliefs, insights, perspective, attitude, cognitive restructuring ... etc.' about nature of knowledge and learning can be assigned to epistemological beliefs. Looking from this point, it is seen that epistemological beliefs have a fundamental role in interpretation of knowledge and cognitive imaging (Pajares, 1992). The analysis of this, beliefs about nature of mathematics have an important place in epistemological beliefs.

One of the ways to endear mathematics to the students that can introduce the nature of mathematics to them, shows stages mathematics passed through to the present, offer how to use it in

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daily life, makes mathematics less fearful is the History of Mathematics (HM) (Baki, 2008; Fauvel, 1991; Lingard, 2000). Without doubt, relating mathematics teaching with historical events and daily life helps student develop a positive attitude towards mathematics (Baki, 2008). With addition of HM into education, students will see HM as a bond connecting several fields of mathematics to each other, will have an experience about presentation of mathematics, and will see mathematics as a human interference (Ofir, 1991; Reimer & Reimer, 1995). Also, in mathematics lessons enriched by HM students will see that mathematics is a science always renewing itself and developing, has a cultural dimension and how it gives direction to our world of ideas and forms it (Baki, 2008).

Teachers and students generally think mathematics is formed by only numbers and operations, the aim of problem solving is finding the correct answer and learning mathematics passes through memorizing (Kayaaslan, 2006; Picker & Berry 2000; Rock & Shaw, 2000). Shortly many negative views such as mathematics is only involved of operations and only clever people can do mathematics are shared by people. As a part of the community students are effected by these negative views (Toluk Uçar et al., 2010). If students are wanted to see mathematics as an attractive field and learn mathematics as doing mathematics himself like a mathematician, students' views about nature of mathematical knowledge must be understood deeply (Rock & Shaw, 2000). So, better ways to improve or change these beliefs can be determined, constituting an obstacle to doing mathematics and therefore learning can be prevented (Toluk Uçar et al., 2010). If nature of mathematical knowledge can be taught to students correctly, students will want to be engaged with it and will want to spend more efforts to put out new things (Aydın, 2010). Few researches about views of elementary students about nature of mathematical knowledge were found in Turkey (Hatisaru, 2009; Toluk Uçar et al., 2010). However, the number of studies about improving nature of mathematics are scarcely any. Students' views about nature if mathematics and mathematical knowledge is considered to be highly effective on students' learning, attitudes and beliefs toward mathematics; it can be said that the number of researches about developing their beliefs towards nature of mathematics should be increased. For these reasons, improving elementary 8<sup>th</sup> class students' view about nature of mathematical knowledge with the help of HM was aimed in this study.

### Method

Case study research method was used in this study. Case studies allow investigate one or more situations, phenomenon or events with samples in limited number deeply (Çepni, 2007).

**Participants.** The study was conducted with 25 students at 8<sup>th</sup> grade at a public elementary school in Trabzon during the spring term of 2010-2011 school year.

**Data collection tools.** The data for this study was collected through semi structured interviews conducted with students. First, four worksheets have been prepared on the nature of mathematical knowledge. The pilot implementation of these worksheets was distributed to 30 students studying the 8<sup>th</sup> grade in another primary school located in the center of Trabzon. Upon the pilot implementation, the views of teachers and experts were extracted and 4 worksheets were finalized. The collection of data lasted for eight class hour. Worksheets were delivered to students and the activities were carried out in pair. Class discussion was hold at the end of each activity. A pre-interview was done with students in this study to determine their prior knowledge before the main application. Also, another interview was done with the same students to examine the effect of the activities and materials developed and to determine the changes in students' ideas. In these interviews, a total of four questions were asked to the students about nature of mathematical knowledge. Six of students

interviewed were chosen among students with high, average and low mathematics grades. Ethically, these students were coded as “S1, S2, S3, S4, S5 and S6”.

**Data analysis.** Data collected via semi structured interviews, were interpreted with content analysis. In the analysis of the interviews, written transcripts were produced by listening to the recorded interviews. During this analysis, the researchers’ comments and the participants’ irrelevant replies were taken off instead of taking all the words individuals uttered. Then, rest of the information was analyzed according to their similarities and presented in tables. To be an example of codes, direct quotes about each code was given. Lastly, common codes formed at the end of arguments were presented in tables.

## Results

In this part, students’ answers to each question in the pre-interview and post-interview were given as tables.

Students’ answers to the question “According to you, what is mathematics?” are as in Table I.

**Table I**

*Ideas of students about mathematics*

| Given Answers  | Pre-Interviews | Post-Interviews |
|--|----------------|-----------------|
| Mathematics has an important role in our life            | S1, S4         | S1, S2          |
| Mathematics explains the secrets of nature               | S1, S2         | S1, S5          |
| Mathematics has an important role among other sciences   | S1, S2, S4, S5 | S2, S3, S4, S5  |
| Mathematics means the good knowledge that must be learnt | -              | S1, S2, S3, S4  |
| Mathematics is an enjoyable science                      | S2             | -               |
| Mathematics is a difficult science                       | S2             | -               |
| Mathematics is a science that must be learnt             | S2             | S2              |
| Mathematics is a science that requires intelligence      | -              | S2, S3, S5      |
| Mathematics is informative                               | S3             | -               |
| Mathematics develops human intelligence                  | S3             | -               |

It is generally seen that what students said on pre- and post-interviews were mostly about the importance of mathematics. During the interviews done after the practice, it is seen that views of students focused on verbal meaning of mathematics and that it is a science requires intelligence. For example, while student S2 indicate in pre-interview that mathematics is also enjoyable in addition to being difficult such as: “*Mathematics is included in each course. It is difficult but also enjoyable, too*”; he expressed mathematics being a science which should be learn, in post-interview by emphasizing verbal meaning of mathematics as follows, “*...I say what, it is a nice branch of science, to be learnt...*”

The answers of students to the question “How do you think that mathematics emerged?” are given in Table II.

**Table II**

*Ideas of students about the emergence of mathematics*

| Given Answers   | Pre-Interviews | Post-Interviews    |
|---|----------------|--------------------|
| Mathematics occurred because of daily life requirements       | S1, S2, S5     | S1, S2, S4, S5     |
| Mathematics occurred as a result of coincidences in life      | S1             | S1                 |
| Emergence of mathematics in ancient Egypt                     | -              | S1, S2, S3, S4, S5 |
| Emergence of mathematics as a result of curiosity             | -              | S2                 |
| Scientists found mathematics with their researches            | S3             | S3                 |
| Many societies contributed to the developments of mathematics | -              | S3                 |

However in pre-interviews it is seen that students don't have any ideas about where mathematics occurred in last interviews it is seen they put into words that mathematics occurred in Egypt. For example, while student S3 said in pre-interview about emergence of mathematics " *That is thing, ancient, there are those who write or something, so scientists had might removed by investigating.*" and S1 said "...*There isn't any example in my mind, but I know that is emerged by change in the life* "; in post-interview all of the students said that mathematics emerged in ancient Egypt. Student S3 stated the emergence of mathematics in post-interview that: "*Here, firstly appeared in Egypt.*"

The answers of students to the question "What do you think about the nature of mathematical knowledge?" are given in Table III.

**Table III**  
*Ideas of students about the nature of mathematical knowledge*

| Given Answers  | Pre-Interviews | Post-Interviews |
|--|----------------|-----------------|
| Mathematics is a definite science  | S1, S2, S4, S5 | S3, S5          |
| Mathematics has a developing structure   | S1, S2, S4, S5 | S1, S2, S4      |
| Always thinking of mathematics as a universal science                            | S1, S2, S3, S4 | S5              |
| Thinking that mathematical knowledge was not universal at past but universal now | -              | S1, S2, S4      |
| Arrival of mathematical knowledge in a changing state from past to present       | -              | S1, S2, S4      |
| Arrival of mathematical knowledge without changing up to now                     | S3             | S3              |
| Mathematics is a science concerned with numbers and operations                   | S2, S3, S4, S5 | S4              |
| Mathematical knowledge is correct  | S3, S4         | S2              |
| Mathematics depends on logic   | S4             | S4              |
| Mathematical knowledge is proved   | S4, S5         | -               |

Before the practice although most of the students were thinking that mathematics is formed of numbers and operations and it came up to now without any changes, it is seen that the number of these students is decreased after the practice. Also after the practice it is determined that most of the students think mathematics not to be universal in past but universal now. While student S1 indicated in pre-interview that he see mathematics as a definite science with the sentences of "...*that is, it has only one source, its all principles are absolute and indisputable...It is universal every time and it will remain always universal...*", it is seen that his views changed in post-interview like this: "*Correctness of our mathematical knowledge, now if you look at past to present, day by day, even so far shows quite change.*"

The answers of students to the question "What do you think about the numbers and symbols used in mathematics? Are the numbers and symbols universal?" are given in Table IV.

**Table IV**  
*Ideas of students about the mathematical symbols and figures*

| Given Answers  | Pre-Interviews | Post-Interviews |
|--|----------------|-----------------|
| Mathematical symbols and figures provide us to solve problems more easily                                  | S1             | -               |
| Mathematical symbols and figures provide us to understand problems better                                  | S1, S2         | -               |
| In different societies mathematical symbols and figures were used different because of language difference | S1, S4, S5     | S2, S3, S4, S5  |
| Usage of mathematical symbols and figures in same meaning among all societies                              | S1, S2, S3     | -               |

It is seen that most of the students noticed mathematical symbols and figures weren't used in same meaning in all societies. While student S4 opines his view related to symbols in pre-interview as: "*Same symbols don't change, numbers can change...*" in post-interview he said that: "*Symbols can vary from past to present, it will vary in the future too.*" It can be seen from this that, student S4's view about

symbols' development changed clearly. While he thought that symbols can't change but numbers can change because of the language difference in the pre-interview; he said that symbols had changed from past to present and it will change in the future.

### Discussion, Conclusion and recommendation

After the practice some students emphasized mathematics is in a form that requires intelligence. As far as the elementary mathematics curriculum was concerned, students learn mathematics by doing mathematics themselves and conscious learning, there is no need to be very clever to be successful in mathematics; in contrast, they can notice that mathematics is a thinking method, needs studying and there is a meaningful unity inside it (Toluk Uçar et al., 2010). Also, teachers mustn't show only the easier way of doing mathematics but also they must emphasize the parts of it that need studying to their students. Teachers must make their students feel that they faced difficulties in their student times themselves and even successful mathematicians may not be so good in all branches of mathematics (Picker & Berry, 2000). Most of the students participated in the study generally emphasized mathematics is an important, useful, enjoyable, meaningful, developing science used in daily life and has a relation with other sciences. We can understand that students are aware of mathematics' importance and its relation with daily life and other sciences. But it is seen that students don't have information about the verbal meaning of mathematics and place where it occurred before the practice. That's why on lessons at school giving information about the verbal meaning of mathematics and where it occurred must be given to students is advised.

It is seen that before the practice most of the students were thinking mathematics is formed by only numbers and operations, it came up to now without any changes, mathematical symbols and figures were used all in same meanings in all societies. This conclusion can be said to be consistent with Aydın (2010)'s study. Aydın (2010) determined in his study conducted by teachers that they see mathematical knowledge as distinct, precise, objective, infallible, can't be interpreted differently and undiscussable. The reason to get same results with that study can be that; students are affected by their teachers' view. So, if teachers have incorrect ideas, it will be inevitable students' thinking like this. So it can be said that, for students who don't have wrong views about nature of science, firstly teachers must be taught correct knowledge about nature of mathematics. For this reason, teacher candidates needed to be shown nature of mathematics in university years. Developing and using practices enriched with HM for the developing and changing nature of mathematics will be a good tool to reach this aim.

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