European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available on-line at: www.oapub.org/edu

doi: 10.5281/zenodo.1239849 Volume 4 │ Issue 6 │ 2018

EVALUATION OF THE TEACHING ENVIRONMENT FOR IMPROVE THE GEOMETRIC HABITS OF MIND OF TENTH GRADE STUDENTS[i]

Zeynep Bahar Erşen[1ii],
Rıdvan Ezentaş[2],
Murat Altun[3]
[1]Afyon Kocatepe University,
Faculty of Education,
Turkey
[2.3]Uludağ University,
Faculty of Education,
Turkey

Abstract:
Geometry is one of the branches of mathematics that we use in many areas of our daily life, perhaps without noticing. For this reason, individuals are geometric thinkers not only in geometry classes; but also in different areas of life. In that case, it is necessary for the individual to acquire geometric habits of mind. The purpose of this study was to introduce the effectiveness of a teaching environment designed for improving the geometric habits of mind of high school students. This research method was designed as a quasi-experimental design. The working group of the study was consisted of 62 students, 31 of which were experimental and 31 of which were control groups. While the experimental group was provided with a teaching environment for improving the geometric habits of mind, multiple choice questions were solved with the control group students. The research data were gathered by pre-test, post-test and permanence test problems developed by the researcher. In the result of the study, it was showed that the designed teaching environment is effective in improving the geometric habits of mind and the permanence of habits. That is, it was determined that there is a significant difference in improving of geometric habits of mind and the persistence of these habits in favor of the experimental group.

Keywords: habits of mind, geometric habits of mind, teaching environment

[i] This article has been produced from her doctoral dissertation titled "*The Design, Implementation and Evaluation of the Teaching Environment for Improve the Geometric Habits of Mind of Tenth Grade Students"*

[ii] **Correspondence e-mail**: zbahar.ozdogan@hotmail.com

1. Introduction

The habit of mind is a thinking attitude that affects the way an individual solves a problem (Costa & Kallick, 2000). Mathematical habit of mind, on the other hand, is a thinking attitude developed by an individual so as to solve an unusual mathematical problem (Goldenberg, 1996; Jacobbe & Millman, 2009). Mathematical habit of mind involves a problem and the strategies developed by the individual to solve the said problem. That is to say, problem-solving underlies the mathematical habit of mind. Indeed, in our current day, regardless of the country of origin, it is the main goal of the education programs to integrate students to the society as members with problem-solving abilities, who can masterfully overcome the problems they face in their daily life. That is because the process of problem-solving requires individuals to make use of multiple habits of mind such as finding a pattern, considering the exceptions, hypothesizing, generalizing, proving, identifying the variants and invariants, thinking critically, thinking creatively, not giving up, taking risks, thinking analytically (Costa, & Kallick, 2000; Driscoll et al., 2007; 2008).

There exist a large number of studies arguing that the mathematical habits of mind should be integrated into mathematics curriculum (Cuoco et al., 1996, Goldenberg, 1996; Hu, 2005; Jacobbe & Millman, 2009; Lim & Selden, 2009; Mark et al., 2010; Marshall, 2004; Seeley, 2014). The results of these studies showed the characteristics of the individuals that make use of the mathematical habits of mind. Even though the characteristics were worded differently in different studies, the main argument stands clear: the individuals should not only know about the mathematical definitions, theorems, algorithms but also be able to use thinking habits similar to those of a mathematician when faced with an unfamiliar mathematical problem. The cognitive and emotional effects of the mathematical habits of mind should also be noted. Some of the mathematical habits of mind regarding cognition can be expressed as seeking patterns, hypothesizing, predicting, sampling, finding alternative solutions, visualizing, reflective thinking, thinking about thinking (metacognition) (Cuoco et al. 1996; Goldenberg, Shteingold & Feurzeig, 2003; Jacobbe & Millman, 2009; Levasseur & Cuoco, 2003; Marshall, 2004; Mazano, Pickering & McTighe, 1993). Mathematical habits of mind regarding emotion can be expressed as not giving up, being determined, showing empathy, being curious, flexibility, being open to learn, doubting, self-discipline (Costa & Kallick, 2000; Leikin, 2007). Apart from the cognitive and emotional aspects of mathematical habits of mind, there exist a number of other more specialized aspects in the literature such as algebraic, geometric, trigonometric, statistical and stochastic (Goldenberg, 1996; Leikin, 2007; Mark et al., 2010). Geometric habits of mind have also been studied in this research.

The concept of geometric habits of mind was coined by Goldenberg (1996), who defined the habits of the individuals possessing such qualities in his project named "Connected Geometry": visualizing, interpreting geometric shapes, defining formally or informally, interchanging visual and verbal information, drawing a conclusion from trials, investigating invariants, deducing, generalizing, reasoning about the algorithms and creating an algorithm, being able to think of the geometric shapes dynamically. Cuoco et al. (1996) highlighted in their research the importance of the interest in working on different geometric systems and proportional reasoning skill, apart from the aforementioned qualities. Following these studies, Driscoll et al. (2007) put forward a research that covered the geometric habits of mind in the most detailed manner, combining the results from both studies. The researchers have analyzed the solutions to the geometric problems asked to the students attending 5th to 10th grade and discussed the solutions with the students. Then, they divided the necessary geometric habits of mind into four different categories: reasoning with relationships, habit of generalizing geometric ideas, habit of investigating invariants, balancing exploration and reflection. Therefore, the theoretical structure of this study is based on this categorization Driscoll et al. (2007) put forward in their paper.

While Driscoll et al. (2007) was doing research about the geometric habits of mind of the students attending primary and secondary school, studies were conducted in Turkey, focusing on the identification and enhancement of geometric habits of mind of the teachers or prospective teachers (Bülbül, 2016; Özen, 2015; Yavuzsoy-Köse & Tanışlı, 2014). Nevertheless, the lack of research on the subject of identifying and enhancing the geometric habits of mind on the future members of our society, the youth, is considered a grave deficiency in the literature in Turkey. Thus, the main purpose of this study is to find an answer to the question: "What are the effects of the designed learning environment on the geometric learning habits of the students attending the 10th grade?" To find a solution to this main problem, following sub-problems was been investigated:

1. Is there a significant difference between the experimental and control group in terms of geometric habits of mind?
    a. Is there a significant difference between the pre-test and post-test average scores of the students in the experimental group?
    b. Is there a significant difference between the pre-test and post-test average scores of the students in the control group?
2. Is there a significant difference between the experimental and control group students in terms of the average scores of the test, the permanence of geometric habits of mind?
    a. Is there a significant difference between the average scoring of geometric habits of mind post-test and permanence test in the experimental group?
    b. Is there a significant difference between the average scoring of geometric habits of mind post-test and permanence test in the control group?

1.1 Geometric Habits of Mind

Geometric habit of mind can be defined as the repertoire an individual possesses in order to solve a geometric problem when faced with one. The term was coined by Goldenberg (1996) in his study titled "Habits of Mind: As an Organizer for the Curriculum". Even though the characteristics of the individuals possessing the geometric habits of mind have been presented in this study, the most extensive research on the subject has been conducted by Driscoll et al. (2007) following a project titled "Fostering Geometric Thinking: A Guide for Teachers, Grades 5-10". According to the researchers, individuals possessing geometric habits of mind have four main habits. These habits are namely: reasoning with relationships, generalizing geometric ideas, investigating invariants, balancing exploration and reflection. The definition of these habits and the general characteristics of the individuals possessing such habits have been explained in the following section.

1.1.1 Reasoning with Relationships

Reasoning with relationships suggests seeking relationships between one, two or three-dimensional geometric shapes (such as congruence, similarity, parallelism, etc.) and being able to reason how to use these relationships in the problem-solving process (Driscoll et al., 2007). Individuals with this reasoning can identify the common/similar or non-similar features between two or more geometric shapes. They can reveal the similarities or differences between these shapes with relevant justifications. They can locate or create geometric sub-shapes within a given geometric shape. They can use symmetry to reason with the geometric shapes. They can also use proportional reasoning to reason with two or more geometric shapes (Driscoll et al., 2008). Proportional reasoning refers to the ability to multiplicatively compare congruent or different measurement spaces and to express this concept mathematically (Clark & Lesh, 2003). Individuals possessing this reasoning ask these questions to themselves in the process of solving a geometric problem:

- How are the given geometric shapes similar to each other?
- How many different ways are there to express the similarity between the geometric shapes?
- What are the different aspects of the geometric shapes?
- Which other shapes comply with the given definition?
- What should I do to the given shape so that it becomes similar to the other one?
- What happens if we look at the relationship between the shapes from another angle?

1.1.2 Generalizing Geometric Ideas

Generalization, around which the school mathematics curriculum revolves, is one of the main goals of the mathematics education (NCTM, 2000; Polya, 1954). It is the process of verifying that the given problem is exceptional and then using this instance to create a generalized rule (Cuoco et al., 1996; Goldenberg, 1996). The process consists of several elements, namely, predicting the "many", "every" or "specific" case, checking if the prediction is correct, drawing a conclusion using the prediction and being able to discuss the results (Driscoll et al, 2008). As to the generalization of the geometric habits of mind, it is about defining and understanding the "general" and "every" case arising from the concept of geometric notion (Driscoll et al., 2007). Individuals with this habit can take notice of the exceptional cases, can experiment with other cases than the exceptional ones and then make generalizations for these new cases. They can see the whole solution set and explain why there is no other solution. They can propose a universal rule for a set of geometric shapes. In a broader sense, they can make inferences about the problem or rules (Driscoll et al., 2008). In addition to these, individuals possessing this habit ask these questions to themselves in the process of solving a geometric problem:

- Does this always happen?
- Why does this always happen?
- Can I find every instance that fits this definition?
- Can I find the cases where this does not happen and if so, can I reformulate my generalization?
- Does this happen for other aspects?

1.1.3 Investigating Invariants

Another characteristic of the individuals with geometric habits of mind is the tendency to investigate invariants (Cuoco et al., 1996; Driscoll et al., 2007; Goldenberg, 1996). Identifying the constants and invariants is one of the most important parts of mathematical research (Leikin, 2007). Invariance, in geometry, refers to the cases/characteristics that stay the same even though the other parts may undergo some changes in a geometric shape. This geometric habit of mind shows which characteristics change and which do not after a geometric shape undergoes some kind of transformation (i.e. reflection, parallel displacement, disintegration, enlarging the shapes, controlled deformation, etc.) (Driscoll et al., 2007) Individuals with this habit can think dynamically when faced with a static instance. They feel curious to find out which characteristics change, and which ones stay the same when a transformation is applied. These people can notice the said characteristics and explain why they changed or stayed the same. They make a prediction on what the effects would be if a point or a shape were to be moved. They take the limited and extreme cases in the transformation process into account (Driscoll et al., 2008). Moreover, individuals with the habit of investigating invariants would ask themselves these questions in the process of solving a geometric problem (Driscoll et al., 2007):

- Which transformations are needed to achieve this view of the shape?
- Is it possible to transform this shape into this other shape using different transformations?
- What has changed? Why?
- What hasn't changed? Why?
- If I apply the same geometric transformation over and over again, what would happen to the given geometric shape?

1.1.4 Balancing Exploration and Reflection

Exploration is reaching a conclusion by adopting various strategies to solve a geometrical problem and reflection is being aware of everything the individual is doing during this process and questioning them. The balance between the questions of "What would happen if I did this?" and "What did I learn by doing this?" is the telltale sign of this habit of mind (Driscoll et al., 2007). Individuals with this habit can draw via prediction and intuition, play with the shape or explore the shape. They can consider the previous, similar cases. They can dwell upon how some of the characteristics of a case, condition or a geometric shape change. They question themselves in every step of the solution process regarding the result. These people can masterfully identify the intermediate steps leading to the solution. They can speculate about the possible outcomes and use creative ways to test their predictions about the results (Driscoll et al., 2008). Moreover, individuals with this habit would ask themselves these questions in the process of solving a geometric problem (Driscoll et al., 2007):

- What would happen if I drew a shape and then added/removed a part of it or used the "backward induction" method?
- What do the things I've done tell me?
- How can my background information help me in solving this problem?
- Which intermediate steps might make it easier for me to reach the results?
- What might be the outcome that I'm thinking of achieving?

2. Methodology

As we were investigating the effects of the learning environment on the geometric habits of mind of the students, we used the quasi-experimental method. This method of research, which is highly used in studies on education, uses a hybrid method where the experimental and control groups are not randomized but the process involves a scientific approach (Campell & Stanley, 1963; Cohen & Manion, 2007).

2.1 Research Group

The selection of the participating students is highly important in finding out and presenting which type of geometric habit of mind a student is using while solving a geometric problem. Since unusual problems were selected for the research, we wanted the students to have a certain level of success. Therefore, we adopted a purposive sampling method, selecting two 10th grade classes from a science high school, located in Afyon. The selection reason for the 10th graders was not only that they had prior knowledge about triangles, quadriaterals, and circles but also that they did not have a nation-wide exam in the near future, hence, not having an exam anxiety.

The experimental group consisted of 17 female and 14 male students while the control group consisted of 13 female and 18 male students, 31 students in total in each group. Both of these groups were taught mathematics by the same teacher and had similar academical success levels. The independent samples t-test results (Table 1) also showed no significant difference between the two groups regarding their average pre-test scores for geometric habits of mind ($t_{(60)}$= 1.449, p > .05).

Table 1: T-Test Findings on Average Pre-Test Scores of Experimental and Control Groups

Test	Groups	N	$\overline{X}$	Ss	Sd	t	p
Pre-test	Experimental	31	16,645	4,667	60	1,449	,153
	Control	31	18,290	4,268			

2.2 Research Design

After identifying the research aims and problems, we established the problems for the 10 graders to enhance their geometric habits of mind in the learning environment. The reason for this is that problem-solving is the main foundation for mathematical habits of mind (Driscoll et al., 2007; Jacobbe & Millmann, 2009). Thus, geometric problems should be the base in the enhancement of geometric habits of thinking. It is aimed in this study to identify and enhance the geometric habits of mind of the students using geometric problems. As noted before, the problems were selected in such a manner that the students would not be able to solve them too easily and so that they were unfamiliar to the students. Otherwise, the geometric habits of mind that the students had already possessed might have surfaced during the study (Cuoco, Goldenberg and Mark; 2010; Driscoll et al., 2007; Leikin, 2007). The researcher guided the students through the solution process, gave directives, encouraged them to use geometric habits of mind so as to reach the solution. Another point taken into account while designing the learning environment is the use of geometric software in the process of investigating the invariants. Studies have proven that individuals should use the software in order to imagine geometric figures as dynamic (Cuoco et al., 1996; Goldenberg, 1996; Leikin, 2007). Hence, a program named GeoGebra was used for the parts that required the usage of the habit of investigating invariants. Another point that was considered while designing the learning environment was to create a space for the students where they can discuss the problems and put forward new ideas. Mathematical communication is one of the elements in process standards in National Council of Teachers of Mathematics (NCTM) (2000). Indeed, one of the competencies and skills that the students are taught in the mathematical education is mathematical communication (MEB, 2013). Finally, students are expected to be aware of the habits of mind they already possess in the enhancement process of geometric habits of mind (Costa & Kallick, 2000). Therefore, whenever a problem was solved by the students, either the researchers or the students explained which habits of mind were used in the problem-solving process and why.

After identifying the necessary characteristics of the learning environment, we consulted three expert mathematical educators and the mathematics teacher of the students about which subjects should be covered and which problems should be selected and then prepared activities. Following the pilot study, the definitive form of the activities and tests were decided. You can see the subjects of the activities and which geometric habits of mind were used in them in the table.

It is evident in the table 2 that the implementation process took a total of 15 weeks. During the first week, the students were informed about the study. During the second, ninth and fifteenth weeks, geometric habits of mind tests were performed. The activities took place during the rest of the weeks. Problem-based learning (PBL) model was adopted during the activity weeks. This model consists of several steps, namely, presenting the problem, investigating the problem, explaining the problem-solving process and discussion (Karataş, 2008).

Table 2: Implementation process

Weeks	Activity Number	Activity Subject	Geometric Habits of Mind to be Improved	Duration
1		Informing about the research and explaining geometric habits of mind		120 mn
2		Implementation of Pre-test		55 mn
3	1. Activity	Angles in triangles	RwR-BER	80 mn
	2. Activity	Triangle disequilibrium	RwR-GGI-BER	
4	3. Activity	Equality of triangles	RwR-GGI-II	80 mn
	4. Activity	Similarity of triangles	RwR-II-BER	
	5. Activity	Bisector	RwR-BER	
5	6. Activity	Median	RwR-BER	80 mn
	7. Activity	Right triangles	RwR-BER	
	8. Activity	Pythagorean-Euclidean relation	RwR-BER	
6	9. Activity	Rectangle	RwR-II	80 mn
	10. Activity	Parallelogram	RwR-II-BER	
	11. Activity	Rectangle	RwR-BER	
7	12. Activity	Square	RwR-GGI-II-BER	80 mn
	13. Activity	Rhombus	RwR-BER	
	14. Activity	Trapezoid	RwR-GGI-II-BER	
8	15. Activity	Length in circles	RwR-BER	80 mn
	16. Activity	Length in circles	RwR-II-BER	
9		Implementation of Post-test		55 mn
15	(After 5 weeks)	Implementation of Permanence-test		55 mn

*RwR: Reasoning with Relationships
*GGI: Generalizing Geometric Ideas
*II: Investigating Invariants
*BER: Balancing Exploration and Reflection

2.3 Data Collection Tools

To find out the effect of the designed learning environment on the students' geometric habits of mind, three tests were administered before and after the implementation process. These tests are, namely, GHMPrT (Geometric Habits of Mind Pre-Test), GHMPoT (Geometric Habits of Mind Post-Test) and GHMPT (Geometric Habits of Mind Permanence Test). While the problems in the tests are determined; it is encouraged to use at least one geometric habit of mind and is considered to be insoluble immediately. In all three tests, 10 open-ended questions were included; each question in the same order in the tests is solved by using the same geometric habits of mind:

Problem 1: It is a problem that requires students to know equality of triangles. However, it is expected that students will reach generalizations for geometric shapes given the sub questions in the question. In this context, the problem requires the use of reasoning with relationships and generalizing geometric ideas.

Problem 2: The problem is for students to use special triangle or special quadrilateral features. However, with the result of making additional drawings in the process of reaching the result; it requires the use of reasoning with relationships and balancing exploration and reflection.

Problem 3: The problem with the folding of rectangles is; to determine the varying or unchanging edge lengths in the solution and to use reasoning with relationships and investigating invariants in this context by making the area account of the triangles.

Problem 4: With an additional drawing in the solution, it is a problem in the kind of proof that the relations between the field relations and the relationship between triangles need to be known. In the process of proof, it requires the use of reasoning with relationships, generalizing geometric ideas and balancing exploration and reflection.

Problem 5: It is intended for students to use the Pythagorean Theory with additional drawings. In this context, it requires the use of reasoning with relationships and balancing exploration and reflection.

Problem 6: In the problem of using angle similarity in triangles, it is expected to use reasoning with relationships.

Problem 7: The problem with the relations between the quadrilaterals / triangles formed in the large rectangle is requires balancing exploration and reflection with an additional drawing, reasoning with relationships by establishing a relationship between the fields. However, the use of investigating invariants is required by specifying the changing / nonchanging properties of the given point in motion.

Problem 8: In the problem of special quadriaterals, students are expected to look for relationships between shapes formed by making new drawings. In this context, it requires the use of reasoning with relationships and balancing exploration and reflection.

Problem 9: The problem involving triangle and quadrilateral is to first make an additional drawing to see the relationship between the fields; for this reason it requires the use of balancing exploration and reflection. Then, the determination of the relationship between the fields requires the use of reasoning with relationships. Moving thought of fixed point / determination of changing or unchanging situations by enlarging a shape edge requires investigating invariants.

Problem 10: The last problem involving quadrilaterals and length in circle is requires balancing exploration and reflection by making new drawings on the figure and reasoning with relationships by establishing relations between the lengths after the drawings.

Expert opinions were taken in order to ensure the validity of geometric habits of mind tests. Also, how the problems in the tests were solved by the students and the in-depth interviews were examined in the pilot study process. Whether these problems fit the indicators of the geometric thinking habits determined by the researcher was examined. In this process, the problems that are prepared by removing the questions being exercises type were given to three specialist mathematics educators and three high school mathematics teachers. Thus, language, level, content and scope validity of geometric habits of mind tests having open-ended problems are provided.

2.4 The role of the Researcher

The purpose of the researcher is to identify geometric habits of mind and then design and implement a learning environment to enhance these habits. The researcher observed and made acquaintance with the students for three weeks before the implementation process. It was observed during this time period that the students were not able to use their geometric habits of mind adequately and tried to reach the solution by using the formulas and problems they had memorized earlier during the class hours. In the learning environment, the researcher took the role of a mentor by answering their questions, asking them questions and giving them feedback. The researcher also made use of the dynamic geometric software in the process so as to enhance the geometric habits of mind of the students. There was an interactive whiteboard as well as a projector in the learning environment. After the implementation process, the researcher analyzed the data and presented the findings to the reader using scientific writing rules.

2.5 Analysis of Data

Four-level scoring scale, suggested by Bülbül (2016) in his doctorate thesis, was used to evaluate the data from the tests. The scoring scale prepared by Bülbül (2016) is as follows:

Score 0: No habit was used.

Score 1: Only 1 habit was used but the correct solution was not attained.

Score 2: More than one habit was used, but the solution was not attained.

Score 3: One or more habits were used, and the problem could be solved.

After evaluating the results from the pre-test, post-test and substantivity test using the scale above, a software named IBM SPSS Statistics 22 was used to compare the data from the experimental and control groups. The geometric habits of mind of the students were compared first among the group and then across the two groups. Hence, it was aimed to present the analysis of the quantitative data from the geometric habits of mind tests and the enhancement of the students in the designed learning environment. Every student's score of GHMPrT, GHMPoT, and GHMST was calculated separately. Next, Shapiro-Wilks test was conducted to see if each geometric habits of mind test result belonging to the experimental and control groups was distributed normally. If the group size is smaller than 50 samples, Shapiro-Wilks test is used to find this out (Büyüköztürk, 2013). As shown in Table 3, the data groups show a normal distribution.

Table 3: Comparing the scores of groups with the Shapiro-Wilks Test

Groups	Tests	N	Shapiro-Wilks	p
Experiment	Pre-test	31	0,973	0,604*
	Post-test	31	0,951	0,170*
	Permanence test	31	0,962	0,336*
Control	Pre-test	31	0,939	0,075*
	Post-test	31	0,969	0,503*
	Permanence test	31	0,942	0,096*

*p > .05

Statistical analyses of the scores obtained in the experimental process are as follows:

- While comparing the average scores of the geometric habits of mind pre-test, post-test and substantivity test, One-Way ANOVA for Repeated Measures analysis was performed, and the effect size was calculated, as the data to be compared satisfied the conditions to have normal distribution, to have the same amount of variance between any two measures and to have the same data source (Can, 2017),
- While comparing the average scores of the geometric habits of mind post-test by checking the pre-test scores' effect on the groups, One-Way ANCOVA analysis was performed and the effect size was calculated, as the data to be compared satisfied the conditions to have normal distribution and same variance levels, to have a linear relationship between the dependent variable and control variable, to have a homogenized regression coefficient and to not have a significant difference of control variable in different groups (Can, 2017),
- While comparing the average scores of the geometric habits of mind post-test and substantivity test, Two-Way ANOVA for Mixed Measures analysis was performed and the effect size was calculated, as the data to be compared satisfied the conditions to have normal distribution for every data, to have a homogenized variance across the groups, to not have a significant difference of covariance in different groups (Can, 2017).

Eta squared (effect size) shows how much of the total variance in the dependent variable the independent variable or factor can represent and is valued between 0.00 and 1.00. Eta squared values are interpreted as small for .01, medium for .06 and large for .14 (Büyüköztürk, 2013).

3. Results

In this section, the progress of the geometric habits of mind of the students in experimental and control groups is studied. As the data groups have shown normal distribution, the progress of the students on the problem-solving performance in experimental and control groups was studied using one-way ANOVA for repeated measures while considering the scores of the tests.

To find out whether the students in the experimental group have shown progress in the geometric habits of mind, one-way ANOVA for repeated measures was used while considering the scores they received from the related geometric habits of mind tests. The values obtained are presented in Table 4 and 5.

Table 4: Descriptive statistics results of the scores from the pre-test, post-test and permanence test taken by the students in the experimental group

Variables	N	$\bar{X}$	Sd
Pre-test	31	16,645	4,666
Post-test	31	24,452	3,443
Permanence test	31	21,387	3,303

As is evident from the Table 4, test scores of the students in the experimental group have improved from the pre-test ($\bar{X}$= 16.645) to post-test ($\bar{X}$= 24.452). Even though the substantivity test scores ($\bar{X}$=21,387) of these students have decreased compared to the post-test scores, it can be seen that the score is still higher than the pre-test scores.

Table 5: One-way ANOVA for repeated measures results of the students in the experimental group for the pre-test, post-test and permanence test

	Sum of Squares	df	Mean Square	F	Sig.	Meaningful Difference	η^2
Between Groups	957,914	30	31,930	76,077	,000	1-2,	,757
Within Groups	959,118	2	712,539			1-3,	
Error	378,215	60	9,366			2-3	
Total	2295,247	71,728					

According to Table 5, it can be seen that there is a significant difference between every test the students in the experimental group have taken ($F_{(2.60)}$=76.077, p<.05) and that the effect size is large (η^2=0.757). According to this, the students scored higher in the post-test and permanence test than in pre-test. However, the average scores obtained from the substantivity test is lower than the average post-test scores. It can be deduced from this result that the learning environment enhanced the geometric habits of mind of the students in the experimental group and even though there is a significant difference between the post-test and permanence test, it can be seen from the positive difference of the permanence test from the pre-test that the learning environment has an effect on the permanence. To find out whether the students in the control group have shown progress in the geometric habits of mind, one-way ANOVA for repeated measures was used while considering the scores they received from the related geometric habits of mind tests. The obtained data are presented in Table 6 and 7.

Table 6: Descriptive statistics results of the scores from the pre-test, post-test and permanence test taken by the students in the control group

Variables	N	$\bar{X}$	Sd
Pre-test	31	18,290	4,268
Post-test	31	20,839	3,407
Permanence test	31	15,613	3,393

As can be seen from Table 6, control group students' average scores from the pre-test ($\bar{X}$=18.290) was lower than the average scores of the post-test ($\bar{X}$= 20.839). However, the average substantivity test scores of these students ($\bar{X}$=15.613), which was performed 5 weeks after the implementation process, are lower than both the pre-test and post-test results.

Table 7: One-way ANOVA for repeated measures results of the students in the control group for the pre-test, post-test and permanence test

	Sum of Squares	df	Mean Square	F	Sig.	Meaningful Difference	η²
Between Groups	616,645	30	20,555	20,378	,000	1-2,	,404
Within Groups	423,376	2	222,263			1-3,	
Error	623,290	60	10,907			2-3	
Total	1663,311	92					

According to Table 7, it can be seen that there is a significant difference between every test the students in the experimental group have taken ($F_{(2.60)}$=20.378, p<.05) and that the effect size is medium (η^2=0.404). According to this, the students scored higher in the post-test than in substantivity test and in pre-test. However, the average scores obtained from the substantivity test is lower than the average pre-test scores.

One-way covariance analysis (ANCOVA) was performed in order to see if there was a significant difference between the average post-test scores of the experimental and control group students. The obtained data are presented in Table 8 and 9.

Table 8: The real test scores of the groups and post-test scores corrected according to the pre-test scores

		Post test		Correct Post test	
Groups	**N**	$\bar{X}$	**Ss**	$\bar{X}$	**Ss**
Experiment	31	24,452	3,443	24,715	0,568
Control	31	20,839	3,407	20,575	0,568

Table 8 shows the independent variant's averages that were calculated by taking the control variable pre-test scores' effects into account. Looking over the pre-test scores' effects, average post-test score ($\bar{X}_{post\text{-}test}$= 24.452) for the experimental group have risen ($\bar{X}_{correctpost\text{-}test}$= 24.715) while average post-test score ($\bar{X}_{post\text{-}test}$= 20.839) for the control group have fallen ($\bar{X}_{correctpost\text{-}test}$= 20.575). ANCOVA analysis was performed in order to see if the corrected average score difference between the two groups was significant. The obtained data are presented in Table 9:

Table 9: ANCOVA results of the post-test scores corrected according to the pre-test scores

	Sum of Squares	df	Mean Square	F	Sig.	η²
Pre-test (Regression)	123,202	1	123,202	12,518	,000	
Groups (Post-test)	**256,706**	**1**	**256,706**			
Error	580,669	59	9,842			
Total	906,194	61		**26,083**	**,000**	**,307**

According to the results of Table 9, there is a significant difference between the two groups' corrected post-test scores, for the benefit of the experimental group ($F_{(1.59)}$=256.083). Effect size was calculated to be .307. The findings obtained show that the designed learning environment had a significant effect on the students' development of geometric habits of mind.

Two-way ANOVA for mixed measures was performed in order to see if there was a significant difference among average scores that were obtained from the post-test and substantivity test taken by the students in both groups. The obtained data are presented in Table 10.

Table 10: Two-way ANOVA results of the students in the experimental and control group for the post-test and permanence test

		Sum of Squares		df	Mean of Square	F	Sig.	η²
Between Groups	1679,846		61					
Group (Experiment/Control)		682,911		1	682,911	41,101	,000	,407
Error		996,935		60	16,616			
Within Goups	948,500		62					
Measurement		532,653		1	532,653	84,182	,000	,584
Group*Measurement		**36,202**		**1**	**36,202**	**5,721**	**,020**	**,087**
Error		379,645		60	6,327			
Total	2628,346		123					

Upon inspecting Table 10, it is evident that there is a significant difference between the experimental and control group students in terms of their average post-test and permanence test scores ($F_{(1.60)}$= 41.101, p< .05). This finding supports the results of the one-way ANOVA for repeated measures for both groups. In addition, the findings from two-way variance analysis for mixed measures show that looking at the group-measure common effect, it can be deduced that the score decay of the experimental group is significantly lower than that of the control group ($F_{(1.60)}$=5.721, p<.05).Considering that the effect size is .087, it is suggested that the process has a significant effect on the students. Hence, it can be concluded that the learning environment designed to enhance the geometric habits of mind for the students have a significant effect on the permanence of the habits.

4. Discussion and Conclusion

The average pre-test scores of both groups show that the students use their geometric habits of mind on an intermediate level. Considering that these students attend a science high school, the obtained results are lower than the expected levels. Indeed, the fact that the average scores for the geometry section in the LYS (Undergraduate Placement Exam) are fairly low for all students (ÖSYM, 2014; 2015; 2016) show that students do not have adequate levels of geometric habits of mind and that these habits should be enhanced.

According to the results obtained from research, experimental group students' average score of the post-test is higher than that of pre-test. It was also found out that the average substantivity test scores, which took place 5 weeks later than the implementation process, are lower than the average post-test scores but higher than that of the pre-test. The students in the experimental group experienced significant differences in terms of test scores and the effect size was large. The average scores they've received from the permanence test and the post-test are higher than that of the pre-test. Hence, it can be deduced that the designed learning environment enhanced the students' geometric habits of mind. The findings of this study support the view put forward by several studies before, which claim that appropriate learning environments enhance the geometric habits of mind (Charbonneau et al., 2009; Cuoco et al., 1996; Driscoll et al., 2008; Goldenberg, 1996; Gordon, 2011; Hu, 2005; Jacobbe & Millman, 2009). Bülbül (2016) also found out in his study with prospective teachers that these prospective teachers also performed better at the end of the implementation process at the geometric habits of mind. Similarly, the studies focusing on enhancing mathematical habits of mind also demonstrated that the habits of mind of the students have enhanced over time (Guenther, 1997; Hu, 2005; Marshall, 2004).

Average pre-test scores of the control group students have increased in the post-test. However, the average permanence test scores of these students, which was performed five weeks after the implementation process, are lower than both the pre-test and post-test results. One-way ANOVA for repeated measures results of the average scores show that there's a statistically significant difference between the scores of the students for every test and that the effect size is medium. According to this, the students scored higher in the post-test than in permanence test and in pre-test. However, the average scores obtained from the permanence test is lower than the average pre-test scores. We can deduce from this result that learning environments focused on multiple choice tests have no effect on learning permanence.

According to the study, there is a significant difference between the two groups' corrected post-test scores, for the benefit of the experimental group. Two-way ANOVA for mixed measures performed to see if there was a significant difference among average scores that were obtained from the post-test and permanence test performed on the students in both groups showed that group-measure common effect indicates the score decay of the experimental group for the substantivity test is significantly less than that of the control group. Hence, we can conclude that the learning environment designed to enhance the geometric habits of mind for the students have a significant effect on the substantivity of the habits. It is evident from these findings that the learning environment designed for the 10th-grade students in order to enhance their geometric habits of mind has succeeded in enhancing such habits and increased the permanence. Moreover, it was found out that problem-based learning and/or usage of dynamic geometric software have a positive effect on the learning substantivity (Dods, 1997; Uslu, 2006; Üstün and Ubuz, 2004).

5. Recommendations

- Using the findings from this study, future studies may be conducted on the students with different levels of education such as elementary school, middle school or high school.
- Another habit of mind that is highlighted in other studies is the algebraic habit of mind (Cuoco et al., 1996). Geometric and algebraic habits of mind can be studied together.
- Following the studies conducted on the cognitive aspect of the geometric habits of mind, the emotional aspect of the said habits is also surfacing in the literature (Costa and Kallick, 2000; Marshall, 2004). New studies can be designed where both aspects are studied simultaneously.
- Further studies can also do a content analysis on with which habits of mind the geometric problems in the textbooks for elementary school, middle school or high school students can be solved.

References

1. Bülbül, B. Ö. (2016). *Matematik öğretmeni adaylarının geometrik düşünme alışkanlıklarını geliştirmeye yönelik tasarlanan öğrenme ortamının değerlendirilmesi* (Yayınlanmamış doktora tezi). Karadeniz Teknik Üniversitesi, Eğitim Bilimleri Enstitüsü, Ortaöğretim Fen ve Matematik Alanları Eğitimi Anabilim Dalı, Matematik Eğitimi Bilim Dalı, Trabzon.
2. Büyüköztürk, Ş. (2013). *Sosyal bilimler için veri analizi el kitabı* (18. Baskı). Ankara: Pegem Akademi.
3. Campell, D. T., & Stanley, J. C. (1963). *Experinmental and quasi-experimental designs for research*. Chicago: Rand McNally & Company.
4. Can, A. (2017). *SPSS ile bilimsel araştırma sürecinde nicel veri analizi* (5. Baskı). Ankara: Pegem Akademi.
5. Charbonneau, P. C., Jackson, H. A., Kobylski, G. C., Roginski, J. W., Sulewski, C. A., & Wattenberg, F. (2009). Developing students' "habits of mind" in a mathematics program. *PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies*, 19(2), 105-126. http://dx.doi.org/10.1080/10511970802409040
6. Clark, K., & Lesh, R. (2003). Whodunit? Exploring proportional reasoning through the footprint problem. *School Science and Mathematics, 103*(2), 92-98.
7. Cuoco, A., Goldenberg, E., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior, 15*(4), 375–402.
8. Dods, R. F. (1997). An action research study of the effectiveness of problem-based learning in promoting the acquisition and retention of knowledge. *Journal for the Education of the Gifted, 20*(4), 423-437.

9. Driscoll, M. J., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering geometric thinking: A guide for teachers grades 5-10*. Portsmouth, NH: Heinemann.
10. Driscoll, M. J., DiMatteo, R. W., Nikula, J., Egan, M., Mark, J., & Kelemanik, G. (2008). *The Fostering Geometric Thinking Toolkit: A Guide for Staff Development.* Portsmouth, NH: Heinemann.
11. Goldenberg, E. P. (1996). "Habits of Mind" as an organizer for the curriculum. *Journal of Education, 178*(1), 13–34.
12. Goldenberg, E. P., Shteingold, N., & Feurzeig, N. (2003). Mathematical habits of mind for young children. *Teaching Mathematics Through Problem Solving: Prekindergarten Grade, 6*, 51-61.
13. Gordon, M. (2011). Mathematical habits of mind: promoting students' thoughtful considerations. *Journal of Curriculum Studies, 43*(4), 457-469.
14. Guenther, S. J. (1997). *An examination of fifth grade students' consideration of habits of mind: A case study*. Disseration Abstracts International, (UMI No. 9841295).
15. Hu, H-W. (2005). *Developing Siblings and Peer Tutors To Assist Native Taiwanese Children In Learning Habits Of Mind For Math Success.* Disseration Abstracts International, 256B (UMI No. 3179886).
16. Jacobbe, T., & Millman, R. S. (2009). Mathematical habits of the mind for preservice teachers. *School Science and Mathematics, 109*(5), 298-302.
17. Leikin, R. (2007). Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks. *In the Proceedings of the Fifth Conference of the European Society for Research in Mathematics Education* (pp. 2330-2339). Larnaca, Cyprus.
18. Levasseur, K., & Cuoco, A. (2009). *Mathematical Habits of Mind*. National Council of Teachers of Mathematics.
19. Lim, K. H., & Selden, A. (2009). Mathematical habits of mind. In S. L. Swars, D. W. Stinson and S. Lemons-Smith (Eds.). *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Atlanta, GA: Georgia State University.
20. Mark, J., Cuoco, A., Goldenberg, E. P., & Sword, S. (2010). Developing mathematical habits of mind. *Mathematics Teaching in the Middle School, 15*(9), 505-509.
21. Marshall, A. R. (2004). *High school mathematics habits of mind instruction: student growth and development.* Dissertation Abstracts International, 115B, (UMI No. 1421654)
22. Marzano, R. J., Pickering, D., & McTighe, J. (1993). *Assessing student outcomes: Performance assessment using the dimensions of learning model.* Alexandria, VA: Association for Supervision and Curriculum Development.
23. Matsuura, R., Sword, S., Piecham, M. B., Stevens, G., & Cuoco, A. (2013). Mathematical habits of mind for teaching: Using language in algebra classrooms. *The Mathematics Enthusiast, 10*(3), 735-776.
24. National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

25. ÖSYM. (2014). 2014 – ÖSYS Yerleştirme Sonuçlarına İlişkin Sayısal Bilgiler. http://dokuman.osym.gov.tr/pdfdokuman/2014/OSYS/yerlestirme/2014-OSYS-YerlestirmeSonuclar%C4%B1naIliskinSayisalBilgiler23072014.pdf was accessed on 01.09.2017.
26. ÖSYM. (2015). 2015 – ÖSYS Yerleştirme Sonuçlarına İlişkin Sayısal Bilgiler. http://dokuman.osym.gov.tr/pdfdokuman/2015/OSYS/2015-OSYSYerlestirmeSonucSayisalBilgiler23072015.pdf was accessed on 01.09.2017.
27. ÖSYM. (2016). 2016 – ÖSYS Yerleştirme Sonuçlarına İlişkin Sayısal Bilgiler. http://dokuman.osym.gov.tr/pdfdokuman/2016/LYS/YerlestirmeSayisalBilgiler10082016.pdf was accessed on 01.09.2017.
28. Özen, D. (2015). *Ortaokul matematik öğretmenlerinin geometrik düşünmelerinin geliştirilmesi: Bir ders imecesi* (Yayınlanmamış doktora tezi). Anadolu Üniversitesi, Eğitim Bilimleri Enstitüsü, Matematik Eğitimi Anabilim Dalı, Matematik Eğitimi Bilim Dalı, Eskişehir.
29. Uslu, G. (2006). *Ortaöğretim matematik dersinde probleme dayalı öğrenmenin öğrencilerin derse ilişkin tutumlarına, akademik başarılarına ve kalıcılık düzeylerine etkisi.* Balıkesir Üniversitesi, Fen Bilimleri Enstitüsü, Matematik Eğitimi Anabilim Dalı, Balıkesir.
30. Üstün, I. ve Ubuz, B. (2004*). Geometrik Kavramların Geometer's Sketchpad Yazılımı ile Geliştirilmesi.* Eğitimde İyi Örnekler Konferansı (17 Ocak 2004). İstanbul: Sabancı Üniversitesi.
31. Yavuzsoy-Köse N. ve Tanışlı, D. (2014). Primary school teacher candidates' geometric habits of mind. Educational Sciences: *Theory & Practice, 14*(3), 1220-1229. http://dx.doi.org/1229.10.12738/estp.2014.3.1864