Effective Word-Problem Instruction:

Using Schemas to Facilitate Mathematical Reasoning

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In a fourth-grade general education classroom, Mrs. Blanton posted her math lesson's objective: Students will solve division word problems. *During her instruction, Mrs. Blanton says, "In a word problem, the word* share *tells you to divide." Mrs. Frank, a special education teacher, provides small-group instruction to Mrs. Blanton's students with learning disabilities. During Mrs. Frank's intervention time, she showed students the word problem of the day:* On Wednesday, the coffee shop had 108 customers. The bookstore had 65 customers. How many more customers did the coffee shop have on Wednesday? *Mrs. Frank reminds her students to use the Math Key Words Poster hanging in her resource room. The poster indicates that* more *means addition.*

Many general and special education teachers across the U.S. teach word problems by defining problems as a single operation (e.g., "Today, we're working on subtraction word problems") and linking key words (e.g., *more, altogether, share, twice*) to specific operations (e.g., *share* means to divide). Unfortunately, teaching students to approach word problems in these ways discourages mathematical reasoning and frequently produces incorrect answers. In Table 1, we list eight common key words, identify the operation typically associated with each, and provide word problems that illustrate how reliance on key words can result in incorrect answers. Neither of these approaches—defining problems in terms of a single operation or linking key words to specific operations—has evidence to support its use.

<Insert Table 1 about here.>

In contrast, other approaches do promote mathematical reasoning and substantially boost word-problem performance among students with learning disabilities (Fuchs et al., 2010; Griffin

& Jitendra, 2009; Xin et al., 2011). Two practices that have emerged from high-quality research studies as particularly effective for word-problem instruction are: (a) attack strategies, which provide students with a general plan for processing and solving word problems (Montague, 2008; Xin & Zhang, 2009), and (b) schema instruction, in which students learn to categorize word problems within problem types (i.e., schemas based on the word-problem's mathematical structure); apply an efficient solution strategy for each word-problem schema; and understand the meaning of word-problem language (Fuchs et al., 2014; Jitendra & Star, 2012). In this article, we focus on word problems commonly seen within textbooks and high-stakes assessments used within the U.S.

The Attack Strategy

An attack strategy is an easy-to-remember series of steps students use to guide their approach to solving word problems. A helpful attack strategy spans across schemas and grade levels. Researchers have determined that students' use of an attack strategy is effective for improving word-problem performance (Case et al., 1992; Fuchs et al., 2014; Jitendra, Griffin, Deatline-Buchman, Sczesniak, 2007; Jitendra & Star, 2012; Montague, 2008; Xin & Zhang, 2009). Some attack strategies address the first phase of word-problem solving—interpreting the word-problem's meaning. During this phase, students read the problem, identify the question, and determine central idea of the problem (i.e., the schema or problem type). An attack strategy is important because many students skip this phase; instead, students will haphazardly select numbers from the word problem and rely on key words to identify an operation. Some attack strategies address the second phase of word-problem solving—finding the missing quantity. The second phase involves setting up a number sentence or using a graphic organizer, performing calculation(s), labeling the number answer, and checking whether the answer makes sense. In some cases, attack strategies address both phases.

In Figure 1 several different attack strategies are presented. The first four strategies make use of acronyms, which help students remember the attack strategy's steps. An acronym is a mnemonic: a pattern of letters, ideas, or associations to help students remember something. Researchers have learned that mnemonics can help students with learning disabilities remember important information (e.g., Uberti, Scruggs, & Mastropieri, 2003) such as the steps of a general word-problem attack strategy. Although mnemonics can be helpful, attack strategies that do not make use of acronyms can also be effective. Students' repeated use of the attack strategy facilitates retention. Although variations in attack strategies exist, the first part of word-problem solving across attack strategies is a thorough reading of the problem.

<Insert Figure 1 about here.>

Whichever attack strategy the teacher selects, it is critical that the teacher explicitly models the attack strategy while explaining how it works. The teacher must also scaffold student learning of the attack strategy by decreasing levels of support until the attack strategy becomes a natural part of a student's word-problem reasoning. Moreover, the teacher must also provide many opportunities for practice of the attack strategy with instructive corrective feedback. The exact amount of modeling, practice, and feedback depends on a student's prior knowledge and skills and the quality of the teacher modeling and corrective feedback. Later in this article, we provide an example of Mrs. Frank modeling the RUN attack strategy as part of schema instruction.

Additive and Multiplicative Schemas

Schema instruction is a demonstrably effective instructional practice for promoting stronger word-problem performance for students with learning disabilities across grade levels (e.g., Fuchs, Craddock, et al., 2008; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, Hoff, & Beck, 1999; Jitendra & Star, 2011; Powell et al., 2015). Whereas defining word problems by key words or operation has no research to support use for students with learning disabilities, schema instruction has a rich research base. Two categories of schemas that have broad usage for teachers are the additive and multiplicative schemas. These schemas can be used to solve word problems from kindergarten through eighth grade.

Additive Schemas

The three major additive schemas are combine, compare, and change problems. Each schema involves addition or subtraction concepts and procedures. Together, the three additive schemas (combine, compare, change) can be used to understand and solve any additive word problem. In Figure 2, a definition, an equation with graphic organizer, an example problem, and variations for each schema is provided.

<Insert Figure 2 about here.>

Combine problems. *Combine* problems put together two or more separate parts to make a sum or total (Part + Part = Total). Combine problems may also be called *total* or *part-part-whole* problems. In the upper elementary grades and middle school, combine problems often involve three or four parts (see variations in Figure 2). Combine problems require students to solve for the total or to find one of the parts. The top of Figure 3 provides two worked examples of a combine word problem: one requires the student to solve for the total (i.e., sum unknown problem), and the other requires students to solve for one of the parts (i.e., part unknown problem). Several validated schema instructional programs (e.g., Fuchs et al., 2009; Powell et al., 2015) employ the attack strategy RUN: *R*ead the problem; *U*nderline the label (i.e., what the problem is mostly about); and *N*ame the problem type. What follows is an example of Mrs.

Frank teaching the second problem in Figure 3.

<Insert Figure 3 about here.>

Mrs. Frank: *We have a mix of numbers and words. It's a word problem! We need to RUN through it! First, let's R: read the problem together.*
Students: *"Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?"*
Mrs. Frank: *We read the problem. Now, let's U: underline the label. What's this problem about?*
Students: *Apples.*
Mrs. Frank: *Do we have to find the red apples or the green apples? Look at the question.*
Students: *Green apples.*
Mrs. Frank: *So, let's underline "green apples." Now, we N: name the problem type. Is this a combine, compare, or change problem?*
Students: *Combine.*
Mrs. Frank: *Why is this a combine problem? Do we have parts put together for a total?*
Students: *Yes. We have red and green apples combined for a total.*
Mrs. Frank: *It is a combine problem. Let's use the combine equation, P1 + P2 = T, to organize the word-problem information and solve the problem. What's our combine equation?*
Students: *P1 plus P2 equals T.*
Mrs. Frank: *Let's read the problem again. "Lyle has 29 red and green apples." 29 is a number. Do we need 29 in the combine equation?*
Students: *Yes! It's the total.*
Mrs. Frank: *29 is about both red and green apples. It is the total. Let's write 29 under T. Now, keep reading.*
Students: *"If 11 of the apples are red…"*
Mrs. Frank: *11 is also a number. Do we need 11 in the combine equation?*
Students: *Yes. 11 talks about one of the parts.*
Mrs. Frank: *11 is one of the parts, so let's write 11 under P1. What should we write under P2?*
Students: *Question mark!*
Mrs. Frank: *That's right. We mark the missing information with a question mark. Now, let's solve this equation. You could start at 11 and add to 29. You could subtract 11 from 29. Your choice!*
Students: *18 green apples*
Mrs. Frank: *There are 18 green apples. Remember, we always make sure to write a number answer and a label answer. Good work!*

The combine equation (P1 + P2 = T) aids as an efficient solution strategy because students do not often understand how to organize the numbers presented in word problems. Note that, after setting up the equation 11 + ? = 29, students may add or subtract to solve the problem, depending on whether the missing information is one of the parts or the total. This shows that teachers should not describe this problem as an addition problem or subtraction problem during instruction; the deeper understanding of the problem is that it is a combine problem.

Compare schemas. In *compare* problems, two sets are compared for a difference (Bigger – Smaller = Difference). Compare problems may also be called *difference* problems. Students may be asked to solve for the difference, the greater set, or the lesser set. To teach compare problems, such as the problem in Figure 3, teachers should start with the RUN attack strategy. Then, teachers can use a graphic organizer to organize the word-problem information related to the compare schema. Teachers could also use a compare equation (B – s = D or G – L = D) to guide students in organizing word-problem information.

Change problems. In *change* problems, an amount increases or decreases (i.e., changes) over time because something happens to change the starting amount (Start +/– Change = End). Change problems with an increase may be called *join* problems whereas change problems with a decrease may be called *separate* problems. Change problems can ask students to solve for an unknown start, change, or end amount. Change problems in the upper elementary and middle-school grades often involve multiple changes (see variations in Figure 2). In the worked examples of change problems of Figure 3, a teacher starts with an attack strategy and then uses a change equation or change graphic organizer. Teachers should introduce these solution strategies (i.e., equation or graphic organizer) separately but allow students to choose the solution strategy they favor for daily use.

Multiplicative Schemas

The three common multiplicative schemas, involving multiplication or division concepts, are equal groups, comparison, and proportions or ratios (see Figure 4 for definitions, graphic organizers, example problems, and variations). Students can use these three multiplicative schemas to represent and solve word problems in the upper elementary and middle-school grades.

<Insert Figure 4 about here.>

In *equal groups* problems, a group or unit is multiplied by a specific number or rate for a product. Equal groups problems may also be called *vary* problems. The unknown may be the groups, the number or rate for each group, or the product. In the worked example in Figure 5, the groups are unknown. A teacher should use an attack strategy to identify that the question is asking to determine the number of cartons of eggs. Then, the teacher models how to solve the problem. The equal groups graphic organizer allows for organization of the word-problem information. When solving the equation of $? \times 12 = 60$, students may multiply (i.e., what times 12 equals 60?) or divide (i.e., 60 divided by 12 equals what?). For this reason, teachers cannot describe these types of word problems as a multiplication or division problem. Instead, presenting word problems such as these through the equal groups schema promotes mathematical reasoning by encouraging students to solve the word problem algebraically (i.e., by balancing the two sides of the equation).

<Insert Figure 5 about here.>

With *comparison* problems, a set is multiplied a number of times for a product. Even though the unknown may be the original set, the multiplier, or the product, students are most often asked to find the product in comparison problems. The worked example in Figure 5 is an example of a typical comparison problem. Teachers could use a graphic organizer to organize the information from the word problem. In addition, presenting this problem using a number line, with the set of 7 multiplied 3 times, could be helpful for students to understand the comparison of the word problem. The set of 7 is multiplied 3 times for a product of 21: 21 is compared to 7 as a multiple of 7.

With the *proportions or ratio* schema, students explore the relationships among quantities. This exploration helps students understand proportions, percentages, unit rate, or ratios. The unknown may be any part of the relationship. The worked examples in Figure 5 reflect the solving of a typical proportion or ratio word problems. When teaching both word problems, teachers should start with an attack strategy and then move to using a solution strategy (e.g., graphic organizer) that helps students understand how to organize the information presented in the word problem.

Grade Levels and Timelines for Introducing Schemas

As mentioned previously, an attack strategy should be introduced and practiced alongside schema instruction. Attack strategies are relatively simple and can be learned quickly; in contrast, understanding word-problem schemas and using a solution strategy (e.g., equation or graphic organizer) associated with each schema requires complex reasoning and a detailed set of skills. Developing mathematical reasoning related to the schemas takes sustained instruction that often spans the entire school year.

Additive schemas appear in mathematics materials as early as kindergarten, but typical schema introduction for additive schemas may start in first grade and continue across the elementary grades, depending upon the prior knowledge of students. Within a school year, we recommend introducing the additive schemas separately and providing mixed schema practice as new schemas are modeled and practiced. Among the three additive schemas, we recommend teaching combine problems first. This is because, when solving for missing parts in combine problems, the conceptual basis is the same no matter which of the parts is missing. Thus, the combine problem type is a relatively easy schema for establishing an understanding of the conceptual and procedural aspects of schema instruction.

With the additive schemas, we recommend teaching the compare problem type next. Compare problems are the most difficult of the three additive schemas. Teaching the compare problem schema after the combine schema allows students to benefit from the foundation of schema instruction achieved with combine problems. Teaching compare problems next means that students only need to distinguish between combine and compare problems (rather than among all three problem types). Teaching change problems last makes sense because the change problem's central idea (increasing or decreasing) is the most story-like and intuitive of the three schemas.

In Table 2, we provide a sample timeline, in weeks, for teaching the three additive schemas (Fuchs et al., 2014; Powell et al., 2015). This timeline assumes the teacher is providing modeling and practice of word problems 2 or 3 times a week. Before schema instruction begins, the timeline includes an introductory unit in which the teacher teaches math skills foundational for schema instruction: single- or multi-digit addition and subtraction with and without regrouping; solving equations with missing information in any position (e.g., $4 + ? = 6$, $2 + 4 = ?$, $? - 4 = 2$, $6 - ? = 2$, $6 - 4 = ?$); interpreting graphs and figures to find important information; and strategies for checking whether answers are reasonable. After the introductory unit, schema instruction begins with a dual focus on the attack strategy and word-problem schemas.

<Insert Table 2 about here>

Unlike additive schemas, which are usually introduced and addressed within the same school year, equal groups and comparison schemas are typically featured during the elementary grades while the proportions or ratios schema is addressed more commonly in middle school. The equal groups schema is often introduced first because it represents the earliest explanations of multiplication and division (e.g., 3×2 is "three groups with two in each group"). Equal groups problems may initially be introduced in second or third grade. In third or fourth grade, the comparison schema should be explicitly taught. After the comparison schema is introduced, mixed practice should provide students with opportunities to distinguish between the equal groups and comparison schemas. In the middle school years, students should learn the proportions or ratio schema, with continued practice across the other additive and multiplicative schemas. The multiplicative schemas and additive schemas can be used to solve word problems with whole numbers or rational numbers. For example, the variations column in Figure 4 presents several multiplicative word problems with rational numbers.

Three Major Components of Effective Schema Instruction

Effective schema instruction incorporates the principles of explicit instruction, which have been shown to be necessary for students with learning disabilities (Gersten et al., 2009). This includes providing explanations in simple, direct language; modeling efficient solution strategies instead of expecting students to discover strategies on their own; ensuring students have the necessary background knowledge and skills to succeed with those strategies; gradually fading support; providing multiple practice opportunities; and incorporating systematic cumulative review. As with attack strategies, the number of practice opportunities differs within schema instruction depending on the student's incoming knowledge and skills, as well as the quality of teacher modeling, explanations, and corrective feedback.

Teaching What Each Schema Means

To explain the three components of effective schema instruction, we use the compare problem type, which is often the most difficult of the schemas for students to understand. Difficulty with the compare problem arises at least in part because its structure relies on subtraction that is conceptualized as a difference between two numbers. This is relatively or entirely unfamiliar to many students because subtraction is taught in schools primarily, or even exclusively, as taking away.

For Mrs. Frank, our special education teacher, her lesson's goal is that students solve the problem introduced at the beginning of this article (see Figure 6). Before jumping to solving this compare problem, Mrs. Frank first presents intact compare stories with no missing quantities using concrete objects and actual student names. For example, Mrs. Frank introduces the compare schema by asking two students, Tina and Seth, to stand back to back, as she says: "Tina and Seth are students in my class. Tina is 43 inches tall. Seth is 48 inches tall. Seth is 5 inches taller than Tina." Mrs. Frank then puts the compare graphic organizer (see the one aligned with height in Figure 2) on the board and leads a discussion in which she models and explains how to identify the boxes into which the bigger, smaller, and difference numbers go. Students discuss filling in the graphic organizer with a variety of compare stories, while Mrs. Frank gradually transfers responsibility to the students, all the time providing corrective feedback.

When students are secure in their understanding of the central idea of the compare schema, Mrs. Frank proceeds by introducing the compare equation. Mrs. Frank uses the compare equation of $B - s = D$ *in which B stands for the bigger quantity, s for the smaller quantity, and D for the difference between the quantities. Mrs. Frank explains how the compare equation maps to the graphic organizer, and students use intact stories to practice filling in the graphic organizer and the equation. Mrs. Frank presents the equation and graphic organizer not only to confirm students' understanding of the word-problem schema but also to help them organize the numbers in word problems.*

Teaching a Solution Strategy for Each Schema

After students understand the meaning of a schema (e.g., compare problems compare two amounts for a difference), students learn to select a solution strategy and use the solution strategy to organize the information from the word problem. Teaching a solution strategy involves modeling from the teacher and practice opportunities in which students receive feedback from the teacher.

Now that Mrs. Frank's students understand what the compare schema means and have mastered the RUN attack strategy within combine schema instruction, Mrs. Frank explicitly models how to solve compare problems. Initially, she uses a word problem with a difference missing. Students complete the same set of activities with the graphic organizer and equation, writing missing information into the graphic organizer and using a blank or question mark to represent the missing information in the equation. Gradually, Mrs. Frank omits the concrete manipulatives, integrates novel names into problems, and substitutes a hand gesture for easy reference to the graphic organizer (one of her hands parallel to the floor at about nose height; the other parallel to the floor at about chest height).

To practice an efficient solution strategy, Mrs. Frank begins to use the compare equation more often than the graphic organizer. She instructs students to write the compare equation as soon as students identify the word-problem schema. In Figure 6, the compare equation is $B - s = D$*. First, Mrs. Frank helps the students identify that the coffee shop is the bigger amount (marked with a B above "coffee shop") and the bookstore is the smaller amount (marked with an s above "bookstore"). She then models how to rewrite the equation with quantities from the word problem as replacements for B, s, and D, using a question mark or a blank to stand in for the missing quantity (*$108 - 65 = ?$*). Then, she works with the students to do the computation in* *different ways (e.g., 108 – 65 or 65 + ? = 108). Mrs. Frank concludes by writing the answer (? = 43 more customers) and checking the reasonableness of the answer (108 – 65 = 43). As this instruction occurs, Mrs. Frank provides many practice opportunities for students and provides focused affirmative and corrective feedback.*

After students learn to recognize word problems as belonging to schemas and are consistently using an efficient solution strategy (i.e., equation or graphic organizer) to organize the necessary word-problem information, the next phase of instruction involves explicitly teaching word-problem specific vocabulary and language.

Teach Important Vocabulary and Language Constructions

Word-problem solving relies heavily on reading and understanding language. Typically developing students often understand important math vocabulary prior to school entry and gradually learn to treat this language (e.g., *all* or *more*) in a special, task-specific way involving more complicated constructions about sets (*in all* and *more than)*. Many teachers assume that students have the necessary language comprehension to understand word problems and the problem's schema. But for students with learning disabilities, this is a shaky assumption.

A strong focus on vocabulary and language is therefore important, especially for students with learning disabilities. Examples of vocabulary and constructions that require explicit instruction, focused on the meaning of the language, are (a) joining words (e.g., *altogether*, *in all*) and superordinate categories (e.g., *animals* mean both *dogs* and *cats*) in combine problems; (b) compare words (e.g., *more*, *fewer*, *than*, -*er* words) and adjective -*er* versus verb -*er* words (e.g., *bigger* vs. *teacher*) in compare problems; and (c) cause-effect conjunctions (e.g., *then*, *because*, *so*), implicit change verbs (e.g., *cost*, *ate*, *found*), and time passage phrases (e.g., *3 hours later*, *the next day*) in change problems. We also recommend a focus on confusing crossproblem constructions (e.g., *more than* vs. *then … more*) and "tricky" labels (e.g., questions with superordinate category words).

For multiplicative schemas, students should learn how words often featured in additive problems (e.g., *more*) may be used within multiplicative problems (e.g., "How many times *more* flowers did Danica pick?"). It is also important for students to understand how to compare quantities with different units (e.g., *minutes* and *hours*), and how in proportions, the units must be a focus of the organization of the problem (i.e., *minutes* compared to *minutes*). For multiplicative problems, students must also learn math-specific vocabulary, such as *ratio*, *rate*, and *percentage*, the interpretation of such terms within word problems, and the variety of ways fractions and multiplicative relationships can be expressed.

We emphasize that word-problem specific language instruction should not teach students to rely on key words for recognizing schemas. As illustrated in Table 1, key words do not help students become word-problem thinkers, and reliance on key words fails to produce correct answers much of the time. We recommend teaching students specifically how and why "grabbing numbers and key words" to form number sentences frequently produces wrong answers.

Teaching students to avoid using key words is accomplished in three ways. First, the teacher explicitly teaches how math words mean different things in the context of a story, so reading the full word problem is necessary to distinguish among meanings. Reading the entire word problem is one thing that students do not always do, and it is one of the reasons an attack strategy is necessary. For example, *sharing* a quantity in equal parts may refer to multiplicative problems (e.g., "Max had 80 dog biscuits and shared them equally among 10 dogs"), whereas *sharing* part of a unit or collection may refer to additive problems (e.g., "Max had 80 dog biscuits and shared 40 of them with his dogs"). Second, the teacher demonstrates solving problems using key words, while eliciting student discussion about how and why this approach produces mistakes (e.g., *more* does not reliably mean add; *share* does not reliably mean divide). Third, the teacher structures activities in which the class analyzes worked problems from "last year's class" to identify how key words can lead students astray.

Multi-Step Word Problems

For solving one-step word problems, students learn to use a single schema. Solving word problems, however, is not always a one-step activity. To challenge students and engage students in mathematical reasoning, multi-step word problems are posed in many textbooks, on high-stakes tests, and in many authentic situations. Fortunately, when students understand word-problem schemas, solving multi-step word problems is much easier. This is because multi-step problems can incorporate more than one schema. For example, "Nathan bought 12 glazed donuts and 16 chocolate donuts for his class. The class ate 23 donuts. How many donuts does Nathan have left?" In this multi-step problem, students first use the combine schema to calculate that Nathan bought 28 donuts. Then, they apply the change schema to determine the change in number of donuts (i.e., 28 – 23 = ?).

Multi-step problems can also combine additive and multiplicative schemas. For example, "Nathan bought 12 glazed donuts and 16 chocolate donuts for his class. Each donut costs $1.10. How much did Nathan spend?" Students may first use the combine schema (i.e., 12 + 16 = 28 donuts) and then the equal groups schema (28 donuts × $1.10 each = ? cost). Note that there are other approaches to solving this problem. Some students may calculate the cost of the glazed donuts using an equal groups schema and then calculate the cost of the chocolate donuts using an equal groups schema. Finally, students may use the combine schema to determine the total cost of the glazed and chocolate donuts.

Key words, as mentioned near the start of this article, also fail as a strategy for solving multi-step word problems. For example, "For a bake sale, Katie baked 52 cupcakes but *shared* 4 of the cupcakes with her brother before taking the cupcakes to the sale. Buzz baked 42 cupcakes. How many cupcakes could Katie and Buzz sell *altogether* at the bake sale?" In this problem, some students may – without reading the problem – interpret *share* as meaning division or *altogether* as meaning addition. Neither word, processed in isolation and tied to an operation, produces a correct answer to this multi-step problem.

Summing Up: What to Do (and Not Do)

Schema instruction can be a powerful tool for helping students understand and solve word problems. Schema instruction facilitates mathematical reasoning by helping students understand the underlying structures within word problems that will be used across grade levels and with whole and rational numbers. To close, we summarize several key dos and do nots for teaching schemas.

Do not teach students to solve word problems by isolating key words and linking those words to operations. Don't say things like "*share* tells us to divide." Teaching students what *share* means helps students understand the conceptual schema of the word problem but telling students to divide whenever they see *share* is error fraught. In a similar vein, **do not** define word problems by an operation. Do not say, "Today we're working on *division* word problems." There is no such thing as a "subtraction" word problem because some students may use addition to solve such a problem; others may use subtraction. Defining a word problem by operation undermines conceptual understanding.

On the other hand, to promote mathematical reasoning related to word problems, **do** explicitly teach word-problem solving. Students with learning disabilities benefit from explicit instruction on effective strategies for solving word problems. **Do** allocate sustained instructional time across the school year for teaching word problems. **Do** teach an attack strategy to help students understand how to work systematically through a word problem. **Do** teach the additive and multiplicative schemas, emphasizing what the schema means. **Do** use equations, graphic organizers, and hand gestures to help students understand the schema's mathematical structure and organize word-problem information. **Do** include multi-step word problems that mix schemas. **Do** provide cumulative review across schemas, which mixes problems with and without irrelevant information, with and without problems that contain important information in graphs and figures, and with missing information in all slots of the schema's equation. Finally, **do** provide explicit instruction on word-problem vocabulary and language constructions that provide students access to the meaning of word problems.

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Table 1

Sample Key Words, Associated Operations, and Key Word Fails

Key word	Associated operation	Problem in which the key word strategy fails
altogether	addition	Alice bought 4 cartons of eggs with 12 eggs in each carton. How many eggs does Alice have *altogether*?
more	addition	Colin had some crayons. Then, he bought 12 *more* crayons. Now, he has 90 crayons. How many crayons did Colin have to start with?
fewer	subtraction	Paulo picked apples. Zach picked 12 *fewer* apples. If Zach picked 20 apples, how many apples did Paulo pick?
left	subtraction	Liz shared 55 candies equally with 3 friends. After sharing, how many candies were *left* over?
each	multiplication	Miles had 3 trays of building blocks with the same number of blocks on *each* tray. If Miles had 75 blocks altogether, how many were on *each* tray?
double	multiplication	Margaret bought *double* the songs as her sister. If Margaret bought 12 songs, how many songs did her sister buy?
share	division	Sal collected 18 quarters to *share* equally among his friends. After sharing, he had 3 quarters remaining. How many quarters did Sal share?
divide	division	Cam *divided* 5 pieces of paper into fourths. How many pieces of paper does Cam have now?

Table 2

Sample Additive Schema Intervention

Week	Schema	New information introduced to students
1	-	Addition; subtraction
2	-	Solving equations; labeling charts and graphs
3	Combine	Attack strategy; Total unknown
4	Combine	Total unknown
5	Combine	Part unknown
6	Combine	Problems with three or four parts
7	Compare	Difference unknown
8	Compare	Difference unknown
9	Compare	Lesser unknown
10	Compare	Greater unknown
11	Compare	Review combine and compare
12	Change	End unknown
13	Change	Change unknown
14	Change	Start unknown
15	-	Review combine, compare, and change
16	-	Review combine, compare, and change

Find the problem
Organize information using a diagram
Plan to solve the problem
Solve the problem

Jitendra & Star (2012)

Read the problem
Underline the question
Name the problem type

Fuchs et al. (2014)

Detect the problem type
Organize the information using the conceptual model diagram
Transform the diagram into a math equation
Solve for the unknown quantity and check your answer

Xin & Zhang (2009)

Search the word problem
Translate the words into an equation or picture
Answer the problem
Review the solution

Gagnon & Maccini (2001)

Read the problem out loud
Look for important words and circle them
Draw pictures to tell what is happening
Write down the math sentence
Write down the answer

Case et al. (1992)

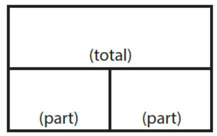
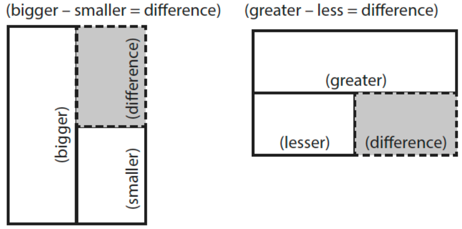
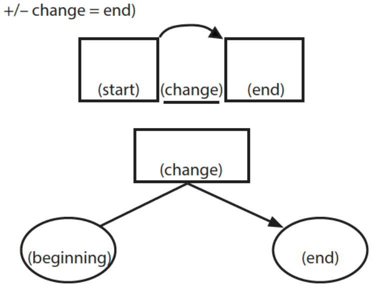
Read (for understanding)
Paraphrase (your own words)
Visualize (a picture or diagram)
Hypothesize (a plan to solve the problem)
Estimate (predict the answer)
Compute (do the arithmetic)
Check (make sure everything is right)

Montague (2008)

Read and retell the problem to discover the problem type
Underline and map important information onto the schematic diagram
Decide whether to add or subtract to solve
Write the mathematics sentence and solve it
Write the complete answer
Check the answer

Jitendra et al. (2007)

Figure 1. Sample attack strategies.

Schema and Definition	Equations and Graphic Organizers	Examples			Variations
Combine (Total; Part-part-whole) Parts combined for a sum	P1 + P2 = T (part + part = total) 	*Sum unknown:* Lyle has 11 red apples and 18 green apples. How many apples does Lyle have altogether?	*Part unknown:* Lyle has 29 red and green apples. If 11 of the apples are red, how many green apples does Lyle have?		*More than two parts:* Lyle has 34 apples. Of the apples, 11 are red, 18 are green, and the rest are yellow. How many yellow apples does Lyle have?
Compare (Difference) Sets compared for a difference	B – s = D (bigger – smaller = difference) G – L = D (greater – less = difference) 	*Difference unknown:* Sasha wrote 85 words in her essay, and Tabitha wrote 110 words. How many fewer words did Sasha write than Tabitha?	*Bigger/greater unknown:* Tabitha wrote 25 more words than Sasha. If Sasha wrote 85 words, how many words did Tabitha write?	*Smaller/lesser unknown:* Tabitha wrote 110 words in her essay. Sasha wrote 25 words fewer than Tabitha. How many words did Sasha write?	(None)
Change (Join; Separate) An amount that increases or decreases	ST +/– C = E (start +/– change = end) 	*End (increase) unknown:* Jorge had $52. Then, he earned $16 babysitting. How much money does Jorge have now? *End (decrease) unknown:* Jorge had $52. Then, he spent $29 at the ballpark. How much money does Jorge have now?	*Change (increase) unknown:* Jorge had $52. Then, he earned some money babysitting. Now, Jorge has $68. How much did Jorge earn babysitting? *Change (decrease) unknown:* Jorge had $52 but spent some money when he went to the ballpark. Now, Jorge has $23. How much did Jorge spend at the ballpark?	*Start (increase) unknown:* Jorge has some money, and then he earned $16 for babysitting. Now, Jorge has $68. How much money did he have to start with? *Start (decrease) unknown:* Jorge had some money. Then, he spent $29 at the ballpark and has $23 left. How much money did Jorge have before going to the ballpark?	*Multiple changes:* Jorge had $78. He stopped and bought a pair of shoes for $42 and then he spent $12 at the grocery. How much money does Jorge have now?

Material collected from: Griffin & Jitendra, 2009; Fuchs et al., 2014; Fuchs, Seethaler, et al., 2008; Fuchs et al., 2010; Jitendra, 2002; Kintsch & Greeno, 1985; Van de Walle, Karp, & Bay-Williams, 2013.

Figure 2. Additive schemas.

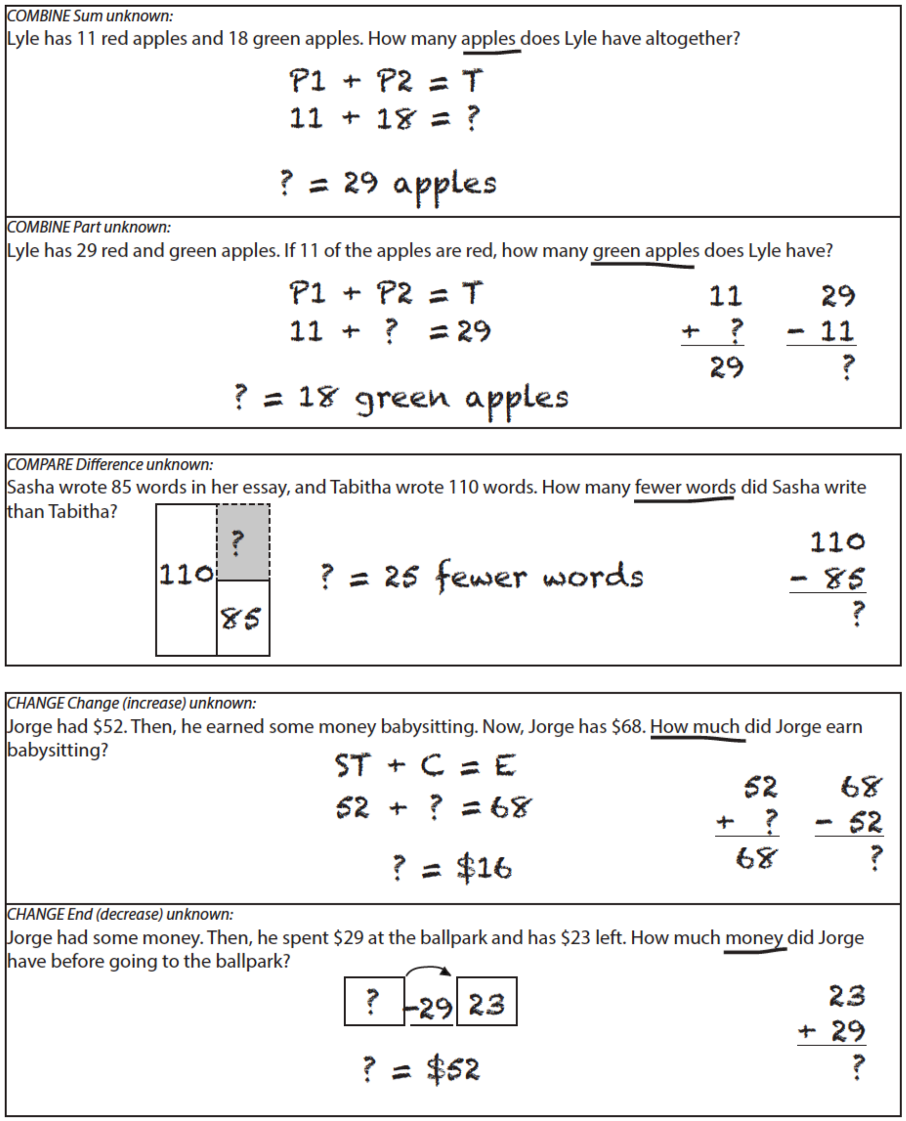
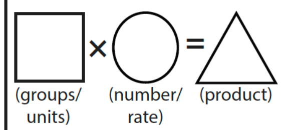
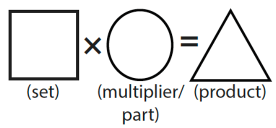
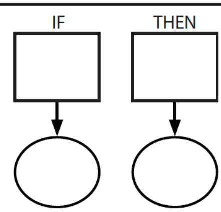
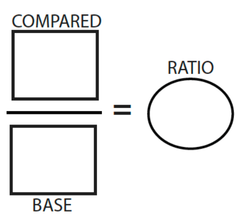


Figure 3. Worked examples of additive word problems.

Schema and Definition	Graphic Organizers	Examples			Variations
Equal Groups (Vary) A number of equal sets or units		*Product unknown:* Maria bought 5 cartons of eggs with 12 eggs in each carton. How many eggs did Maria buy?	*Groups unknown:* Maria bought 60 eggs. The eggs were sold in cartons with 12 eggs each. How many cartons of eggs did Maria buy?	*Number unknown:* Maria bought 5 cartons of eggs for a total of 60 eggs. How many eggs were in each carton?	*With rate:* Maria bought 5 cartons of eggs. Each carton cost $2.95. How much did Maria spend on eggs?
Comparison One set as a multiple or part of another set		*Product unknown:* Malik picked 7 flowers. Danica picked 3 times as many flowers. How many flowers did Danica pick?	*Set unknown:* Danica picked 3 times as many flowers as Malik. If Danica picked 21 flowers, how many flowers did Malik pick?	*Times unknown:* Malik picked 7 flowers. Danica picked 21 flowers. How many times more flowers did Danica pick?	*With fraction:* Malik picked 25 red and yellow flowers. If 1/5 of the flowers were yellow, how many were red?
Proportions (Percentages; Unit Rate) Relationships among quantities		*Subject unknown:* Sally typed 56 words in 2 minutes. How many words could Sally type in 7 minutes?	*Object unknown:* Sally typed 56 words in 2 minutes. How many minutes would it take Sally to type 192 words?		*With percentage:* Watson received an 80% on his science quiz. If the test had 40 questions, how many questions did Watson answer correctly?
Ratio		*Base unknown:* Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 15 cookies, how many brownies did he bake?	*Compared unknown:* Justin baked cookies and brownies. The ratio of cookies to brownies was 3:5. If he baked 25 brownies, how many cookies did he bake?	*Ratio unknown:* Justin baked 15 cookies and 25 brownies. What's the ratio of cookies to brownies?	*With unit rate:* Paula bought 5 boxes of markers. She spent $9.75. What is the price of one box of markers?

Material collected from: Jitendra, DiPipi, & Perron-Jones, 2002; Jitendra & Star, 2011; Jitendra et al., 2009; Van de Walle et al., 2013; Xin, Jitendra, & Deatline-Buchman, 2005; Xin & Zhang, 2009.

Figure 4. Multiplicative schemas.

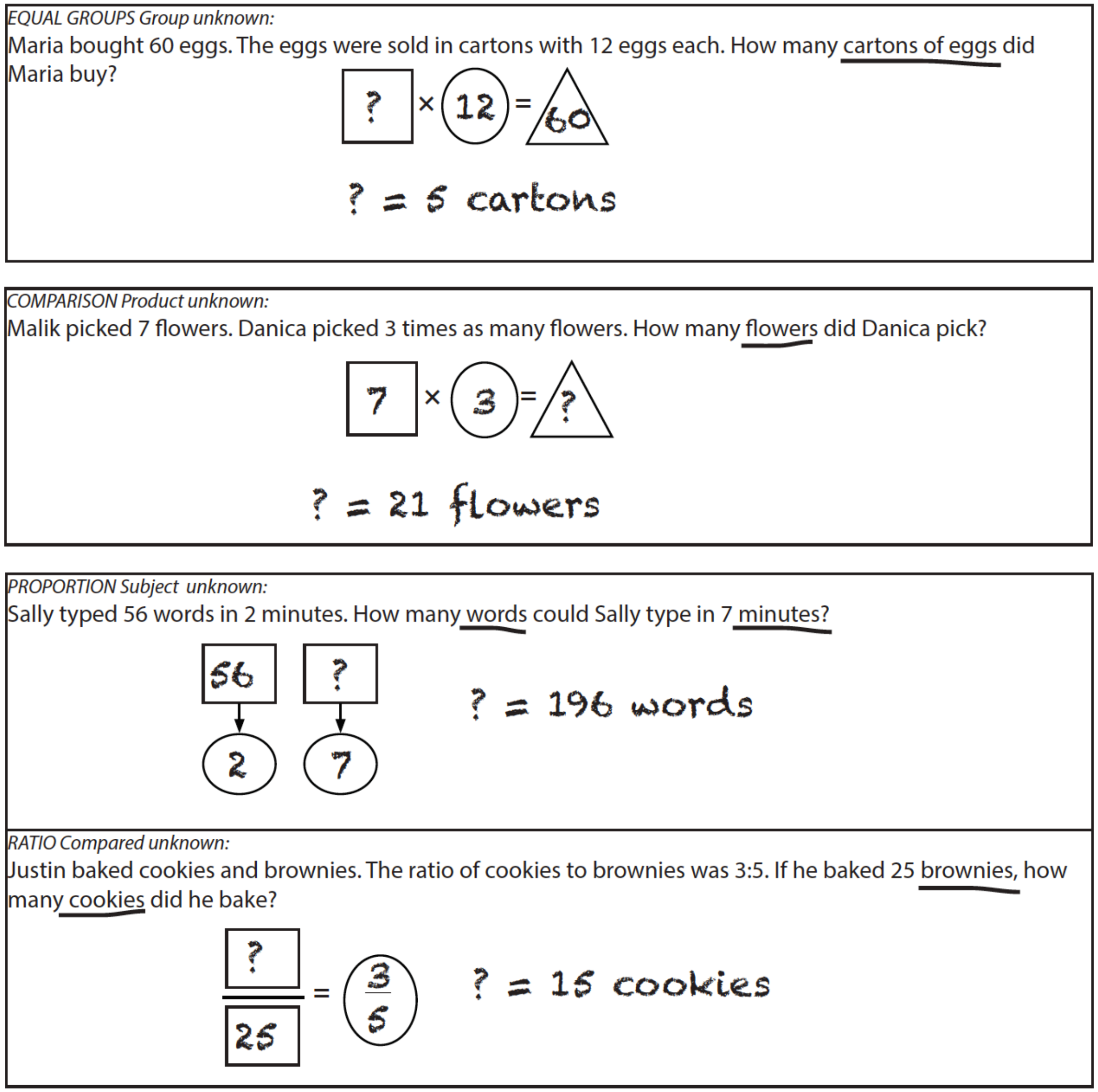


Figure 5. Worked examples of multiplicative word problems.

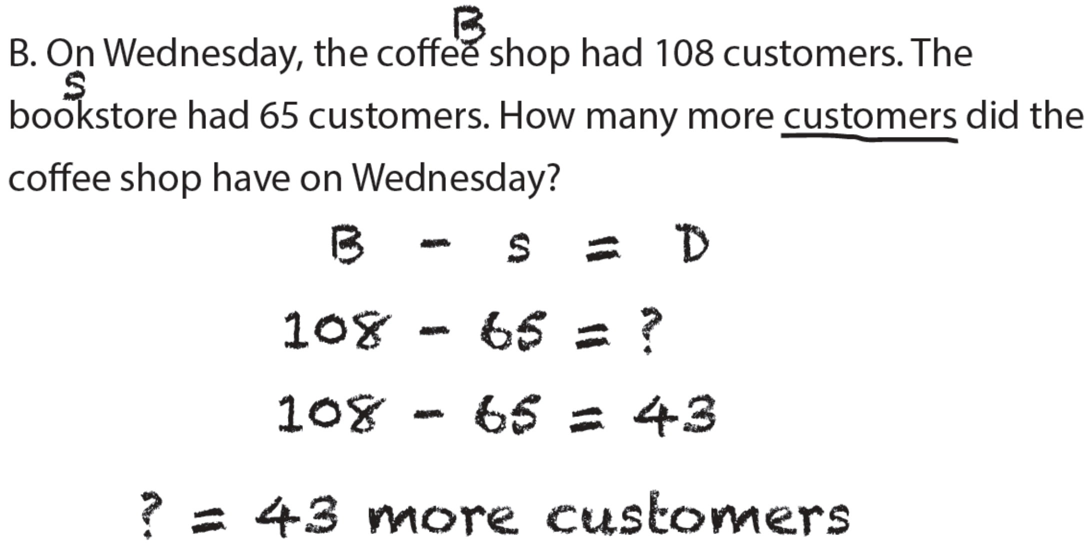


Figure 6. Sample compare problem.