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Teaching Students to Understand and Solve Word Problems

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Abstract

Many elementary students have difficulty solving mathematical word problems. To alleviate trouble with word problems, we designed a word-problem intervention for 3rd-grade students with mathematics difficulty. In this intervention, tutors help students understand the three word-problem schemas for addition and subtraction word problems. Tutors also help students learn how to set up and solve equations. This is important because students use equations to represent the word-problem schemas. We describe the intervention program and the daily activities to enable teachers to use the practices in their own mathematics teaching.

Teaching Students to Understand and Solve Word Problems

To demonstrate competency in mathematics, students need to understand how to set up and solve word problems. Word problems include a combination of words and numbers that require interpretation by the student (e.g., *Sadie has 14 dolls. Hayden has 21 dolls. How many dolls do the girls have together?*). In Texas, as early as third grade, students learn to set up and solve word problems to establish mathematics proficiency (e.g., TEKS 3.4A). Many students, however, experience difficulty with word problems (Jitendra et al., 2013; Kong & Orosco, 2015). To support students who struggle to solve word problems, we developed and refined a word-problem program (i.e., Pirate Math) specifically for students in Texas.

We designed the Pirate Math program to help third-graders who experience difficulty solving word problems set up and solve three types of addition and subtraction word problems. Several of the students in our study received special education services in mathematics but most received mathematics instruction in the general education classroom. Therefore, the program is designed for students with word-problem difficulty and can be delivered in a variety of settings (e.g., interventionists in special education or teachers in general education). Originally developed at Vanderbilt University (Fuchs et al., 2008; Powell et al., 2015), Pirate Math has successfully improved the word-problem solving performance of students in second and third grade. Like pirates who search for a treasure marked with an "X," students in the Pirate Math program find "X" by solving equations (e.g., 6 + X = 13) created to represent the structure of the word problem (e.g., Reece had 6 notebooks. Then, her aunt gave her some notebooks for her birthday. Now, Reece has 13 notebooks. How many notebooks did she get for her birthday?). This practice aligns with TEKS 3.5A in which students are expected to represent addition and subtraction problems with equations.

Addition and Subtraction Schemas

For addition and subtraction word problems, there are three word-problem schemas (i.e., problem types). In lieu of describing a word problem as "addition" or "subtraction," a schema allows a student to understand the underlying structure of the word problem. The operation does not define the word problem – the schema does.

The three word-problem schemas include Total, Difference, and Change (Fuchs et al., 2008; Powell, 2011; Riley & Greeno, 1988). In Total problems, also called combine or part-part-whole problems, students combine amounts for a total (e.g., *Donna ordered 7 pizzas. Michelle also ordered some pizzas. If they ordered 15 pizzas altogether, how many pizzas does Michelle have?*). Students may solve Total problems where the total is missing or one of the parts is missing. In Difference problems, also called compare problems, students compare two amounts for a difference (e.g., *Andrew bought 25 records. John bought 18 records. How many fewer records did John buy?*). Students may solve Difference problems where the difference is missing, the greater amount is missing, or the amount that is less is missing. In Change problems, students start with an amount and the amount increases or decreases to a new end amount (*e.g., Kristi picked 22 flowers. Then, her friend Laura gave her 17 more flowers. How many flowers does Kristi have now?*). Students may solve Change problems where the start amount is missing, the change amount is missing, or the end amount is missing.

Overview of Lessons

The Pirate Math intervention includes over 50 one-on-one lessons, implemented three times a week, with each session lasting about 30 min. Each lesson consists of (1) Math Fact Flashcards, (2) tutor-led activity about the equal sign (i.e., Equation Quest), (3) tutor-led lesson featuring schema instruction (i.e., Buccaneer Problems), (4) Shipshape Sorting, and (5) Jolly

Roger Review. The use of a set of brief activities proves an effective strategy for students with mathematics difficulties to sustain attention and learn content material (Zheng, Flynn, & Swanson, 2013).

In Math Fact Flashcards, the tutor shows flashcards to the student during two, 1-min timings. The student graphs the highest score from the two trials on a bar graph. In Equation Quest, the tutor and student complete activities related to solving equations and the meaning of the equal sign. In the Buccaneer Problems, the tutor provides scaffolded instruction to the student to set up and solve three additive word-problem schemas: Total, Difference, and Change. In Shipshape Sorting, the tutor reads aloud word-problem cards for 1 min as the student identifies the problem type as Total, Difference, or Change. In Jolly Roger Review, the student participates in a paper-and-pencil review, which consists of completing 9 addition and subtraction math problems in 1 min and a word problem in 2 min. Table 1 features the general outline of the Pirate Math lessons.

Table 1

Pirate Math Unit and Lesson Overview

Unit	Days	Topics Covered
Introduction	1-4	 Solve basic addition problems, with and without regrouping Solve basic subtraction problems, with and without regrouping Label graphs (i.e., bar graphs, pictographs, pictographs with multiplier, tables) Introduce and discuss meaning of the equal sign
Total	5-16	 Introduce Total problems (P1 + P2 = T) Solve Total problems with total missing (e.g., 5 + 4 = X) Solve Total problems with one part missing (e.g., 5 + X = 9) Introduce three-part Total problems (P1 + P2 + P3 = T) Solve three-part Total problems with total or one part missing (e.g., 5 + 4 + 2 = X; 5 + X + 2 = 11) Solve Total problems with graphs (i.e., bar, pictographs, pictographs with multipliers, tables) Use cubes to balance both sides of the equal sign Use cubes to solve missing addend problems Draw pictures to balance both sides of the equal sign Draw pictures to balance both sides of the equal sign

- Solve equations by balancing sides and isolating the X
- Solve Total equations by balancing sides
- Practice computation (addition and subtraction with and without regrouping)

Difference

17-33

- Introduce Difference problems (G L = D)
- Introduce and practice identifying compare words (e.g., more, fewer, less, taller, smaller, faster)
- Solve Difference problems with difference missing (e.g., 8-4=X)
- Solve Difference problems with the greater amount missing (e.g., X 4 = 4)
- Solve Difference problems with the less amount missing (e.g., 8 X = 4)
- Solve Total problems (i.e., total missing, part missing, three-part Total problems with total or part missing)
- Solve Total and Difference problems with graphs (i.e., bar graphs, pictographs, pictographs with multipliers, tables)
- Use cubes to balance both sides of the equal sign with subtraction problems
- Solve Total and Difference equations by balancing sides
- Draw pictures to solve missing addend, minuend, and subtrahend problems

Change 34-42

- Introduce Change increase and Change decrease problems (ST \pm C = E)
- Review the word more and how more can be used in a Difference or Change problems
- Solve Change increase and Change decrease problems with end amount missing (e.g., 8 + 5 = X; 8 5 = X)
- Solve Change increase and Change decrease problems with change missing (e.g., 8 + X = 13; 8 X = 3)
- Solve Change increase and Change decrease problems with start amount missing (e.g., X + 5 = 13; X 5 = 3)
- Introduce double Change problems (e.g., 8 + 5 + 5 = 13; 10 5 5 = X; 10 + 5 5 = X)
- Solve double Change problems with two increases, two decreases, and one increase and one decrease
- Solve Total, Difference, and Change problems with and without graphs
- Use cubes to balance both sides of the equal sign with addition and subtraction problems
- Solve Total, Difference, and Change equations by balancing sides
- Solve equations with more than two addends (e.g., 2 + 2 + 2 = X + 4)
- Solve equations with minuend (e.g., X 5 = 10 7) or subtrahend missing (e.g., 8 X = 10 7)
- Choose numbers for equations and solve (e.g., X =)

Review 43-51

- Solve Total, Difference, and Change problems with and without graphs
- Solve Total, Difference, and Change equations by balancing sides
- Solve equations with more than two addends (e.g., 2 + 2 + 2 = X + 4)
- Solve equations with minuend (e.g., X 5 = 10 7) or subtrahend missing (e.g., 8 X = 10 7)
- Choose numbers for equations and solve (e.g., X _ = _)

Activity #1: Math Fact Flashcards

Research suggests that fact fluency proves an important skill for future success in mathematics (National Mathematics Advisory Panel, 2008). We introduce Pirate Math lessons with a fluency-building activity where students are presented with addition and subtraction flashcards with minuend and subtrahends ranging from 0 to 9. On Days 1 through 4 of the program, the tutor teaches the student a counting-up strategy for learning addition and subtraction facts. The student is prompted to implement the counting-up strategy throughout the lessons (Tournaki, 2003). In the flashcard activity, the student has 1 min to complete each of two trials. After the initial 1 min trial, the tutor and student count the number of flashcards answered correctly. The tutor also provides immediate, corrective feedback to the student by reviewing the counting-up strategy for any noted errors. Prior to starting the second 1 min trial, the tutor challenges the student to beat his or her previous score. The tutor encourages the student as a way to alleviate the tedious nature of the fluency building activity and motivate him to improve his or her score (Fuchs et al., 2008). At the end of the second 1 min timing, the tutor and student graph the highest score (see Figure 1). The graph serves as a self-regulation tool for setting future goals and monitoring student progress (Fuchs et al., 1997).

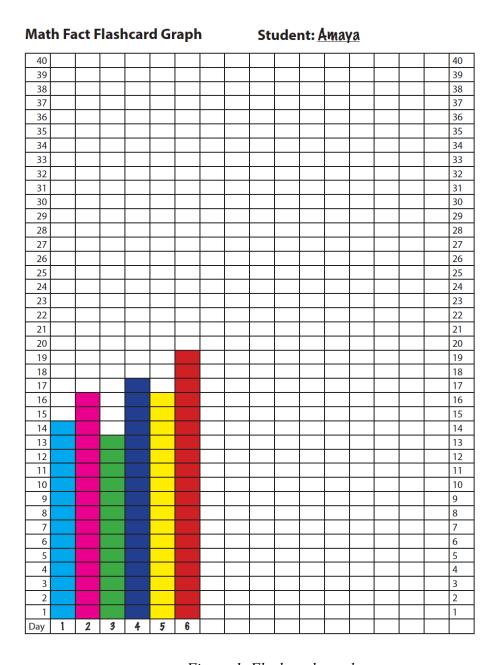


Figure 1. Flashcard graph.

Activity #2: Equation Quest

Students often misinterpret the meaning of common mathematical symbols (Driver & Powell, 2015). For example, students may associate the equal sign as an operational indicator (e.g., "do something" or "write the answer") instead of understanding the equal sign as a relational symbol indicating balance between two sides of the equal sign (Powell, 2012). To

equip students with a better understanding of the equal sign, the Pirate Math curriculum incorporates explicit instruction on the equal sign through an activity called Equation Quest. During this 2-min activity, the tutor reintroduces the common symbol and teaches the student to understand the meaning of the equal sign as *the same as*. Through a sequence of activities, the student learns that the equal sign acts as a balance between two sides of an equation and does not solely signal a calculation.

Equation Quest follows the concrete-representational-abstract (CRA) framework (Miller & Hudson, 2006) to teach the student about the equal sign, first using standard equations (e.g., 4 +7 = X) followed by non-standard equations (e.g., X = 8 + 2 or 5 + 4 = X + 7). The combination of standard and nonstandard equations helps students develop better pre-algebraic reasoning (McNeil & Alibali, 2005; Powell, Driver, & Julian, 2015). In the initial concrete phase, the tutor and student work with manipulatives (i.e., cubes) to determine if the number of cubes on the left side of the equation is the same as the number of cubes on the right side of the equation (see Figure 2). The tutor prompts the student to add or subtract cubes from one side of the equation to make both sides the same. As the lessons progress, the tutor and student enter the representational phase of the framework where the student draws shapes to represent the equations. After the student masters the concept of the equal sign using cubes and drawings, the abstract phase ensues. In the abstract phase, the student only uses numbers and symbols to determine if the left side of the equation is the same as the right side of the equation. The student learns a sequence of steps to balance an equation, using variables (e.g., "X") to represent missing numbers. To provide consistency across lessons and student participants, the tutor maintains the language of the equal sign (e.g., the same as) throughout the Pirate Math lessons.

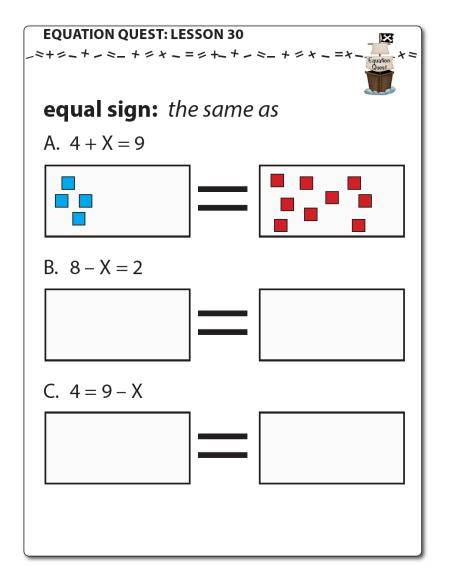


Figure 2. Sample Equation Quest activity.

Activity #3: Buccaneer Problems

The third activity consists of tutor-led schema instruction through Buccaneer Problems. Days 1 through 4 of the Buccaneer Problem lessons include a review of addition and subtraction skills using the *counting up* strategy to assist with fact-retrieval difficulties and regrouping for double-digit calculations. On Day 5, the student begins to apply the learned fluency and equation-solving skills to solve word problems. Each lesson includes a review of previously learned material, is highly interactive, and requires the student to verbalize his or her thinking.

When approaching any word problem, the student learns to think through the problem before solving it by first checking for a table or a graph and numbering it (if applicable), and then *RUNning* through the problem. To *RUN*, the student must *Read* the problem, *Underline* the label and cross out irrelevant information, and *Name* the problem type (i.e., choose the correct schema to use) by asking questions about the problem. To determine if a problem is a Total problem, the student asks: *Are two or more parts being put together for a total?* For Difference problems, the student asks: *Are two amounts being compared for a difference?* For Change problems, the student asks: *Is there a starting amount that increases or decreases to a new amount?* The student then employs a written series of steps to solve the problem. The steps are specific to each of the three schemas, but for all problems, the student is taught to use an equation to represent the problem and to mark "X" to represent the missing information. For the young pirate, "X" represents the treasure for which they are searching.

Total problems. The Total schema first is introduced on Day 5. The missing information (i.e., "X") may be the total or one of the parts. After checking for a table or a graph and RUNning through the problem to organize and prepare to solve, the student uses five steps to find the solution of Total problems: (1) Write P1 + P2 = T (i.e., part 1 + part 2 = the total), (2) Find T, (3) Find P1 and P2, (4) Write the signs, and (5) Find X. When these steps are combined with the equation-solving skills learned in Equation Quest, the student's paper looks similar to Figure 3. For Total problems with more than two parts, the student is taught to simply alter the equation (i.e., P1 + P2 + P3 = T).

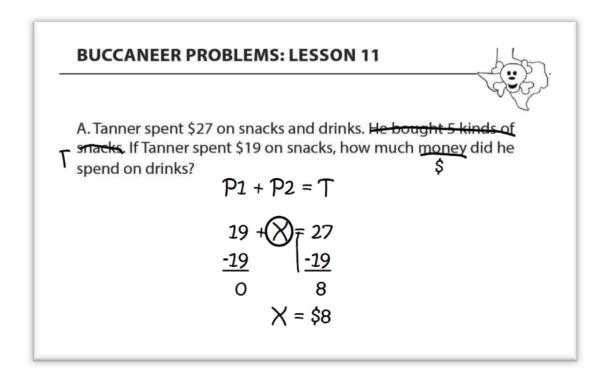


Figure 3. Total problem example.

Difference problems. In Difference problems, the student learns to compare an amount that is greater and an amount that is less to find the difference. The missing information (i.e., "X") for Difference problems may be the amount that is greater, the amount that is less, or the difference. The most important problem attribute used for identifying this schema and correctly solving the problem is the compare sentence. The student learns to find the compare sentence by looking for a compare word (e.g., words like *more*, *less*, or *fewer*, or other words like *older*, *shorter*, or *faster*), and then using the compare sentence to determine which quantities are greater and less, and whether the difference is given or missing. There are six steps to solving a difference problem: (1) Write G - L = D (i.e., amount that is greater – amount that is less = difference), (2) Put brackets around the compare sentence and label G and L, (3) Find D, (4) Find G and L, (5) Write the signs, and (6) Find X (see Figure 4).

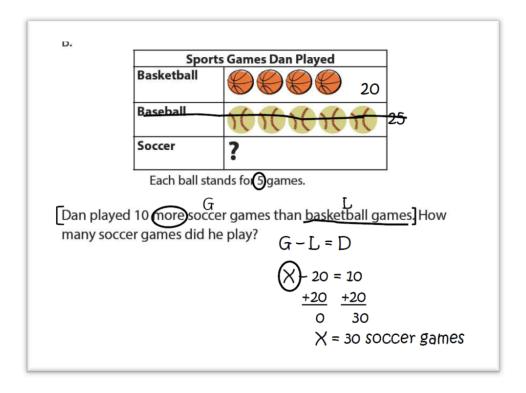


Figure 4. Difference problem example.

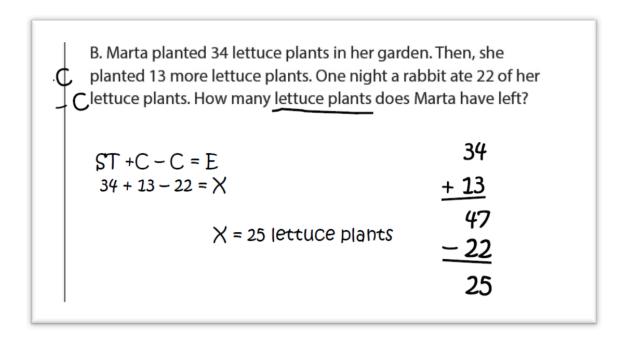


Figure 5. Change problem example.

Activity #4: Shipshape Sorting

Shipshape Sorting is a timed activity that allows the student to practice identifying word-problem schemas learned during the Buccaneer problems. Before the sorting activity begins, a mat with four squares is placed in front the student. Each square is labeled with a word-problem type letter (i.e., T for Total, D for Difference, or C for Change) or the question mark symbol (see Figure 6). The tutor reviews the three word-problem schemas, and explains the directions to place each word-problem card on the square with the corresponding problem type letter (i.e., T, D, or C). If a student is uncertain about the problem type, he should place the card on the square with the question mark symbol. The tutor reminds the student to sort the word-problem cards and to not solve any of the word problems. The tutor sets the timer for 1 min and reads the first word-problem card aloud before handing it to the student. The tutor waits for the student to place the card on the mat before reading the next word-problem card. After 1 min, the tutor reviews the sorting cards on the mat by providing immediate, corrective feedback to the student on at least

three of the word-problem cards. Shipshape Sorting provides a valuable opportunity for the student to practice identifying word-problem schemas. Efficiently and accurately sorting word-problem schemas proves helpful for selecting the appropriate equation (i.e., Total, Difference, or Change equation) to solve each word problem.

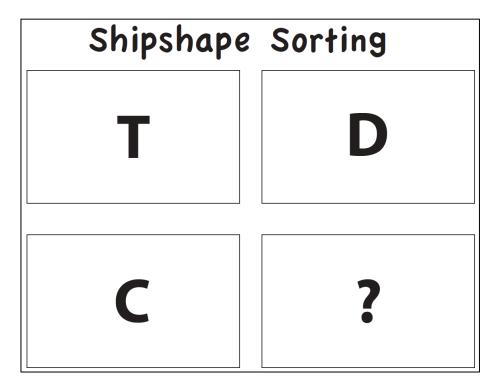
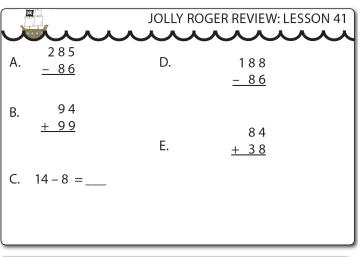


Figure 6. Sorting math.

Activity #5: Jolly Roger Review

The Jolly Roger Review includes a brief, timed review of the lesson content. Low-stakes practice testing outside the classroom has been shown to improve test scores for students across grade and achievement levels (Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013). Specifically, practice testing directly improves test-taking ability by providing student exposure to the test-taking environment and indirectly by promoting knowledge of the material (Dunlosky et al., 2013). Low-stakes practice testing proves especially beneficial for students when provided in conjunction with immediate feedback (Dunlosky et al., 2013). The Jolly Roger Review

activity incorporates such components of low-stakes practice testing. First, the student has 1 min to answer up to nine computational math problems (i.e., single and double-digit addition and/or subtraction problems) or write appropriate equations for the three word-problem schemas (e.g., Total equation: P1 + P2 = T). Next, the student has 2 min to complete a word-problem using the appropriate schema steps taught during the Buccaneer Problem activity (see Figure 7). The student performs the timed review autonomously and then receives content-rich feedback from the tutor, which reinforces mastered content. The Jolly Roger Review activity promotes independent practice of previously learned material and preparation for the test-taking environment.



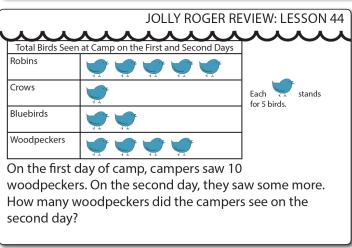


Figure 7. Sample Jolly Roger Review.

Motivation

Students with learning difficulties often need motivation to remain attentive and to increase positive behavior associated with learning (Fuchs et al., 2008). Token economy systems have demonstrated reduced problematic behavior and regulated attention in third-grade students with learning disabilities (Fuchs et al., 2008; Higgins, Williams, & McLaughlin, 2001). The Pirate Math program incorporates a token-based reward system and goal-driven tasks to promote student attention and encourage student motivation. At the beginning of each lesson, the tutor reviews the Pirate Math rules (e.g., listening to the instructor, staying in the seat, working hard) with the student. When a student follows the Pirate Math rules, he receives gold coins throughout the lesson. At the end of lesson, the student counts the number of gold coins earned and colors the appropriate number of coins on a treasure map (see Figure 8). When the student colors all of the coins on the treasure map, he reaches the treasure chest and receives a prize of his or her choosing from the treasure box. The behavioral component of the Pirate Math program decreases challenging behaviors and fosters an environment conducive to learning.

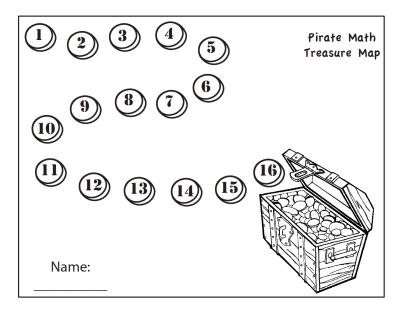


Figure 8. Treasure map.

Initial Results

During the 2015-2016 school year, we tutored third-grade students with mathematics difficulty in public schools in Austin, Texas. After screening over 1,100 general education students, we defined mathematics difficulty as performance below the 13th percentile on a test of word problems. Of the 133 students that we identified with mathematics difficulty, we provided individual tutoring to 88 of these students. A few of these students received special education services but the majority received instruction in general education without a diagnosis of a mathematics disability. At posttest, students receiving the version of Pirate Math described in this article demonstrated a 21-point gain (from pretest) on an assessment of single- and double-digit additive word problems, whereas students not receiving Pirate Math only demonstrated an 8-point gain. These results indicated that students who receive Pirate Math improve significantly on word-problem measures. We are continuing this research during the next few school years and look forward to sharing results with teachers across Texas and the United States.

Pirate Math in Your Classroom

In this article, we described the Pirate Math program and the strategies embedded within the program. Even though our focus is third grade, we believe teachers across the elementary grades could implement several of the Pirate Math strategies in their classrooms. For example, teachers should introduce the additive schemas (i.e., Total, Difference, Change) to students and help students understand word problems by schema rather than operation. Beyond third grade, teachers could introduce the multiplicative schemas (Xin, Jitendra, & Deatline-Buchman, 2005). Teachers should also provide students with an attack strategy (e.g., RUN) in which students are encouraged to read a word problem before doing any other work. The attack strategy also helps students work methodically through any word problem. Additionally, teachers should consider

using a fluency practice activity to help students who have difficulty with automaticity of mathematics facts. When students become more fluency with math facts, the computation within word problems becomes easier. Teachers should also provide instruction about the equal sign meaning *the same as*, which would help students solve different types of addition and subtraction equations and establish pre-algebraic reasoning. All of these strategies could be used to contribute to a deeper understanding of word problems in mathematics.

References

- Driver, M. K., & Powell, S. R. (2015). Symbolic and nonsymbolic equivalence tasks: The influence of symbols on students with mathematics difficulty. *Learning Disabilities**Research and Practice, 30, 127–134. doi:10.1111/ldrp.12059
- Dunlosky, J., Rawson, K. A., Marsh, E. J., Nathan, M. J., & Willingham, D. T. (2013).

 Improving students' learning with effective learning techniques promising directions from cognitive and educational psychology. *Psychological Science in the Public Interest*, 14, 4–58. doi:10.1177/1529100612453266
- Fuchs, L. S., Fuchs, D., Karns, K., Hamlett, C. L., Katzaroff, M., & Dutka, S. (1997). Effects of task-focused goals on low-achieving students with and without learning disabilities.
 American Educational Research Journal, 34, 513–543. doi:10.3102/00028312034003513
- Fuchs, L. S., Fuchs, D., Powell, S. R., Seethaler, P. M., Cirino, P. T., & Fletcher, J. M. (2008). Intensive intervention for students with mathematics disabilities: Seven principles of effective practice. *Learning Disability Quarterly*, *31*, 79–92.
- Fuchs, L. S., Seethaler, P. M., Powell, S. R., Fuchs, D., Hamlett, C. L., & Fletcher, J. M. (2008).
 Effects of preventative tutoring on the mathematical problem solving of third-grade students with math and reading difficulties. *Exceptional Children*, 74, 155–173.
 doi:10.1177/001440290807400202
- Higgins, J. W., Williams, R. L., & McLaughlin, T. F. (2001). The effects of a token economy employing instructional consequences for a third-grade student with learning disabilities:

 A data-based case study. *Education and Treatment of Children*, 24, 99–106.
- Jitendra, A. K., Rodriguez, M., Kanive, R., Huang, J.-P., Church, C., Corroy, K. A., & Zaslofsky, A. (2013). Impact of small-group tutoring interventions on the mathematical

- problem solving and achievement of third-grade students with mathematics difficulties. *Learning Disability Quarterly*, 36, 21–25. doi:10.1177/0731948712457561
- King, J. E., & Orosco, M. J. (2016). Word-problem-solving strategy for minority students at risk for math difficulties. *Learning Disability Quarterly*, 39, 171–181. doi:10.1177/0731948715607347
- McNeil, N. M., & Alibali, M. W. (2005). Knowledge change as a function of mathematics experience: All contexts are not created equal. *Journal of Cognition and Development*, 6, 285–306. doi:10.1207/s15327647jcd0602 6
- Miller, S. P., & Hudson, P. J. (2006). Helping students with disabilities understand what mathematics means. *Teaching Exceptional Children*, *39*, 28–35. doi:10.1177/004005990603900105
- National Mathematics Advisory Panel. (2008). Foundations for success: The final report of the national mathematics advisory panel. Washington, DC: U.S. Department of Education.
- Powell, S. R. (2011). Solving word problems using schemas: A review of the literature. *Learning Disabilities Research and Practice*, 26(2), 94–108. doi:10.1111/j.1540-5826.2011.00329.x
- Powell, S. R. (2012). Equations and the equal sign in elementary mathematics textbooks. *The Elementary School Journal*, *112*, 627–648. doi:10.1111/j.1540-5826.2011.00329.x
- Powell, S. R., Driver, M. K., & Julian, T. E. (2015). The effect of tutoring with nonstandard equations for students with mathematics difficulty. *Journal of Learning Disabilities*, 48, 523–534. doi:10.1177/0022219413512613
- Powell, S. R., Fuchs, L. S., Cirino, P. T., Fuchs, D., Compton, D. L., & Changas, P. C. (2015). Effects of a multitier support system on calculation, word problem, and pre-algebraic

- learning among at-risk learners. *Exceptional Children, 81*, 443–470. doi:10.1177/0014402914563702
- Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities and of solving problems. *Cognition and Instruction*, *5*(1), 49–101. doi:10.1207/s1532690xci0501 2
- Tournaki, N. (2003). The differential effects of teaching addition through strategy instruction versus drill and practice to students with and without learning disabilities. *Journal of Learning Disabilities*, *36*, 449–458. doi:10.1177/00222194030360050601
- Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of mathematical word problem solving instruction on middle school students with learning problems. *Journal of Special Education*, *39*, 181–192. doi:10.1177/00224669050390030501
- Zheng, X., Flynn, L.J., & Swanson, H.L. (2013). Experimental intervention studies on word problem solving and math disabilities a selective analysis of the literature. *Learning Disability Quarterly*, *36*, 97–111. doi:10.1177/0731948712444277