

Toward Common Ground: A Framework for the INVESTIGATION of Mathematics Methods Courses

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We present an analysis of the practices of one mathematics teacher educator in the context of an elementary methods course. Our analysis of the 51 tasks used in the course revealed a content structure characterized by four components: mathematical knowledge, psychology of mathematics learning, teachers' didactic actions, and reflection. The epistemic nature of the tasks in these content areas is described. We also observed a number of structuring frameworks, largely informed by theory, that were presented to the students as tools for completing course tasks. We conclude with a general framework for the investigation of other methods courses.

Objectives

Teachers have a number of ways to check the degree to which their practice is aligned with local and national objectives set out for elementary mathematics teaching. Aside from assessing their students' understanding in the classroom, there are a number of standards documents that can assist them to stay "on track," such as the *NCTM Standards* (NCTM, 2000), which provides teachers a comprehensive vision of elementary mathematics instruction in terms of content and process objectives. The same cannot be said of mathematics teacher educators (MTEs), those who prepare future elementary teachers of mathematics in the context of teacher education programs. Few, if any, guidelines exist with respect to content, curriculum, or pedagogical approach for the MTE, and as a result, MTEs engage in widely different practices, with virtually no communication among them (Osana, Sierpinska, Bobos, & Kelecsenyi, 2010). Furthermore, while the research on the practices of MTEs has grown over the last decade (e.g., Even & Ball, 2009), it provides at the same time analyses that are at a large and small grain size. For example, cross-cultural comparisons of the mathematics components of teacher training programs are available (e.g., Pope & Mewborn, 2009) as well as psychological descriptions of individual preservice teachers' thinking (Newton, 2008; Tirosh, 2000), and detailed, narrative accounts of single MTEs engaging preservice teachers in specific tasks (see Chapman, 2009). Other studies have examined the practices of MTEs by using their syllabi as data (Taylor & Ronau, 2006), but the limitation of this approach lies in the challenge of determining the types of knowledge their tasks actually or intend to generate, resulting in a superficial or disconnected analysis that often generates more questions than answers.

Such varying perspectives and levels of analysis make it difficult to compare the practices of MTEs in their methods courses, which is the objective of our current research. Little is known about what MTEs do as they prepare future teachers of mathematics, including the reasons for what they do and the effects of their practices. Before one can even determine the effects of their specific practices, however, one needs a common language and organizing framework to analyze and document them. Thus, the objectives of the current study is twofold: (a) to present a detailed analysis of the practices of one MTE, and (b) to present a framework that can be used to document the practices of other MTEs in the context of their own methods courses.

The present study is part of a larger, ongoing project in which we are analyzing the data from five other university sites. We are currently using the framework presented in this paper, called

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the Mathematics Teacher Educator Framework (MTEF), to document the practices of the other five MTEs from whom we collected data. In so doing, we are refining, and may possibly expand, the framework so that it is general enough to examine the practices of any other MTE in the context of an elementary methods course. This will lead the way for more meaningful dialogue to take place among MTEs and allow for common ground on which to conduct future research in the area. While the findings presented in this paper are the beginnings of a work in progress, we nevertheless believe that our insights at this stage may prompt other MTEs, as well as scholars in the field, to think about adopting the common language we propose through the MTEF to communicate more productively about the preparation of mathematics teachers.

Theoretical Framework

The approach we take in our analysis of MTEs' practices is through an examination of the tasks they use with preservice teachers, those that are required of them in class as well as assignments that are to be completed outside of class. A theoretical framework that we have found particularly useful in our examination of tasks is derived from Chevallard's (2002) "praxeology theory." A praxeology is a theoretical model of a practice, and as such, a praxeology of a practice describes it using four dimensions: tasks, techniques, technology, and theory. One can begin talking about the existence of a "practice" in the execution of a class of tasks if (a) the *tasks* have been divided into types; (b) there are *techniques* for tackling each type of task; (c) there is a conceptual framework and *technological* tools to justify the purposes of the tasks and the validity of the techniques; and (d) there is a discourse that systematizes and brings theoretical coherence into the previous elements of the practice. The discourses in (c) and (d) are necessary for the practitioners to communicate their practice to others and to assess whether a task has been satisfactorily completed.

So far, the Chevallard's (2002) framework of "praxeology" has been usefully applied primarily in the study of practices in mathematics teaching (e.g., Barbé et al., 2005; Sierpinska, Bobos, & Knipping, 2008). We argue, however, that the same theory can be used to analyze our observations of MTEs' practices in their methods courses. In this paper, we show how we used Chevallard's model of Anthropological Theory of the Didactic (ATD) to analyze the tasks collected from one MTE in an elementary teacher-training program at a Canadian university.

Method

Participant and Settings

The mathematics teacher educator in the present study (referred to in this paper as MJ) was a full-time, tenured faculty member in education at a large university in Canada. For the 10 years prior to our data collection at this institution, MJ was responsible for teaching two required elementary mathematics methods courses, each offered during 13-week semesters. The first of these courses, offered every fall, provides the context for the present study. MJ's research background was in educational psychology, which included a minor in mathematics education.

Data Collection and Analysis

The primary source of data for the present study was 51 tasks used in MJ's methods course. We created a rubric based on the ATD model, and used it to code each task as we made a first pass through the data. The rubric contained two major categories: (a) the institutional status of the task, which contained the subcategories of in-class activity, take home assignment, and in-class test, and (b) ATD analysis, which contained the four subcategories specified by

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Chevellard's (2002) ATD model, namely task type, technique, technology, and theory.

We also interviewed MJ to obtain more detail about each task and to tap into her justifications for including the tasks in her course. We conducted a total of 15 interviews with MJ over the course of one year, with the duration of each ranging from half an hour to two hours. The interviews were semi-structured, and included questions such as, "Why did you choose this task in particular? Why did you design it this way? What do you intend for the students to learn by engaging in this task? How was it implemented in the class?" Four of the interviews were audio recorded; detailed notes were taken during the remaining interviews. Finally, a third source of data used in our analyses was the collection of documents provided to students in the course, which included the syllabus, all handouts and tests, and lecture slides.

The transcriptions of the audio recorded interviews and the interview notes were analyzed using grounded theory techniques (Strauss & Corbin, 1998). These analyses were used to supplement the initial coding of the tasks to generate further categories in the rubric, which, through several passes through the interview transcripts and careful examinations of course documents, allowed us to describe the course and to create the MTEF in its current state. More specifically, when coding for task type, we coded the data for the content that was targeted by each task (e.g., psychological principles underlying children's thinking). This allowed us to address the structure of the course content. When coding for techniques, we searched for ways suggested by MJ to go about working through the task (e.g., "use your knowledge of problem types and apply it to extend the given third-grade activity"). We also coded the data for the techniques associated with each task, which directed us to search for tools suggested by MJ that justified the use of the techniques (e.g., a set of principles for analyzing a classroom lesson). Finally, we coded the data for evidence of any discourse that was intended to systematize and bring together theoretical coherence to the technologies recommended for each task.

Results and Discussion

In this section, we illustrate MJ's practice by presenting the content structure of the course (*task type*), the *techniques* suggested by MJ to tackle the tasks, the *technologies* MJ required her students to use as tools in their use of techniques, and finally, the *theories* used to justify the technologies. Our analysis of the tasks using the ATD framework allowed for an additional theme to emerge, called *epistemic actions*, which describes the kinds of knowledge that MJ intended to foster using the tasks she chose. Note that in this section, we refer to the preservice teachers in MJ's course as "students."

Task Types and Techniques

The content structure of MJ's course consisted of four content categories: (1) mathematical knowledge for teaching, (2) psychology of mathematics learning, (3) the teacher's didactic actions, and (4) reflection. We describe each of these categories below.

Mathematical knowledge for teaching. The first category – mathematical knowledge for teaching (MKT) – is defined here as knowledge of the concepts and principles of school mathematics (1). In MJ's course, MKT included knowledge of numbers and their properties, models of mathematical operations, and the ability to solve the problems in the elementary mathematics curriculum. There were 17 MKT tasks in MJ's course, which constituted 33% of all the tasks analyzed. A sample MKT task is presented in Figure 1.

Psychology of mathematics learning. We defined the category of Psychology of Mathematics Learning (PML) as knowledge of how children learn mathematics, which included, in MJ's course, (a) how children learn to count and understand number, and (b) the types of strategies

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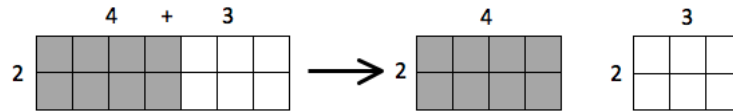
children acquire as they learn to solve problems with whole numbers. The same number of PML tasks was assigned by MJ as MKT tasks – that is, we coded 17 PML tasks, which made up 33% of all tasks in the data set. A sample PML task is presented in Figure 2.

Is the statement below true or false? Justify.

On one of Mr. Barr's lessons on multiplication in his fifth-grade classroom, he writes the following on the board:

$$2 \times 7 = (2 \times 4) + (2 \times 3)$$

One of his students raises his hand and asks, "Where does the extra 2 on the right hand side come from?" Mr. Barr then draws the following picture and says, "See? It was there all along!"



Statement: Mr. Barr is using a picture to illustrate the associative property.

Figure 1. Sample MKT task (adapted from Sowder, Sowder, & Nickerson, 2011).

Is the statement below true or false? Justify.

Jonathan used the following method to solve $234 \div 3 = \square$.

$$\begin{array}{l} \text{What is } 234 \text{ divided by } 3? \\ 70 + 70 + 70 = 210 \\ 20 \div 3 = 6 \text{ R}2 \\ 4 \div 3 = 1 \text{ R}1 \\ \text{So, } 234 \div 3 = 77 \text{ R}3 = 78 \end{array}$$

Statement: This method will work for any division problem with whole numbers (divisor $\neq 0$).

Figure 2. Sample PML task.

Teacher's didactic actions. There were 11 tasks (22% of all the tasks) that we placed in the category of Teacher's Didactic Actions (TDA), which we defined as the knowledge and skills needed to perform pedagogical actions, such as identifying the didactic objective of an activity, using a pictorial representation to explain an algorithm, and producing tasks intended to mobilize a specific mathematical concept. We present a sample TDA task in Figure 3.

Reflection. In the fourth category of task type, Reflection, we placed 6 of MJ's tasks, which constituted 12% of all 51 tasks. Reflection tasks were those that targeted the students' beliefs about the goals and objectives of the course in relation to their development as teachers. More specifically, MJ required her students to think about why subject matter knowledge is important for elementary mathematics teachers, how a "good" teacher uses mathematics in the classroom, and to identify a new kind of knowledge (namely, the professional body of mathematical knowledge that is "pedagogically useful," Ball & Bass, 2000) and distinguish it from the "plain old math," a term used by MJ to describe what she believed reflected her students' conceptions of the subject-matter. One such task assigned by MJ was to write a brief "journal entry" on the first day of class describing her students' thoughts about various aspects of a teacher's subject-matter knowledge, including their beliefs about the types of mathematical knowledge necessary for elementary school teachers. Subsequently, MJ required her students to read a non-technical article (Ball, Hill, & Bass, 2005) on the topic of mathematical knowledge for teaching and to compare Ball et al.'s account to the beliefs expressed in the journals of their peers, which were

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anonymously posted on the course's website.

Printing Pages

Book pages are printed on large sheets of paper called *forms*. Each form contains the same number of pages, so the number of pages in a book will always divide by that number with no remainder. After the forms are printed, they are cut apart so the pages can be bound.

? How many pages can be on a form? Explain your answer.

Book	Pages
1. How Come the Best Clues Are Always in the Garbage?	176
2. How Can I Be a Detective if I Have To Babysit?	160
3. Who's Got Gertie? And How Can We Get Her Back!	176
4. How Can a Frozen Detective Stay Hot on the Trail?	168
5. What's a Daring Detective Like Me Doing in the Doghouse?	192
6. How Can a Brilliant Detective Shine in the Dark?	200
7. What's a Serious Detective Like Me Doing in Such a Silly Movie?	192

As a teacher, you want to use the context of the problem to extend students' knowledge of multiplication and division – that is, to see how they think about other problem types. Write one word problem, in the context of this situational problem, that you might use with your students to meet this objective.

Figure 3. Sample TDA task (situational problem from Kestell & Small, 2004).

Across all the tasks in the four content categories, we observed MJ specifying a number of techniques for her students in the course. These techniques included using a checklist; recalling a definition; applying a principle or property; reading a text; describing observations; and comparing actions, tasks, and strategies; and were made explicit in all the tasks MJ assigned.

Technologies and Theories

Technologies can involve terminology, number facts, mathematical definitions, types of strategies used by children to solve problems, and didactic principles. Some technologies are loose collections of facts or principles, and others cohere as comprehensive frameworks. In classifying the technologies in MJ's course, we observed that they fell into two categories: (1) related to the mathematics itself (e.g., principles and properties of involving operations with whole numbers), and (2) related to teaching actions and children's learning. The majority of the technologies in MJ's course took the form of "structuring frameworks"; they were collections of principles, tightly bound together, to be used as tools for completing tasks.

To illustrate, MJ presented a number of mathematical properties, such as commutativity, associativity, and distributivity, as tools for thinking about a teacher's actions. The task provided in Figure 1 shows how MJ required her students to use their knowledge of the distributive property of multiplication over addition to interpret a teacher's actions during a lesson. In addition, a large component of MJ's course centered on frameworks of problem types and children's thinking taken from Cognitively Guided Instruction (CGI; Carpenter et al., 1996), a mathematics professional development program for elementary teachers. In several of her tasks, MJ required students to use the CGI taxonomy of children's strategies to interpret and evaluate their mathematical work. Another task MJ asked her students to complete was to use the taxonomy to evaluate the relative difficulty of two division word problems – one measurement division and one partitive division – while taking into account a specific strategy to be used in each case. As a final example, MJ required her students to use a framework presented by Hiebert et al. (2007) on the dimensions of effective learning environments in mathematics. In one task, she required her students to analyze a videoclip of a fourth-grade lesson on area and perimeter and to analyze the teacher's actions using Hiebert's framework. In the assignment, MJ states: "This is not necessarily a perfect lesson... Use the Hiebert reading to point out, where applicable, ways the teacher creates an effective environment and ways he can improve his teaching."

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Our analysis revealed that MJ's technologies were informed by a deep theoretical understanding of teaching and learning in mathematics, but were left unjustified by theory. In particular, MJ noted specific reasons for not including theory in the course, such as a lack of time and pressure from the students themselves to know "the answer" and the one "right" way to teach. MJ was sensitive to the many "layers of complexity" in teaching mathematics, and as such, there were some tasks in her course that were left open to her students' interpretation. More specifically, MJ expected her students to use the structuring frameworks in completing the course tasks, but was less concerned with their analyses or creations *per se* – as long as they engaged in analytical thinking (in her case, used the frameworks consistently, applied the correct definitions, and attended to the information provided in the task), the task was left open to the students' interpretation of it. This relatively open-ended nature of many of her tasks caused, in her experience, great discomfort and uncertainty in her students. As such, MJ indicated to us that she was unwilling to add to the uncertainty by also requiring them to use theory to justify their actions in the context of the course tasks.

Epistemic Actions

We used our analysis of the 51 tasks to discern their epistemic nature – that is, from the results of our examination of the types of tasks emerged a picture of the type of knowledge MJ was intending to impart to the students in her course. Across all four content categories (MKT, PML, TDA, and Reflection), we observed seven epistemic actions intended by the tasks. These epistemic actions were: identify, produce, discuss/reflect, assess, model, solve, and explain. The actions differed according to the content with which they were associated. For MKT tasks, for example, students were required to *produce* a representation of a number with base-ten blocks; *identify* which property of an operation, among those they learned in class, was used in a teacher's didactic action or a student's solution; *identify* the type of a given problem based on a list of types of problems presented in class; and *solve* a mathematics problem. In the PML category, sample actions were to *identify* a child's strategy; *produce* an example of a given problem; use knowledge of children's thinking *reflect* on parents' false beliefs about mathematics learning and to *produce* an argument to convince them otherwise; *discuss* the relative difficulty of a problem; and *assess* the counting skills of a child. With respect to tasks of type TDA, for instance, students were asked to *produce* a problem of a given type; *explain* a standard algorithm to a child; and *identify* the didactic objective of a given curricular activity. Finally, in the Reflection category of task type, students were asked to *reflect* on why teachers need to know mathematics and how they use their knowledge during teaching; and to *identify* a new category of mathematical knowledge, namely "mathematical knowledge for teaching," of which they should have become aware by reading an article by Ball et al (2005).

The most frequently occurring action observed in the data was "identify," which accounted for 33% of all the epistemic actions found in the 51 tasks. This was nearly twice as many as the number of actions in the next frequent category, "produce," which made up 17% of all actions. "Discuss/reflect" and "assess" were actions accounted for 16% and 14% of all epistemic actions, respectively, followed by the final three (i.e., model, solve, and explain) which together constituted 20% of all actions. An additional epistemic action emerged from our analysis, and that was of justification, which was systemic throughout the course and was required of students in almost all tasks. Indeed, 18 of the 51 tasks (35% of the total) required the students to provide a written justification of their claims – oral justifications were required in almost all other cases. MJ expected the students' justifications to be systemic, or in other words, based on a system of

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concepts, properties, and principles in the areas of MKT, TDA, and PML, which were, in MJ's course, accepted as true. The choice of the concepts and the truth of these properties, principles, and statements was not questioned or debated.

Contributions: The Mathematics Teacher Educator Framework

Cochran-Smith (2003) argued that the responsibility of preparing qualified teachers rests primarily with teacher educators, who are becoming increasingly active in studying their own practices. Because of this, she continues, one sees, "the emergence of new terminology and new contexts for doing and making public the work of teacher education" (p. 9). We present here what we call the Mathematics Teacher Educator Framework (MTEF), an organizing model that we believe provides a common discourse to describe and communicate the work of the MTE. The MTEF emerged from our analysis of the tasks used in MJ's methods course and, while it is relatively restrictive because it is based on only one site, we present it here as a first of its kind.

The MTEF is anchored in a task analysis informed by the ATD model (i.e., task, technique, technology, and theory), through which one can describe the content structure of a methods course. Currently, the model contains four content areas (i.e., MKT, PML, TDA, and Reflection), but as our data analysis on the larger project unfolds, additions and reconfigurations to this content structure are inevitable. From the present task analysis, we were able to discern the epistemic nature of the tasks used in MJ's course, which provides a view into the knowledge that she was trying to impart to her students. Given that uncovering the knowledge objectives of MTEs is by no means straightforward, we consider this aspect of the MTEF to be a particularly important contribution to the literature in mathematics teacher preparation.

Once the epistemic actions in any given methods course are determined, one can begin to classify them according to the degree to which they call for higher level thinking. In MJ's course, the tasks that called for "identification" were of lower level than those that called for "reflection" (Vygotsky, 1987). Indeed, in one of our interviews with MJ, we noticed that she was surprised that she had given so much weight to relatively low-level tasks. We used this opportunity to discuss with her how to increase the presence of reflection, generalization, and abstraction in her tasks. Reflective practice and adopting an inquiry stance toward teaching have been identified as key areas of growth for the developing mathematics teacher (Doerr et al., 2010; van Es & Sherin, 2008). The epistemic nature of tasks gleaned from the MTEF can itself be used as a tool for reflection, as MTEs may not be aware of the effects their own practices may have on the development of preservice teachers (cf. Torff & Sternberg, 2001).

Endnotes

1. We note that our conceptualization of MKT is more restrictive than that offered by Ball et al. (2008), who defined MKT as a combination of subject matter and pedagogical content knowledge (Shulman, 1987). We restrict our characterization of MKT to knowledge of school mathematics and the "specialized content knowledge" described by Ball et al. as the mathematical knowledge unique to teaching.

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