

AN ANALYTIC FRAME FOR EXAMINING TEACHERS' COLLABORATIVE MATHEMATICS WORK TO DEVELOP SPECIALIZED CONTENT KNOWLEDGE

Matthew P. Campbell
Oregon State University
campbmat@onid.orst.edu

Rebekah Elliott
Oregon State University
elliotttr@science.oregonstate.edu

A common practice in mathematics professional development to enhance teachers' content knowledge is to engage them in solving and discussing mathematics tasks. Yet, what is entailed in these experiences and how teacher discussions afford or constrain opportunities to develop teachers' knowledge of mathematics is not fully understood. In this paper, the authors provide a research-based framework to analyze teachers' mathematical productions (verbal reasoning and written inscriptions) used in collaborative mathematical work that illustrates the complexity of considering teachers' opportunities to learn and how discussion of mathematics may or may not be resources for learning.

Introduction

To better understand the mathematical knowledge teachers need to prepare them for their work, Ball and colleagues (Ball, Thames, & Phelps, 2008) have analyzed detailed records of practice to identify the content and pedagogical knowledge demands of mathematics teaching developing a practice-based theory of mathematical knowledge for teaching (MKT). From this work, some of the *mathematical* demands that are unique to the field of teaching have been outlined and include: recognizing what is involved in using a particular representation, linking representations to underlying ideas and to other representations, and giving or evaluating mathematical explanations (Ball et al., 2008). These demands become even more prevalent when teachers facilitate the development of students' mathematical understanding by selecting high-level mathematics tasks, orchestrating student discourse around the task, advancing students' mathematical justifications, and helping students seek connections between previous and new knowledge.

Mathematics professional development (PD) is considered one setting in which teachers may deepen their content knowledge used for teaching (Ball & Cohen, 1999). While mathematics content in PD is often overlooked or superseded by other issues of teaching (Hill, 2004), one common way to keep mathematics central is to engage teachers with activities and discussions around mathematics tasks (Jaworski, 2007). These activities often situate mathematical work in examinations of student work or representations of practice such as narrative or video cases (Kazemi & Franke, 2004; Silver et al., 2007). While *practice-based* (Ball & Cohen, 1999) activities are invaluable to mathematical and pedagogical learning, there is still much to learn about how teachers collectively do mathematics in ways that support their mathematical learning (Suzuka et al., 2010). This paper highlights research on teachers' collective opportunities to learn mathematics and shares a framework for analysis of teachers' mathematical work.

The most common rationale for engaging teachers in mathematical work is to advance teachers' mathematical content knowledge. However, this goal takes on varying levels of focus and specificity across studies. Recent research and development efforts have advanced that the purpose of doing mathematics in professional education (teacher preparation and PD) is to develop teachers' specialized content knowledge (SCK) needed to meet the demands of teaching (Ball et al., 2008; Elliott et al., 2009a; Lo, Grant, & Flowers, 2008; Suzuka et al., 2010). Programs like those discussed by Lo and her colleagues (2008) and Suzuka, Sleep, Ball, and

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their colleagues (2010) engage teachers in solving mathematics tasks and analyzing solutions, alternative strategies, and multiple representations in order to enhance teachers' content knowledge and connect it to the demands of teaching.

To develop teachers' knowledge, researchers have also asserted the need for teachers to justify their own mathematical reasoning. Simon and Blume (1996) considered how pressing on preservice teachers' mathematical justifications supported the emergence of learners' understandings of mathematical concepts. Other researchers have found that teachers' and teacher-leaders' justifications invoked or developed MKT through the unpacking of key mathematical ideas (Elliott, Lesseig, & Kazemi, 2009). Such mathematical work provides teachers with both a deeper understanding of the mathematics they use in the classroom as well as more facility to use this knowledge in a classroom in which students' reasoning is central (Stylianides & Ball, 2008).

Fostering meaningful mathematics learning for teaching in PD is no easy task. One prevailing issue that impacts this work is that doing mathematics with teachers is different from doing mathematics in the classroom (Elliott et al., 2009a). This is due in part to the fact that teachers tend to hold mathematical knowledge differently than students. While teachers may have previous experience with mathematical content, they often do not have a deep understanding of or facility with developing justifications that explain mathematical ideas (Lo et al., 2008). To meet the mathematical demands of teaching, teachers need not only *a* method for solving a math task, but a repertoire of solutions and justifications that unpack why methods work. Yet, a number of studies have found that teachers were less likely to positively evaluate justifications that use non-standard methods, those potentially more likely to be accessible to students who are not formally trained in proof, or to distinguish arguments that provided a mathematically valid reason rather than a thorough explanation of *how* a solution was developed (Bieda, 2010; Knuth, 2002).

Despite the struggles researchers have uncovered on engaging teachers in mathematical work that unpacks key mathematical ideas the importance of providing teachers with opportunities to do this work in PD is not diminished. Indeed, these struggles point to a need for more research to be done to understand the potential learning opportunities made available to teachers during collaborative mathematics work. If we expect teachers to have opportunities to learn mathematics for teaching in PD, leaders need understandings of what makes a discussion productive and how discussion might stay focused on key mathematical ideas. Similarly, the research community needs tools for analyzing discussions that coordinate argumentation with attention to teachers' specialized content knowledge. Detailed images of mathematics discussions among teachers could serve professional educators learning how to facilitate PD oriented to SCK and researchers who are trying to build a knowledge base on teacher learning.

We advance that researchers need opportunities to share the analytic work entailed in examining collaborative mathematical activity in order to construct more rigorous means of evaluating how conversations advance teachers' learning of mathematics for teaching. To this end, we share our coding system and examine the utility of it. Using data from a teacher-leader research project, we will illustrate how the framework examines what is entailed in teachers' mathematical discussions and how key mathematical ideas are potentially drawn upon. The framework and the data illustrations are a basis for a discussion on the import of frameworks that examine the depth of conversation, key mathematical ideas, and teachers' opportunities to learn MKT, in particular SCK. As such, we intend that this discussion will open up a dialogue for other researchers to consider how we might investigate teachers' collective mathematical work.

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Context

The seminars from which data for this paper are drawn are part of a five-year research and development project, *Researching Mathematics Leader Learning* (RMLL) investigating how teacher-leaders develop mathematically rich learning environments for teachers. The project's framework and design evolved to focus on developing teachers' SCK as a key purpose for engaging in mathematics during PD. RMLL seminars provided opportunities for teacher-leaders, who were also teachers of mathematics, to do mathematics, discuss the mathematics, and consider the mathematical entailments of the tasks they solved (other activities were part of RMLL seminars, but not central to this paper). For this paper we will refer to RMLL participants as teachers since we investigate their collective mathematical work and not activities central to their leading. Participants in this study ($n = 70$) were volunteers involved in one of two phases of RMLL seminars and were becoming, or already were, leaders in their schools, districts, and regions charged with facilitating mathematics PD. Most participants were current part- or full-time classroom K-12 teachers. A few participants were full time leaders of mathematics PD.

Using data on participants' mathematical productions in small and whole group discourse, RMLL researchers have focused on justification as a means to understand the mathematical work at play and the normative aspects of these interactions (Elliott et al., 2009b). This previous work inventoried what participants reported as acceptable justifications, what participants' solution methods entailed, and made claims about how justifications allowed for an understanding of the underlying mathematics (Elliott, Lesseig, & Campbell, 2010). The intent of our previous work was to provide images of mathematical justification productions that invoke and develop SCK.

We have come to realize that a focus on teachers' solution methods and justifications without considering the collective talk elaborating teachers' solutions severely limits how we might understand doing mathematics in PD and teachers' opportunities to learn. As a result, we consider teachers' justification episodes through a new lens drawing on research on mathematical discussions of both students and teachers.

Theoretical Considerations

We are interested in the collective work of teachers solving mathematics tasks with a facilitator. Guided by a situated learning perspective (Putnam & Borko, 2000; Wenger, 1998), this study examines the entailments of learners' participation in collective activity. Specific to our study, we investigate the collective work of a group of teachers solving mathematics tasks and the potential resources made available through their work to learn mathematics for teaching.

Previous studies have considered the nature of students' mathematical discussions as well as the mathematical learning opportunities they afford or constrain. In an account of students' collective mathematical learning, Cobb (2002) considers the mathematical ideas that students take up, entailments of classroom discussion, and how students use tools and inscriptions. We saw these foci as central in how to examine the collective participation of teachers in mathematical activity. Considering community discourse in terms of the three foci allows for the documentation of emergent practices, which account for mathematical learning of the community (Cobb, 2002). In his study Cobb uses Toulmin's (1958) scheme for argumentation to examine classroom discussion. Similarly, Forman and colleagues (1998) in their analysis of a mathematics classroom advance that Toulmin's argument analysis is a valuable tool for examining the mathematical reasoning used to solve tasks. In general, both research projects use a classification of utterances as *claims*, *warrants*, and *backings* to examine the structure of conversation to infer what we consider to be depth of a conversation. *Claims* are assertions in

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need of argumentative support because the audience has called them into question or the rules for social interaction require such support. *Warrants* explicate how conclusions have been drawn from accepted givens. *Backings* strengthen the acceptability of warrants, essentially serving as the “why” that refers to the structure of the key mathematical ideas at play.

Considering the benefits of the approach to analyzing classroom mathematical practices used by Cobb (2002) and Forman and colleagues (1998), we took into consideration recent studies that have taken a similar theoretical and analytical stance in examining teacher discourse around mathematics tasks. Steele’s (2005) analysis of mathematical and pedagogical discussions in a mathematics methods course also used Toulmin’s (1958) argument model as a basis for analysis. In addition to the focus on claims, warrants, and backings (referred to as “bridge statements” and “evidence”), Steele also considered the potential responses to an argument adding “challenges” and “connections” as elements of the community’s discourse structure and important for considering the depth of a discussion. Implicit in Steele’s analysis is how deep conversations afford teachers opportunities to learn mathematics, although he does not take this up directly. Steele’s work instead considers how argumentation may be driven by the underlying disciplines informing the conversation to explain how mathematical discussion differ from pedagogical discussions. We have found Steele’s analysis helpful to consider the depth of conversation, and we use a similar analytic tool to explicitly examine learning opportunities for teachers.

Researchers who directly consider the entailments of teachers’ discussions have reported conflicting results when examining the nature of mathematical and pedagogical conversations to advance teachers’ learning. Crespo (2006) dubbed teachers’ talk during mathematical work to be *exploratory*, in that it tended to be interactive, consisting of unprompted interruptions, disagreements, and tentativeness, and pedagogical talk as *expository*, more definitive in nature. Exploratory discussion allowed teachers to make public numerous mathematical ideas, but here the analyses did not explore the nature of the mathematical ideas, nor how ideas contributed to teacher learning. Crockett (2002), in another study, noted that teachers working on a mathematics task were so invested in reaching a correct answer that they missed out on examining the mathematical value and assumptions in the problem. Steele, Crespo, and Crockett provide valuable insights on teacher discourse of mathematics. However, each shares slightly different levels of detail and conflicting results for supporting teacher learning through teachers doing and discussing mathematics.

To fill the knowledge gap in understanding how teachers doing mathematics in PD may contribute to their development of SCK, we came to understand that our analyses must take up discursive interactions as mathematical justifications unfold in relationship to drawing on key mathematical ideas. Research on students’ and teachers’ mathematical discourse allowed us to conceptualize teachers’ math argumentation accounting for both how and what math ideas are justified.

Methods

Our work looking at teachers’ discussions around mathematics tasks and our consideration of how to analyze such work has been an iterative process. We developed our coding scheme based on the theoretical considerations uncovered in the literature and by reviewing data first individually, then together. An initial selection of video-data that included a variety of teachers across our two-phase research design, two different mathematics tasks that center on algebraic reasoning, and instances from early and late in RMLL seminars were examined to develop the window. After a code window was developed, an individual researcher would code the data and

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a second researcher would verify codes. Any differences in coding were resolved through discussion to arrive at 100% agreement.

Figure 1 illustrates the coding window used to analyze teachers' mathematical discussions in both small and whole group settings. Data were analyzed using the video analysis software, *Studiocode*. Participants' written mathematical inscriptions were also examined to look closely at what is referenced in video and the mathematical nature of the inscription.

Each video episode was coded as follows: Each discussion was coded into *idea units* based on the main speaker and the claim being argued or challenged. Each idea unit was then further described using elements of the argument structure: claim, warrant (referred to here as *evidence-how*) and backing (referred to here as *evidence-why*). Evidence-why codes were distinguished by levels of justification (Simon & Blume, 1996) to describe the backing for a solution. In addition to those elements of an initial argument we have also included challenges to a claim (e.g., press for justification, stumbling) and connections (e.g., to other solutions, across multiple representations) from the main speaker or the rest of the group. Finally, we coded when teachers explicitly identified key math ideas or implicitly discussed key math ideas embedded in their arguments. In using this code, we are not suggesting that an individual teacher knows a particular key idea. Instead, we are highlighting the ideas that are made public that have potential to serve in the development of SCK. Because our teachers, at times, would narrate their experience by talking about themselves as learners or how the work connected to other experiences we coded their narrations as *meta-talk* and *accounts of experience* to capture teachers' disclosures made during math activity.

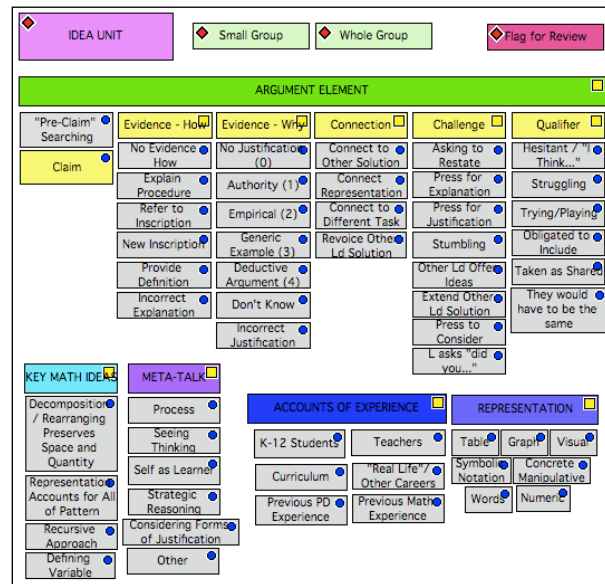


Figure 1. *Studiocode* Coding Window

Discussion of an Illustration of Teachers'

Collaborative Mathematical Work

To illustrate how a consideration of both teachers' justification productions and the argumentation structure of a teacher group support the interpretation of complex discourse of teachers in PD, analyses from our larger project will be discussed here. Data for the discussion in this paper came mid-way through the RMLL seminars where participants were engaged in collaborative discussions of solutions to a math task (the *Staircase* task; see figure 2). The *Staircase* task is a commonly used task in PD with the potential to highlight important mathematics ideas such as understanding how models can be decomposed and rearranged while preserving space and quantity and mapping relationships among representations, in particular how the composition of a figure is accounted

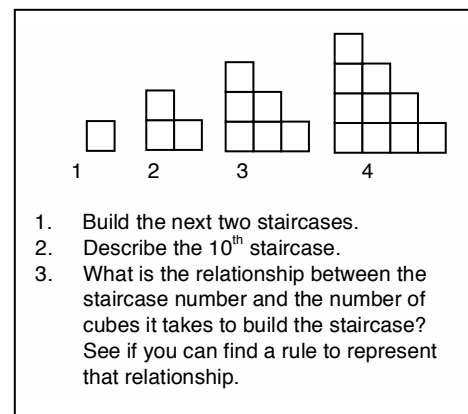


Figure 2. The *Staircase* Task

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for in a symbolic expression.

The episode shown here is a discussion among three Phase I participants: Wendy, Hal, and Erica (all pseudonyms). The transcript of the segment, our coding of this instance, as well as the relevant inscriptions can be found in Figure 3. In the episode, Wendy is presenting her solution to the *Staircase* task, which she mentions relates to a solution that Hal just presented. She begins by verbally recounting the way she recomposed the visual model in order to create a “duplicate” staircase. In the process of drawing a connection between her visual model and her symbolic solution, Wendy stumbles. Her next move we code as a new solution, because it uses her inscription differently. Wendy considers the re-conceptualizing of the inscription and its correspondence to a symbolic expression. Her restated idea exhibited a more transparent link between her representations and the idea of a duplicate staircase (and, thus, needing to divide by two). This move makes available the idea that decomposition preserves quantity. Hal’s revoicing of Wendy’s idea provides evidence that this connection has been taken up publicly.

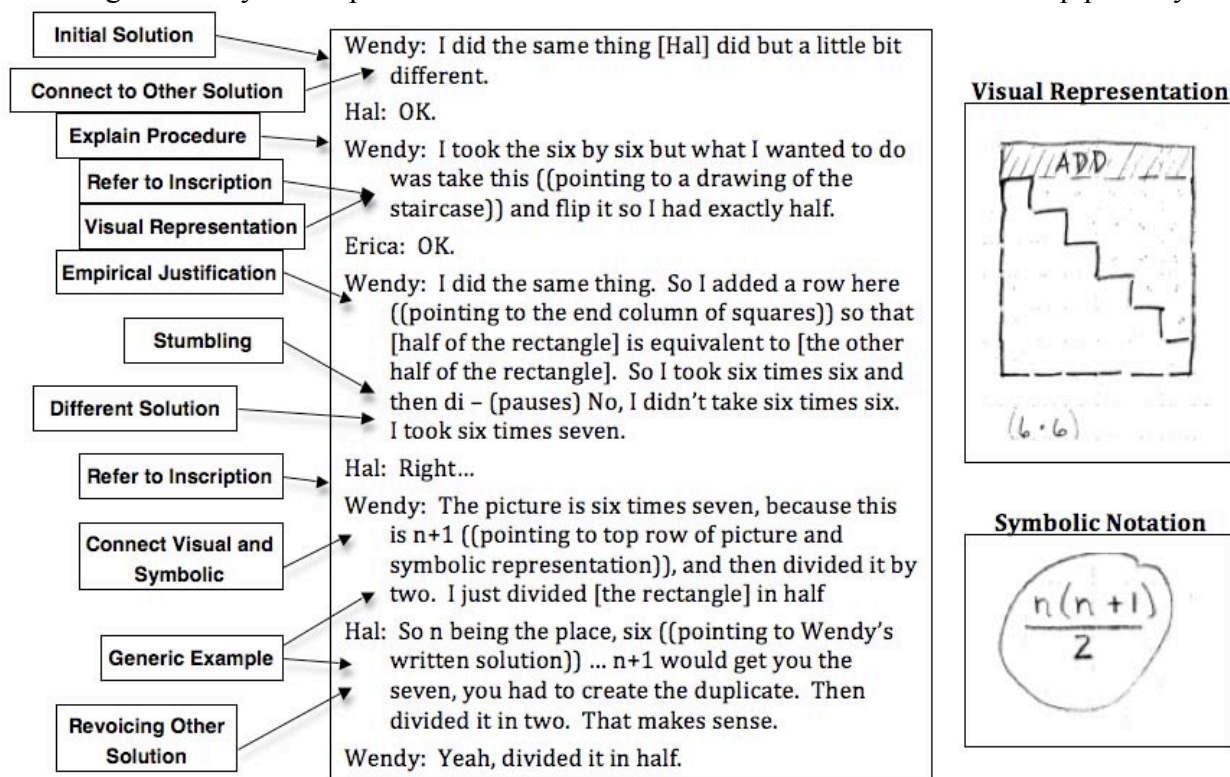


Figure 3. Teachers' Discussion and Inscription Around *Staircase* Task

What does this say about doing math in PD? In Wendy's first solutions we do not see her connect to a symbolic expression in a transparent way but she provides an explanation with a mention of another solution. We see that stumbling acts as a challenge that has opened up her reasoning as she connects her model to a symbolic expression such that the decomposition of the figure corresponds and is accounted for symbolically. Hal's later revoicing allows us to see his reasoning connected to her reasoning. As we examine the claims that these teachers are making – how they get supported, challenged, and used in the collective space – we are able to see the mathematical content that is emerging in the collective through the argument, interactions, and use of inscriptions. Further analyses of subsequent data will allow us to trace mathematical ideas across teacher discussion and consider what mathematical ideas are put on the table by this

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group, which seem to be taken up and unpacked through considerable discussion of claims, warrants (explaining how), backings (explaining why), challenges, connections and qualifiers. This analysis allows us to move our examination beyond an inventory of solutions and representational use. We are teasing out what is entailed in doing mathematics, the role of inscriptions and discourse. Subsequent data analysis will also examine the role of facilitation in teachers' collective mathematical work as the small group moves to a whole discussion.

Conclusion

In this paper we have provided the theoretical and analytic considerations for examining teachers' collective mathematical work accounting for argumentation, levels of justification, and key mathematical ideas. Previous research has accounted for teachers' argumentation without incorporating attention to justification or key mathematical ideas emerging in teachers' collective mathematical work. Important to note in these analysis is the role of our video coding tool, *Studiocode*, that makes it possible to examine over 20 episodes of teachers doing mathematics and provides methods for examining patterns across the 20 episodes to consider how teachers' mathematical learning emerges in the community.

The framework we have presented here affords ways of capturing the complexity of teachers' collaborative mathematics talk. With this framework, we are able to look at a wide range of data to better understand the aspects of collaborative talk that advance mathematical learning. The excerpt of data we chose for this paper also illustrates the framework's ability to highlight the opportunities for key mathematical ideas to emerge in discussion, such as "stumbles", outside challenges, and building connections across representations and solutions. As we continue to move forward with our examinations of teachers' collaborative mathematics talk in order to better understand these opportunities we plan to consider more deeply what stumbles or challenges provide teachers in their mathematical learning and uncover more aspects of collaborative work that facilitate the development of SCK.

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