

## CONNECTING PD TO PRACTICE: USING A TASK IN 7<sup>TH</sup> GRADE MATH<sup>1</sup>

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*In this study, we expand the Cohen and Ball triangle of interactions to explore the relationship of professional development to classroom practice. We consider a case study of one teacher's implementation of a task from professional development in her 7<sup>th</sup> grade classroom. We were specifically interested in how the content and pedagogy of the professional development would be adopted by the teacher. Our findings suggest that this teacher treated pedagogy and mathematical content as separable, which led to problematic implementation of PD practices.*

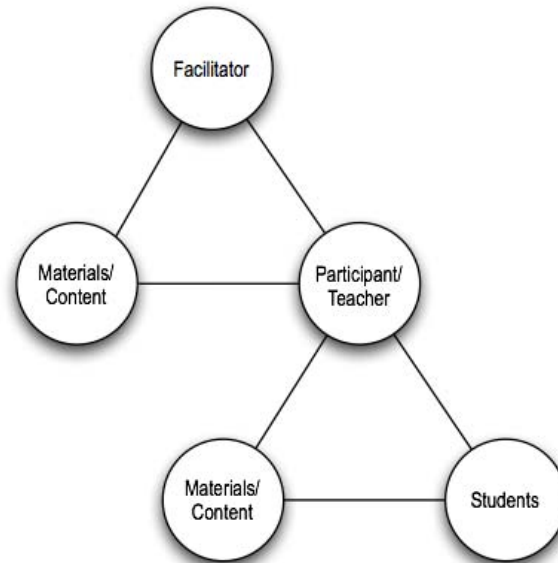
Research and theory around the design of high-quality professional development suggests that using materials like those a teacher would use in her classroom is one particularly effective strategy for influencing teaching (e.g., Banilower, Heck, & Weiss, 2007; Hill, 2004) as is modeling the kind of learning environment that is desired for the K-12 classroom (e.g., Desimone, et al., 2002; Elmore, 2002; Hill, 2004). Further, an emerging body of research highlights the need for high-quality teaching practices to be implemented in ways that are grounded in the mathematics. For example, Wood, Williams, and McNeal (2006) found that teachers who asked questions that pushed students to engage in inquiry and argument led to higher levels of success among mathematics students than teachers who asked questions that only required the students to report their thinking. While both instances involve *questioning*, the first is necessarily grounded in the mathematics and students' reasoning while the latter can be used generically (e.g., 'How did you get your answer?'). Similarly, Kazemi and Stipek (2001) identified the characteristics of classrooms in which there was more push for conceptual learning. Their findings suggest that these classrooms include norms such as: explanations need to include mathematical arguments rather than descriptions; students are expected to understand connections among multiple problem-solving strategies; errors are treated as a means for enhancing learning, thereby serving a generative role; and collaboration features consensus building through mathematical argumentation. Like the Wood et al. study, this study demonstrates that the precise ways in which pedagogical moves are implemented fundamentally shapes the classroom environment. Given what is known about professional development and what is known about teacher practice, we sought to understand what aspects of professional development (PD) a teacher might carry into her classroom and what the movement between the PD and classroom environment might look like.

### Theoretical Framework

We frame our effort by extending the triangle of interactions metaphor introduced by Cohen and Ball (1999, 2001). In their model, the classroom learning environment is shaped by the interactions of three primary elements: teachers, students, and content as embodied in materials. We extend this metaphor to include a second triangle, which represents the same elements interacting in professional development (see Figure 1). The two triangles are joined at the vertex representing the teacher/participant. This vertex is noteworthy because the teacher is not only present in both environments, but also because her interpretation of the PD fundamentally shapes both environments as she brings experiences from her classroom to the PD and experiences from the PD into her classroom. While it is true that the materials from the PD can be taken into the

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classroom, the teacher is responsible for turning the intended (written) curriculum into the enacted curriculum (Remillard, 2005).



**Figure 1. Double triangle of interactions representing the classroom (bottom) and PD environment (top).**

The teacher is ultimately responsible for implementing ideas from PD in her classroom. This study, therefore, documents one teacher's experience in PD and the way this experience shaped her classroom. Since the teacher is the point joining the two triangles it is important to examine the teacher in the professional development context and her classroom context. To frame our analysis, we focused on a specific set of instructional moves as well as the teachers' content knowledge. Specifically, we were interested in the ways the teacher supported connection-making in her classroom as this had been a significant factor in her PD experience. We defined connection-making with a 4-facet framework for connection making. The four facets include:

- *Questioning & Communicating*: in this framework, these serve as tools for articulating, expanding, and refining mathematical ideas. Specific kinds of communication in which we were interested included that which supported exploring connections among representations, problematizing ideas, promoting conjecture-making and testing, and engaging with ideas in ways that move the learner to deeper levels of mathematical understanding.
- *Reasoning with Representations*: in the framework, reasoning with representations is seen as connection making. To reason with a representation, a person must draw on a set of mathematical understandings, embody them in the representation, and communicate to others about those ideas using language and the representation itself.
- *Embracing Multiple Approaches*: this framework builds from the perspective that different people have different solution paths. The path taken depends on knowledge invoked and the solver's knowledge. The value of highlighting these differences is in its effectiveness for introducing all of the learners to new perspectives and promoting their ability to create connections among ideas.
- *Scaffolding*: this framework is built from the perspective that facilitators make a number of moves that support learners in moving from their current levels of

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understanding to new understandings. Scaffolding moves include, but are not limited to, those consistent with the five practices for facilitating discourse around cognitively challenging tasks (Stein, Engle, Smith, & Hughes, 2008). These include anticipating responses, monitoring responses as learners work, select particular approaches to highlight during group discussion, sequence responses to highlight particular ideas, and support learners in making connections between different approaches.

These four facets of connection making formed the basis of our analysis of both the PD and classroom settings as we considered how the teacher translated the PD experience into her classroom practice.

### Methods

Data were collected as part of a larger research project in which we used a mixed methods design to understand teacher learning in PD and the impact it has on classroom experiences and student performance. For this study, we considered one videotaped session of a 14-week mathematics PD experience and one videotaped session from one participant's 7<sup>th</sup> grade classroom. This teacher, Donna, was the only one in our study who invited us to videotape an implementation of a task taken directly from the PD course.

#### *Professional Development*

The PD workshop was a 14-week (total of 42 hours) experience for middle school teachers in which they specifically engaged in content knowledge development. The course engaged the 14 participants in exploring fraction multiplication and division as well as proportional reasoning. Each session, which was taught in the district office in a large, underachieving, urban school district, lasted three-hours and engaged the participants in hands-on, technology-supported engagement with the mathematics. The three stated goals of the PD were (1) understanding referent units (the *whole* to which a fraction refers) in a variety of situations; (2) using drawn representations; and (3) proportional reasoning. To meet these goals, participants were asked to engage with a number of open-middle tasks either as a group or in small groups. For the small group work, each teacher was responsible for preparing a *write-up* that documented the approach taken to solving the task, any dead-ends hit, and a fully discussed solution.

The facilitator, a member of the research team, had extensive experience as a professional developer and was a former high school teacher. Each session was videotaped using two cameras. One was focused on any written work being discussed, while the other focused on the people speaking. These two sources were combined into a single view using picture-in-picture technology to create a restored view (Hall, 2000).

#### *7<sup>th</sup> Grade Class*

As part of the larger research project, we asked several teachers representing a cross-section of abilities on our pre-course assessment to allow us into their classrooms so we could understand how the professional development impacted their teaching. We specifically asked to see examples of a typical lesson and a lesson the teacher felt was like the PD. For this analysis, we consider Donna's implementation of a task she thought was like the PD. The lesson was implemented approximately three months after the completion of the PD. As in the PD, the classroom was videotaped using two cameras and the sources were combined.

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### Data Analysis

Both the PD session and Donna's 7<sup>th</sup> grade lesson video were analyzed using memoing (Strauss & Corbin, 1998). Every instance of any of the four facets of connection making was noted. This included those instances that did not capture our full definition (e.g., the teacher may ask a closed question rather than a generative one). We chose this approach in place of grounded theory to help us focus on those aspects of classroom practice that have been demonstrated to be important for learning as well as those specifically built into the PD experience. Once we had memoed each lesson, we built a model of it to help us understand how the facets of connection making worked in that lesson.

Secondary data, including weekly telephone interviews with Donna completed during the 14-week PD, write-ups of her tasks, and reflections (e.g., specific questions about generalizing the mathematics) collected in the PD sessions were analyzed. We used the same facets of connection making and memoing technique as was used with the video data.

### Results

In this section, we briefly present an overview of Donna's experience in the PD as well as some elaboration on her content knowledge. This is followed by a discussion of Donna's PD-inspired lesson. Data are summarized due to space limitations. Both the PD and the 7<sup>th</sup> grade lesson like the PD focused on a pair of mathematical tasks (shown in Figure 2).

**Problem 2:**

a) Joe the farmer drained his grain silo into a holding bin. The silo was full and held 300 cubic meters of grain. It drained at a rate of 5 cubic meters per hour. How fast would Joe have to drain a silo that held 420 cubic meters of grain so that it took the same amount of time? How fast would Joe have to drain a silo that held 210 cubic meters of grain so that it took the same amount of time? Is the relationship between volume and rate the same across these three examples? If so, describe the general relationship.

b) Grain elevators are large containers used to fill train cars with grain that farmers have harvested. If the elevator dispenses grain at 500 cubic feet per minute, it takes 12 minutes to fill a train car. If the elevator dispenses grain at 200 cubic feet per minute, how long will it take? If the grain elevator dispenses grain at 250 cubic feet per minute, how long will it take? Is the relationship between rate and time the same across these three examples? If so, describe the general relationship.

**Figure 2. Task used in both lessons. Discussion in this paper focuses on task (b).**

### Donna's PD Experience

A typical session of PD opened with the facilitator asking participants to work on a task for a short time that was then discussed in the whole group. Then, participants were introduced to the tasks from which they could select to complete a write-up. During the times in which the participants worked, they could choose to work alone or with a partner.

Donna, who was an 11-year veteran teacher, began the PD with a slightly above average mathematical knowledge in the areas of interest to us. On our pretest, her *z*-score was 0.2 but by the end of the PD, she scored a 1.4, showing much more than the 0.3 growth considered significant. In observations of the class from which these data are drawn, it was clear that she had some confusion about aspects of proportional reasoning and that her definitions for

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proportional relationships were superficial. For example, she spoke of direct proportions as being “up/up” relationships and inverse proportions as “up/down” relationships. In both whole class discussions and one-on-one discussions, the facilitator communicated the need for precision in defining the relationships by saying things like, “I think this speaks really nicely to what that is that we’re talking about. I think just saying ‘as one goes up the other goes up’ and ‘as one goes up the other goes down’ — talking like that — additive and multiplicative — I don’t think we’re really being clear enough.”

In the course of the PD class session, Donna developed a conjecture for herself about inverse variation. She asserted that if you multiply one value (e.g.,  $y$ ) by an amount, then you would need to multiply the other value (e.g.,  $x$ ) by the reciprocal of the amount. As shown in Figure 3, for example, to increase the 12 to 24 requires multiplying by 2, therefore, according to Donna’s rule, the corresponding value to 24 would have to be  $\frac{1}{2}$  of 500. The facilitator worked one-on-one with Donna to explore this conjecture by asking questions about the situation in Figure 2B. The facilitator challenged Donna by asking questions about the t-chart Donna had made (Figure 3) such as, “So, if we take 500 to 200 and then we did 12 to 30, we should see the reciprocal?” Donna and the facilitator worked on this situation together for several minutes using the representations Donna had created and Donna’s own conjectures. In the whole-class debriefing, she scaffolded the entire group’s thinking by again emphasizing the need for more precision in defining proportional situations than simply describing them in terms of the idea of one value increasing or decreasing as the other increases. Through these conversations, we assert that Donna had an opportunity to begin thinking about proportional relationships as being multiplicative in nature.

500	12
250	24
200	30

**Figure 3: Donna’s work for the situation in Figure 2, Question B.**

At the end of the session, participants were asked in the reflection activity whether data shown in a table were directly proportional, inversely proportional, or neither. Donna showed evidence of both properly identifying the situations and providing mathematically grounded evidence for those. For example, her rationale for (appropriately) rejecting one table of data was that it “... is not a proportional relationship because there is not a constant pattern that fits either description ( $xy=k$  or  $y=kx$ ). The multiplicative aspect is not there.” She also provided a coherent definition for constant of proportionality that suggested she may be starting to think differently about direct and inverse relationships than she previously had: “A constant of proportionality is a number that can be multiplied by one term to get the other. Doesn’t apply to inverse?”

Despite the promising evidence that Donna was beginning to sort out proportionality in November, by March when we recorded her classroom, she reported that she was still confused about how and why inverse and direct proportions were different. She said, “...I’m not even absolutely sure that’s ever clearly defined for me... [in my own classroom] I recognized the difference... I’m not sure that it’s defined. And I guess it’s because the materials I work from

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don't define it clearly to me. I probably don't define it clearly to [my students]."

### *Donna's Classroom Lesson*

Donna introduced the two tasks in Figure 2 (parts A and B) to her students by asking them to read aloud and using communication moves to ensure the students understood the problem. She encouraged them to work with "the mathematician next to you". She also used questioning to remind them of the representation they had been using with proportional situations (graphs) but also promoted other approaches saying, "... you can use the graph if you want to. But if you have an idea in your head of how you could visualize this in a different way, some other kind of model, then you can draw that." Donna circulated the room as the students worked asking them questions. Her questioning was largely confined to reporting questions (Wood et al, 2006) in which students explained *how* they did the problem but not *why* that had chosen an approach. For example, she asked one group, "What did you do when you did what you did?" We saw a few frequent questioning patterns in Donna's interactions with her students. One was to ask students to explain what the problem was asking if they were confused about what to do. Another pattern was her invitation to students to use their own strategies when solving a problem. However, this became problematic for Donna when she needed to evaluate students' approaches. Their approaches, at times, differed from what she expected and her only responses were either encouragement or to explain her own thinking—both without supporting students in connecting their understanding to her own. In one case, a student used a solution path Donna had not expected, so she explained her own to him. His response to her was to ask if he could have done it her way. When she said he could have, he commented on his own effort saying, "All that work for nothing." Communication was about evaluation and efficiency, not about engaging in mathematical discussions or pushing students thinking forward as Donna had experienced in the questioning she had in PD.

Donna had suggested to students that they should use representations in her launch of the problem, but as she circulated, she was clearly surprised by some of the representations she saw. In the spirit of supporting multiple approaches, Donna seemed hesitant as she tried to accept the ways students chose to approach the situation. For example, Donna encountered a group that created a bar graph for their inverse proportion. She asked the group, "Is this the kind of graph we've been doing with direct and inverse variation?" A student responded, "We've been doing line graphs, but I didn't feel like the line graph would be comfortable..." Donna responded, "If you think this is going to show you a better picture then go for it." Another student in the group commented, "I don't feel right about doing a bar graph." Donna articulated that students could use different approaches as long as they were comfortable with their answers, and she reminded them that the question asked them to determine which kind of variation (direct or inverse) was represented in the problem. Unlike the PD where mathematical discussions were had about the representations and their mathematical affordances, in Donna's class, the discussions focused on student comfort with no elaboration on that comfort or on aspects of the mathematics.

### *Analysis*

In the PD, the framework of connection making was clearly present with all aspects of the class working together to scaffold learners. The facilitator used questioning to either understand participants' thinking or to push their understanding. She used precise communication in terms of not only incorporating mathematical terms but specifically discussing those terms and why she was focused on the precise use of them. She encouraged the use of representations and focused teachers on explaining them to her and to each other. The whole class debriefings of

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tasks always focused on connecting the various approaches together.

In contrast, when Donna implemented the task in her class questioning stopped at explaining how a problem was worked. Although multiple approaches were tolerated, they were not used to promote thinking and connection making among students. In fact, students saw Donna's approaches as being better than their own and, in the class we observed, they did not get to see any approaches other than their own or ones that Donna talked to them about in their partner work. Communication was not mathematically precise and, while not presented in the data due to space limitations, she pursued using the up/up and up/down explanations for the proportional relationships. She also emphasized the shape of the graph without pursuing any discussion about understanding the graphs in more detail. There was no sense of connecting ideas to each other and Donna did not seem to scaffold students' efforts so that the pieces fit together. In fact, she would give different pairs of students different directions without purpose—for example, one group was told to work the problem another way while another was asked about a different representation. But, there was no follow-up of any kind on these challenges.

### Conclusions

Our goal in this study was to explore aspects of professional development a teacher might carry into her classroom and what the movement between the professional development and classroom environment might look like. In our case, we were offered the opportunity to observe a single teacher engage with a task taken from the PD. We assert that this analysis contributes to our understanding of how a teacher mediates the double triangle of interaction.

Donna was a good teacher in many respects. Her classroom was pleasant and her students clearly liked her. Her mathematics knowledge was clearly above average, despite some problematic understandings. According to her interviews, she enjoyed the PD and thought it was helpful to her teaching, though she could not provide examples of specific ways in which it was helpful.

From the perspective of the double triangle of interaction, however, we can see that the teacher mediation of the two environments plays an important role in the experience the students have. In PD, the task served as a conversation starter. It had been intentionally chosen by the facilitator to highlight particular relationships among the numbers (and the facilitator had discussed this intentionality in the PD). The focus of the task activities was on developing conceptual understanding that was mathematically precise. To do this, the facilitator used all four aspects of the framework for connection making in ways that were consistent with high press practices (Kazemi & Stipek, 2001).

Donna, having experienced this environment as a learner, chose to take the exact materials to her own students. She framed the task as a challenge for the students and used it to engage them in talking to each other and to problem solve. The focus of her interactions was often on correct calculations and the take-away was the shape of the graphs. There was no attention to precise use of mathematical terms nor was there any attention to the multiplicative relationship of the values in the task. Whereas the task had been used to engage participants with mathematical concepts to develop understanding in the PD, in the 7<sup>th</sup> grade classroom, the task was about doing hands-on mathematics with a goal of knowing that the graph shapes were different.

This suggests that while Donna was able to see the moves the facilitator made and take those into her classroom (e.g., she used questioning, supported multiple approaches and representations, etc.). Her lack of tying these to the mathematics in ways that supported a high press learning environment meant that the students missed out on opportunities for learning.

As suggested by the double triangle of interactions, in order for PD to impact classroom

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practice, it must be relevant and apparent to the teacher because she is the one who moves the opportunities from one environment to the other. While Donna valued her experiences as a learner, she did not take them, except in a superficial way, into her classroom practice. This suggests two things. First, the Donna viewed pedagogical moves and content knowledge as being separate from each other—thus her ability to use the same pedagogical moves as the facilitator but in less rigorous ways. Second, it suggests that the goals of PD may need to be reconceptualized from supporting the teacher in her personal development to providing support for developing the learning environment in which the teacher practices. This does not mean telling the teacher how to conduct her practice, rather it means focusing the PD so that the teacher's personal development is situated, for her, in the practice of teaching. For example, we wonder whether Donna's content knowledge development around the relationship of quantities in the inverse proportion would have been more salient to her teaching had we presented her with student reasoning similar to her own and let her think about how she would interpret and respond to that reasoning in her own practice. Pursuing opportunities to more fully integrate the two triangles of interaction may support teachers in better translating their own experience as learners into their practice as teachers.

### Endnotes

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