HOW DO STUDENTS CREATE OPPORTUNITIES TO LEARN MATHEMATICS?: REPRESENTING STUDENTS IN RESEARCH ON CURRICULUM USE

Amanda Jansen
University of Delaware
jansen@udel.edu

The purpose of this paper is to argue for increased attention to students' experiences in research on mathematics curriculum. I call for examining students' roles in the process of curriculum use to complement studies of student data to assess the outcomes of curriculum use. I present a framework that illuminates students' roles in the process of curriculum use.

Opportunities to learn are considered to be the strongest influence on what students learn (Hiebert & Grouws, 2007). One way to think about creating opportunities to learn is through a process of curriculum construction. When students are represented in studies of mathematics curriculum, student performance (and, more rarely, attitudes or dispositions) is assessed as an outcome indicator. However, students are less frequently represented in research studies as explicitly part of the process of constructing curriculum. The purpose of this paper is to argue for an increase in research on how students play roles in creating opportunities to learn mathematics. In this paper, I present a framework for the ways in which researchers have represented or might represent students as part of the *process* of curriculum construction, beyond representing students as outcomes of teachers' curriculum use.

The need for more complex representations of students in research on mathematics curriculum has been requested previously. Smith and Star (2007) argued for broader conceptualizations of impact of mathematics curriculum. They recommended that researchers look beyond achievement outcomes to include students' disposition measures as outcomes and to examine interactions between indicators of impact when studying the effects of curriculum. The Research Advisory Committee [RAC] of the National Council of Teachers of Mathematics [NCTM] (Confrey et al., 2008) called for examining subgroups of students to assess *for whom* various curriculum materials and approaches are more and less effective (and why). The RAC suggested that a more complex representation of students would involve accounting for race, socio-economic class, and gender when studying the effects of curriculum materials. Erickson and Shultz (1992) noted that when researchers have investigated students' experiences in classrooms, they rarely accounted for subject matter. In this paper, I extend these calls for more complex representations of students in research on mathematics curriculum by emphasizing a focus on students' roles in the *process* of creating opportunities to learn mathematics.

Why is it valuable to consider how students are represented in research on mathematics curriculum? In the context of post-No Child Left Behind, so much emphasis is on "what works." Students are represented in terms of achievement scores, as outcomes in the process of implementing instructional innovations (including particular curriculum materials or teaching practices). Yet we realize that students are more than their test scores. Classrooms are social contexts, and all of the participants – including students – affect the unfolding of social events. As the social event of a lesson unfolds, it usually does not do so exactly as a teacher planned; changes are often made to the lesson in the moment, as the participants respond to one another. In this way, among others, students can play a role in constructing what can be learned during a lesson.

In this paper, I present a framework for characterizing how students influence their opportunities to learn mathematics. I take up the following questions in this paper: How have students been represented in research on mathematics curriculum? How *might* students be represented in research on mathematics curriculum? What are the affordances and constraints of these different constructions? My analysis of current research on mathematics curriculum suggests the need for further research on the role of students in the *process* of curriculum use.

Theoretical Framework

To describe my general perspective on research on mathematics curriculum, first I define what I mean by "curriculum." Then, I share some of the purposes for studying mathematics curriculum use in past research, and I discuss how students have been represented in this area of research. The Mathematical Tasks Framework (Henningsen & Stein, 1997) is presented for the insights it provides about how students are constructed and might be constructed in research on curriculum.

Curriculum

Following Stein, Remillard, and Smith (2007), I view curriculum broadly as "the substance and content of teaching and learning – the 'what' of teaching and learning (as distinguished from the 'how' of teaching)" (p. 321). I view curriculum construction as a dynamic & flexible process. Opportunities to learn mathematics in a classroom involve teachers and students participating with written curriculum materials and each other (Remillard, 2005). These learning opportunities are shaped by the tasks in the written materials, how teachers implement the tasks, and how students engage with the tasks.

Purposes of Research on Mathematics Curriculum

Mathematics educators whose research addresses mathematics curriculum have examined (1) comparative effects of new curriculum materials (evaluation studies) and (2) how features of curriculum and pedagogy shape these effects and influence students' learning (Star & Smith, 2007). Across studies that evaluate the effects of curriculum materials, the focus has been either upon demonstrating that newer materials ("Standards-based" materials designed to help teachers implement lessons aligned with the NCTM Standards) do no harm or can more effectively promote students' learning of content aligned with NCTM Standards, compared to other curriculum materials (e.g., Huntley et al., 2000).

Research on mathematics curriculum has evolved toward representing teachers' roles with greater complexity. Researchers from the QUASAR project illustrated that teachers' implementation of mathematics tasks could transform tasks that were written to be challenging to become less demanding or teacher could maintain the level of challenge through task implementation (Henningsen & Stein, 1997). Tarr et al. (2008) studied the effects of the use of middle grades curriculum materials along with instructional practices (Standards-Based Learning Environment) on student achievement; they found that curriculum materials converged with teachers' instructional practices to foster students' achievement. Across studies, results suggest the importance of teachers' roles in constructing students' opportunities to learn from curriculum materials.

There is a need to represent students in research on curriculum with greater complexity as well, including how students help create opportunities to learn from particular curriculum materials. In a recent review of the research literature on how curriculum influences student learning (Stein, Remillard, & Smith, 2007), students' roles are discussed for less than a page (p.

Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.

355), which suggests a lack of research on their role in the process of curriculum use. If students are represented typically at the "end" of the curriculum use process, specifically in terms of what they have learned, students may be represented as having a relatively passive role in curriculum use – as receiving the curriculum. Following Goodlad (1979), it is possible that the teachers' instructional approaches and written curriculum materials (or, in Goodlad's terms, the intended, perceived, and operational curriculum) may not be aligned with what students experience (the received curriculum). Thus, capturing students' perspectives in relation to curriculum use would provide insight on constructing opportunities to learn mathematics. Not only might students receive a curriculum that may not be aligned with what the teacher intends or implements, students' engagement with a mathematical task can influence their learning.

Stages of Curriculum Use and Students' Roles

Stages in the Mathematical Tasks Framework (Figure 1) provide insights about how students have been represented in research on curriculum use. The first stage is the mathematical task as represented in curricular/instructional materials, or the written curriculum. Teachers then plan to use written tasks in the form of lessons. Teachers implement these lessons, which would be the second stage – mathematical task as set up by the teacher in the classroom. The third stage represents students' interactions with mathematical tasks – mathematical task as implemented by students in the classroom. The fourth stage represents students as *outcomes* of curriculum use.

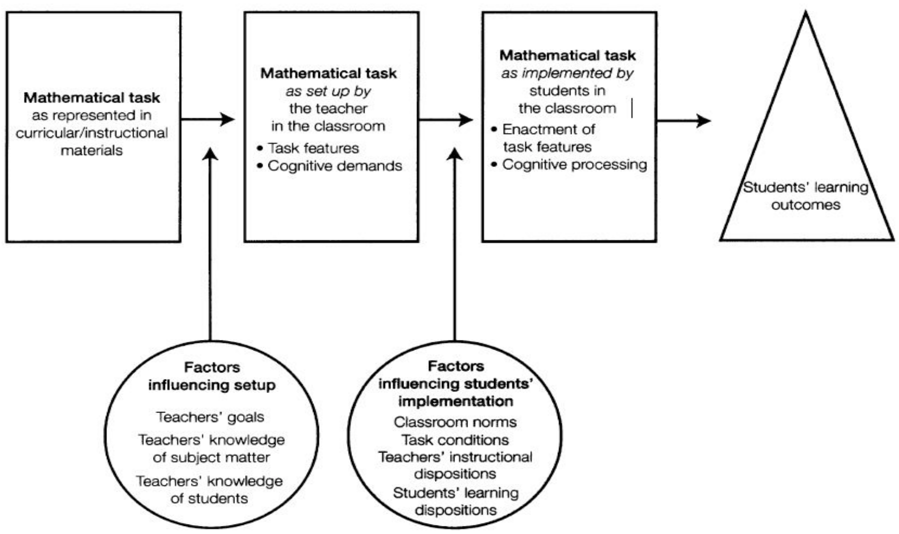

Figure 1. Mathematical Tasks Framework (Henningsen & Stein, 1997, p. 528)

Note that students also appear as factors influencing how teachers set up tasks (teachers' knowledge of students) and how students implement tasks (students' learning dispositions). The Mathematical Tasks Framework suggests that students' roles in the process of curriculum use include influencing teachers' planning and implementation of mathematical tasks, students' engagement with tasks, and, potentially (if there was an arrow creating a cycle from student learning outcomes to teachers' implementation of the task again in the future) students could influence how teachers revised their lessons based upon these mathematical tasks.

The idea that students play a role in influencing opportunities to learn is not new. Borasi (1990) wrote about students as the "invisible hand operating in mathematics instruction" (p. 174), and she illustrated that students' conceptions and expectations can influence their learning. Erickson and Shultz (1992) argued for the need to capture variation in students' subjective experiences of curriculum. Although some mathematics education researchers have examined how student engagement shapes their opportunities to learn (e.g., Nasir & Hand, 2008), these studies are either set outside of school or not in relation to tasks in specific curriculum materials. To support mathematics educators in moving toward more complex representations of students in research on mathematics curriculum, there is a need for a framework to describe students' roles in the process of curriculum use.

Methods

To develop this framework, I purposefully selected research literature to examine for this paper. I used the following criteria to select articles: To discuss how students have been represented to date, I drew upon peer reviewed publications. To consider how students *might* be represented, I drew upon a wider range of texts and looked beyond peer reviewed publications to include conference papers and book chapters. The Mathematical Tasks Framework provided a structure for locating research on students' experiences with curriculum use. I examined research with representations of students in the second stage (mathematical task as set up by the teacher), the third stage (mathematical tasks as implemented by students), and, if there could be a feedback loop between student learning outcomes and selection of mathematical tasks, I examined research that represented students in the process of teachers revising opportunities to learn as well.

Results

In this section of the paper, I explore how students have been represented in the process of curriculum use within a framework (Table 1), and components of this framework serve as inspiration for how students might be represented in future research on the process of curriculum use in mathematics education.

Processes	How *are* students represented in curriculum research?	How *might* students be represented?
When *teachers plan* lessons based upon curriculum materials	• Students' mathematical thinking as resources to draw upon when designing opportunities to learn. • Students' funds of knowledge as resources to draw upon when designing opportunities to learn.	• Students' voices as a factor that influences selection of curriculum materials or other aspects of curriculum design.
When *teachers implement* lessons based upon curriculum materials	• Students as texts to be "read" by teachers during instruction (to inform in-the-moment revisions) • Students' engagement as a factor that influences cognitive demand of mathematical tasks (maintenance or decline).	• Students' engagement as a factor that influences *increase* in cognitive demand.
When *students implement* mathematical tasks	• Variations in how students engage with the same mathematical tasks.	• In terms of strengths in students' engagement in balance with opportunities to improve their engagement.
When *teachers revise* lessons based upon curriculum materials	• Students' mathematical thinking as a factor that influences revisions of opportunities to learn.	• Students' voices (attitudes or other perspectives toward curriculum) can be a factor that influences revisions of opportunities to learn.

Table 1. Students' Roles in Constructing Opportunities to Learn

Students as Resources when Planning Lessons

Students play a role in the process of constructing opportunities to learn when they are draws upon as resources for teachers when selecting tasks or designing and planning to implement mathematical tasks in their classroom. In *Child and the Curriculum*, Dewey (1902) argues for "psychologizing" the curriculum. This process involves constructing educative experiences for learners that originate from the child's present experiences and move into the logic of academic disciplines. Research on curriculum design (designing, selecting, and planning curriculum) is conducted in the spirit of psychologizing the curriculum when it represents students as resources to inform the process.

Some ways that students have been represented as resources in the process of curriculum design or planning to implement tasks has been in the form of local instruction theories (Gravemeijer, 2004) or students' "funds of knowledge" (Bartell et al., 2010). Local instruction theories (Gravemeijer, 2004) describe a reflexive relationship between hypotheses about how students learn, think, and reason about a given topic and the means of instructional support to help students understand that topic. Students' thinking is a resource for task design with the goal of moving students' thinking toward a particular mathematical learning goal. Some researchers promote a view of curriculum design that draws upon home and community-based funds of knowledge (knowledge, skills, and experience found in students' homes and communities). Bartell and colleagues (2010) describe how pre-service elementary teachers designed lessons that built on their own students' funds of knowledge after learning more about the students' lives (e.g., participating in a tour of the students' communities led by community members). In this research, students play a role in curriculum construction because the opportunities to learn in these pre-service teachers' classrooms would differ or be adjusted depending on what they learned about their students' lives outside of school.

Additionally, students could be represented in research on the process of curriculum design in settings where curriculum materials (or tasks or content goals) were chosen (at least in part) based upon students' input. How frequently are students consulted as stakeholders in the process of selecting and designing opportunities to learn? After all, education is *for* students (Levin, 2000), so their voices about the materials and tasks could be represented by research and considered when making instructional decisions. For instance, researchers could investigate the criteria students use to select mathematics curriculum materials or tasks.

Students as Resources when Implementing Lessons

Teachers' interactions with students could influence how teachers implement tasks in the classroom. For instance, teachers "read" their students when implementing mathematical tasks, which can lead to improvised curriculum construction, such as changing tasks or ending a task early (Remillard, 1999). Additionally, mathematical tasks have been observed to either maintain their level of challenge and difficulty (maintain cognitive demand) or become less challenging (cognitive demand decreases) as teachers and students interact (Henningsen and Stein, 1997). Students may pressure teachers to reduce ambiguity in order to reduce their anxiety about being successful on a challenging task. Students shape their learning opportunities as they influence teachers' implementation of tasks.

Future research could examine whether and how students can play a role in raising the level of cognitive demand of a task. It is possible that students could push the level of cognitive demand higher if teachers are open to students' mathematical wonderments. As students have opportunities to pose their own mathematical problems (Brown & Walter, 2004), they may start asking increasingly challenging questions. Students may make connections that the teacher had not intended, perhaps even during a lesson that was not headed toward making connections, taking a lesson that was initially focused on procedural fluency toward a lesson that examines meanings behind and connections between procedures.

Students' Implementation of Mathematical Tasks during Lessons

Prior research has examined how students engage differently with tasks from the same curriculum materials. Two studies of seventh grade students in classrooms that used NCTM Standards-based curriculum materials (Lubienski, 2000; Jansen, 2008) illustrate variations between how different students take up the same mathematical tasks. SES differences appeared to moderate how students engaged with open, contextualized mathematical tasks as set up by the researcher-teacher (Lubienski, 2000). Higher SES students engaged with more confidence and with an eye toward the intended mathematical ideas when working on open, contextual tasks. Lower SES students preferred more external direction and approached problems in a way that led to missing some of the intended mathematical points. Jansen (2008) examined how students' beliefs moderated their participation in classrooms with Standards-based curriculum materials. Students who believed whole-class discussions were threatening avoided talking about mathematics conceptually, yet these students participated by talking about mathematics procedurally. Additionally, students' beliefs about appropriate behavior during mathematics

Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.

class appeared to constrain whether they critiqued their classmates' solutions. Waddell (2010) investigated African-American elementary school students' engagement in classrooms that implemented Standards-based materials over three years and identified ways in which the student's engagement converged with and diverged from the patterns of engagement promoted by their teachers' instructional practices. These studies illustrate that students' engagement and participation, as moderated by SES, cultures, or their beliefs, influences their opportunities to learn from mathematical tasks.

It is helpful to examine a range of patterns in student engagement with mathematical tasks, because teachers would benefit from increased awareness of variations in students' engagement with tasks. Teachers can more effectively support students' engagement if they have knowledge of how students might engage. When researchers investigate student engagement, one dilemma involves how to discuss ways to improve students' engagement without placing blame upon students for not participating productively. (Henningsen and Stein (1997) appear to navigate this dilemma by focusing upon how teachers can influence students' engagement.) Future research should continue to uncover different ways in which students engage with the same mathematical tasks, especially highlighting positive qualities of students' engagement that lead to productive learning opportunities. Results of such studies could support more teachers as they read their students while implementing tasks. Knowledge about how students engage can help teachers as they intervene to support students to engage productively (or encourage students when they do engage productively).

Students as Resources when Revising Lessons

Teachers can use their knowledge of their students to revise their lessons based upon their curriculum materials. Referring back to Gravemeijer's (2004) local instruction theories, these theories are considered to be revisable. Teachers can develop and take these theories as conjectures about how and why curriculum materials or mathematical tasks (and instructional moves to use when implementing the tasks) are effective, then test and modify them in their own classrooms. When teachers learn or develop new understandings about students' mathematical thinking, evidence suggests that they may change their instruction (Fennema et al., 1996). Thus, students are represented in the process of research on mathematics curriculum revisions by descriptions of their mathematical thinking and their influence on curricular (and instructional) decisions.

It is also possible to imagine research in which opportunities for students' learning are revised based on students' feedback. Some researchers have attempted to solicit students' perspectives about their experiences with particular curriculum materials. In a report written by high school students and their teacher (Holt et al., 2001), students shared their perspectives on their experiences in classrooms with Standards-based curriculum materials. These students reported an appreciation of the participatory aspects of these classrooms and reports that the teachers' supportive stance toward the materials improved their experience. Bay, Beem, Reyes, Papick, and Barnes (1999) assessed over 1,000 middle school students' reactions to Standards-based curriculum materials after a year of use by asking students to write letters about their experiences with the materials. Students were generally positive about the materials and appreciated the hands-on activities, real-life applications, and collaborative work. Students' perspectives on curriculum materials have been investigated, but researchers could follow up on this line of work by studying the effects of implementing students' recommendations or the selection of materials chosen with students' input.

Discussion

Just as complex representations of how teachers use curriculum materials provide insight on creating opportunities for students to learn, additional representations of students are needed in research on mathematics curriculum. Currently, students are represented primarily as outcomes of curriculum use. Researchers could continue to investigate the role of students as a factor that influences teachers' planning and implementation of curriculum, as a factor that influences revisions of lessons, and as a diverse group that engages differently with similar tasks. Advances in research on mathematics curriculum could include studies of students as active participants as decision-makers about curriculum and representations of students' strengths when engaging in mathematical tasks (including ways in which students play a role in raising the cognitive demands of tasks).

Explicitly investigating and highlighting students' roles in the process of curriculum construction can provide insight into social processes of learning to consider not only whether but *how* the use of curriculum materials shapes learning. Although there are political pressures to use curriculum materials that are proven to be "effective," it is important to understand the conditions that lead to productive uses of these materials. Students' roles in the process of constructing opportunities to learn are conditions that mathematics educators need to understand more fully to gain a broader perspective on how curriculum materials support students' learning. For these reasons, I hope for more complex representations of students to play an increasingly central role in future research on mathematics curriculum materials.

References

Bartell, T., Aguirre, J., Drake, C., Foote, Mary Q., McDuffie, A. R., & Turner, E. (2010). (2010, October). Preparing preK-8 teachers to connect children's mathematical thinking and community funds of knowledge. In D. B. Erchick, A. Manouchehri, and D. Owens (Eds.). *Proceedings of the thirty-second annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.*

Bay, J. M., Beem, J. K., Reys, R. E., Papick, I., & Barnes, D. E. (1999). Student reactions to standards-based mathematics curricula: The interplay between curriculum, teachers, and students. *School Science and Mathematics*, *99*(4), 182-188.

Borasi, R. (1990). The invisible hand operating in mathematics instruction: Student's conceptions and expectations. In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990s* (pp. 174-182). Reston, VA: National Council of Teachers of Mathematics.

Brown, S. I., & Walter, M. I. (2004) *The Art of Problem Posing* (3rd edition). Mahwah, NJ: Lawrence Erlbaum Associates.

Confrey, J., King, K. D., Strutchens, M. E., Sutton, J. T., Battista, M. T., Boerst, T. A., Smith, M. S., Reed, J. (2008). Situating research on curricular change. *Journal for Research in Mathematics Education, 39*(2), 102-112.

Dewey, J. (1902). *The Child and the Curriculum.* Chicago: University of Chicago Press.

Erickson, F., & Shultz, J. (1992). Students' experience of the curriculum. In P. Jackson (Ed.), *Handbook of Research on Curriculum* (pp. 465-485). New York, NY: Macmillan.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, *27*(4), 403-434.

Goodlad, J. I. (1979). *Curriculum inquiry: The study of curriculum practice*. New York:

McGraw-Hill.
Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning*, *6*(2), 105.
Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, *28*(5), 524-549.
Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 371-404). Greenwich, CT: Information Age.
Holt, D. V., Gann, E., Gordon-Walinsky, S., Klinger, E., Toliver, R., & Wolff, E. (2001). Caught in the storm of reform: Five student perspectives on the implementation of the Interactive Mathematics Program. In J. Schultz & A. Cook-Sather (Eds.), *In our Own Words* (pp. 105-125). Lanham, MD: Rowman & Littlefield.
Huntley, M. A., Rasmussen, C. L., Villarubi, R. S., Santong, J., Fey, J. T. (2000). Effects of Standards-based mathematics education: A study of the Core-Plus Mathematics Project algebra and functions strand. *Journal for Research in Mathematics Education, 31*(3), 328-361.
Jansen, A. (2008). An investigation of relationships between seventh grade students' beliefs and their participation during mathematics discussions in two classrooms. *Mathematical Thinking and Learning, 10*(1), 68-100.
Levin, B. (2000). Putting students at the centre in education reform. *Journal of Educational Change, 1*, 155-172.
Lloyd, G. and Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education 29*(3): 248-274.
Lubienki, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education, 31*(4), 454-482.
Nasir, N.S. & Hand, V. (2008). From the court to the classroom: Opportunities for engagement, learning, and identity in basketball and classroom mathematics. *Journal of the Learning Sciences, 17*, 143-179
Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum Development. *Curriculum Inquiry*, *29*(3), 315-342.
Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75,* 211-246.
Smith, J. P., III, & Star, J. R. (2007). Expanding the notion of K-12 Standards-based mathematics and reform calculus programs. *Journal for Research in Mathematics Education, 38*(1), 3-34.
Stein, M. K., Remillard, J., Smith, M. S. (2007). How curriculum influences student learning. In *Second Handbook of Research on Mathematics Teaching and Learning*, Edited by Frank Lester, pp. 319-370. Charlotte, NC: Information Age Publishing.
Tarr, J. E., Reys, R. E., Reys, B. J., Chavez, O., Shih, J., Osterlind, S. J. (2008). The impact of middle-grades mathematics curricula and the classroom learning environment on student achievement. *Journal for Research in Mathematics Education, 39*(3), 247-280.
Waddell, L. (2010). How do we learn? African American elementary students learning reform mathematics in urban classrooms. *Journal of Urban Mathematics Education*, *3*(2), 116-154.

Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.