INVESTIGATING RELATIONS BETWEEN ABILITY, PREFERENCE, AND CALCULUS PERFORMANCE

Erhan Selcuk Haciomeroglu
University of Central Florida
erhansh@mail.ucf.edu

Eric Chicken
Florida State University
chicken@stat.fsu.edu

The goal of the present study was to report an instrument designed to determine students' mathematical performances and preference for visual or analytic thinking for the calculus derivative and antiderivative tasks as well as examine the relationships among students' cognitive style, cognitive ability, and mathematical performance in calculus. Data were collected from 150 Advanced Placement calculus students. The results suggest that the instrument is measuring an important component of cognition and has the potential to be a measure of performance and preference for visual thinking in calculus.

Introduction

Researchers have been interested in identifying the preference and ability components of cognitive style for several decades (e.g., Clements, 1979; Bishop, 1980, 1989; Hegarty & Waller, 2005). The visualizer-verbalizer distinction in particular has been an area of interest for researchers in various disciplines (e.g., Hadamard, 1945, Kozhevnikov, Hegarty, & Mayer, 2002; Paivio, 1971; Richardson, 1969, 1977). The goal of the present study was to report an instrument designed to determine students' mathematical performances and their preferred mode of thinking for the calculus derivative and antiderivative tasks as well as examine the relationships among students' cognitive style, cognitive ability, and mathematical performance in calculus.

Background

Krutetskii (1976) identified types of mathematical giftedness based on students' preferences for two cognitive processes: verbal-logical or visual-pictorial. Following the work of Krutetskii (1976), Moses (1977), Lean and Clements (1981), Suwarsono (1982), and Presmeg (1985) have recognized that individuals could be placed on a continuum (i.e., degree of visuality) according to their preference for visual processing. In designing instruments —Problem Solving Inventory (PSI) and Mathematical Processing Instrument (MPI)—consisting of algebra word problems to determine students' preferences, and in their work, Moses and Suwarsono defined visuality as the extent to which a learner prefers to use visual processes to solve mathematics problems. That is, visualizers are considered as learners who prefer to think with images and visual strategies, and analyzers (or verbalizers) as learners who prefer not to think with images and visual strategies when there is a choice on a specific task. In this study, we used the visualizer-verbalizer distinction as a lens to determine students' preferred mode of thinking. However, in the remainder of this paper, we have used the terms "analytic" and "analyzer" interchangeably to describe verbal-logical processing or verbalizers.

There have been studies of cognitive abilities and styles in mathematical performance in different content areas. Battista (1990) with high school students found that spatial visualization and logical reasoning were significant factors of geometry achievement and geometric problem solving, and that spatial visualization was related to the use of visual and analytic problem

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solving strategies (i.e., analytic, visual without drawing, and visual with drawing). A similar finding was reported by Ferrini-Mundy (1987), who found a correlation between spatial ability and certain aspects of calculus. However, other research has shown divergent perspectives. Studies including MPI, developed by Suwarsono (1982) and later modified by Presmeg (1985) as a measure of visualizer-analyzer cognitive style have found either a weak relationship or no relationship between either mathematical performance or cognitive abilities and mathematical visuality. For instance, Galindo-Morales (1994) compared mathematical visuality indicated by MPI and performance of students enrolled in three calculus courses using different instructional approaches (i.e., graphing calculator, Mathematica, and no technology) and concluded that there was no significant relationship between the degree of visuality and calculus performance in any of the three groups. Hegarty and Kozhenikov (1999) administered the MPI to measure sixth grade students' problem solving performance and preference for visual thinking. Their results revealed that mathematical visuality did not correlate with problem solving performance and was negatively associated with the cognitive abilities—verbal ability, nonverbal reasoning, and spatial ability.

Our contention is that calculus requires visual thinking and adequate understanding of visual representations, and that aspects of visuality (or visual imagery) that play an important role in calculus performance may not be measured accurately by existing questionnaires consisting of tasks that do not involve calculus. Moreover, research has shown that the nature, complexity, or novelty of a task influences the degree of visuality (or visual imagery) a student uses when solving the task (e.g., Dean & Morris, 2003; Lowrie & Kay, 2001; Massa & Mayer, 2006; Paivio, 1971; Richardson, 1977). Although various tasks and questionnaires have been designed to measure cognitive styles and learning preferences related to the verbalizer-visualizer distinction (e.g., Mayer & Massa, 2003; McAvinue & Robertson, 2006-2007; Riding, 2001), no adequate instrument for Calculus is available. Thus, there is a need for a calculus instrument designed to determine students' mathematical visuality. We believe this demands research examining the role of cognitive abilities and styles in calculus performance. The present study extends existing research on cognitive styles by examining the relationships among calculus students' preferred modes of thinking, cognitive abilities, and mathematical performances and provides measures of visualizer-analyzer style dimension and mathematical performance in calculus.

Method

Participants

The participants were 169 high school students who were enrolled in Advanced Placement (AP) calculus courses at four high schools in two school districts in Central Florida in the United States at the time of the study. All 169 students agreed to participate in the study. Nineteen students who failed to take all tests were excluded from the data. Of the 150 students, 55 percent of the students were males, and 45 per cent were females.

Materials

The six tests, measuring spatial orientation (Cube Comparisons (CC) and Card Rotations (CR)), spatial visualization (Form Board (FB) and Paper Folding (PF)), and logical reasoning (Nonsense Syllogisms (NS) and Diagramming Relationships (DR)) abilities, are part of the KIT of Reference Tests for Cognitive Factors (Ekstrom, French, & Harman, 1976). Cognitive style tests consisted of a revision of Mathematical Processing Instrument for Calculus ([MPIC], Haciomeroglu et al, 2009) and a modified version of Mathematical Processing Instrument

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([MPI], Suwarsono, 1982). The students' scores on the Advanced Placement (AP) Calculus Exam were collected from teachers at the end of the study.

Spatial Ability Measures

The Cube Comparisons Test consists of 21 items and requires the participant to view two drawings of a cube and determine whether or not the two drawings can be of the same cube. The Card Rotations Test consists of 10 items, each of which presents a two-dimensional figure and eight other drawings of the same card. The participant indicates whether each of the eight cards, without reflecting, is the same or different from the original figure. The Form Board Test consists of 24 items. Each item presents five shaded drawings of pieces and requires the participant to decide which of the shaded figures, from two to five, can be used to make the given geometric figure. The Paper Folding Test consists of 10 items each of which illustrate folds made in a square sheet of paper and a hole punched in it. The participant selects one of the five drawings that shows the position of the holes when the paper is completely unfolded.

Logical Reasoning Ability Measures

The Nonsense Syllogisms Test consists of 15 items. Each item is a formal syllogism, in which statements are nonsense and cannot be solved by reference to past learning. The participant determines whether conclusions drawn from the statements show good reasoning. The Diagramming Relationships Test consists of 15 items. In each item, the participant selects one of five diagrams, which illustrates the interrelationships among sets of three objects.

Cognitive Style Measures

Two cognitive style tests, a revision of Mathematical Processing Instrument for Calculus ([MPIC], Haciomeroglu et al, 2009) and a modified version of Mathematical Processing Instrument ([MPI], Suwarsono, 1982), were used to determine the degree to which students preferred visual or analytic thinking.

The MPIC and the MPI consist of two parts. The first part of each instrument is a test consisting of mathematical tasks: there are 10 derivative and 10 antiderivative tasks (i.e., 7 graphic and 3 algebraic tasks in each test) on the MPIC and 8 algebra word problems on the MPI. The second part is a visualizer-analyzer questionnaire consisting of a visual and an analytic solution for each task on the MPIC and at least 3 or more visual or analytic solutions for each task on the MPI. Upon completion of each test, the students were given the visualizer-analyzer questionnaire and were asked to choose for each task a method of solution that most closely describes how they solved the tasks.

In this study, the MPIC was used to measure the students' preference for visual thinking and mathematical performance for derivative and antiderivative tasks presented graphically or algebraically. Thus, it yielded two performance and two visuality scores for each student: performance (P-G) and visuality (V-G) scores from 14 graphic derivative and antiderivative tasks, and performance (P-A) and visuality (V-A) scores from 6 algebraic derivative and antiderivative tasks. The MPI was used to measure mathematical visuality, but not performance because it consisted of algebra word problems and may not reflect the differences in their mathematical performance. The internal reliability of the MPI visualizer-analyzer questionnaire was 0.225. The internal reliability of the V-G (14 graphic tasks) and V-A (6 algebraic tasks) visualizer-analyzer questionnaires were 0.918 and 0.71 respectively.

Calculus Performance Measures

Three calculus performance scores were included in the analyses. The students' scores on the AP Calculus Exam were collected from teachers at the end of study. The AP Calculus Exam is an important standardized test. High school students who perform well can earn college credit and advanced placement. It covers differential and integral calculus topics, and scores are reported on a 5-point scale (5 is the highest and 1 is the lowest). The students' calculus performance was also assessed by the MPIC Derivative and Antiderivative tests presented graphically and algebraically. The internal reliability of graphic (P-G) and algebraic (P-A) tests were 0.801 and 0.36 respectively.

Procedure

All students received standardized instructions and were tested in groups of 12 to 30 in their classrooms. All participating students gave their informed consent and were debriefed at the end of the study. Four school visits were made during semester, and the tasks were administered in the following order: At the first visit, Form Board, Card Rotations, and Diagramming Relationships were administered. At the second visit, Paper Folding, Cube Comparisons, and Nonsense Syllogisms were administered. The students had completed MPI test prior to the third visit, and they were first given MPI questionnaire and then MPIC Derivative test and questionnaire at the third visit. At the fourth visit, MPIC Antiderivative test and questionnaire were administered. The students were willing to participate in the study and enjoyed most of the tests under classroom conditions. We were unable to administer fewer tests per day due to the time restrictions. Results might have been higher under research conditions. The students were given 8 minutes for FB, 4 minutes for ND and DR, and 3 minutes for CC, CR, and PF. Completion of MPIC and MPI was not timed. The total scores for CC, CR, FB, and NS tests were determined by subtracting the number of incorrect answers from the number of correct answers. Since there were 5 response options for each item on PF and DR, the total scores were determined by subtracting one-fourth the number of incorrect answers from the number of correct answers.

Scoring of MPIC and MPI

In determining preference for visual or analytic thinking, the primary goal was to identify the students' methods as visual or analytic; whether their answers were correct or incorrect mattered less than their method(s) in measuring mathematical visuality. On the MPIC visualizer-analyzer questionnaire, to determine the students' visual preference scores for the derivative and antiderivative tasks, they were given a score of 0 for each analytic solution and 2 points for each visual solution. If a solution does not give any indication of method or both methods were used, a score of 1 was given. On the MPI visualizer-analyzer questionnaire, to determine the students' visual preference scores for the algebra word problems, they were given a score of 0 for each analytic solution and 1 point for each visual solution. Thus, for the derivative, the antiderivative, and the MPI questionnaires, the total possible scores were 20, 20, and 8 points respectively.

Graph is shown. Please sketch the graph of the derivative.

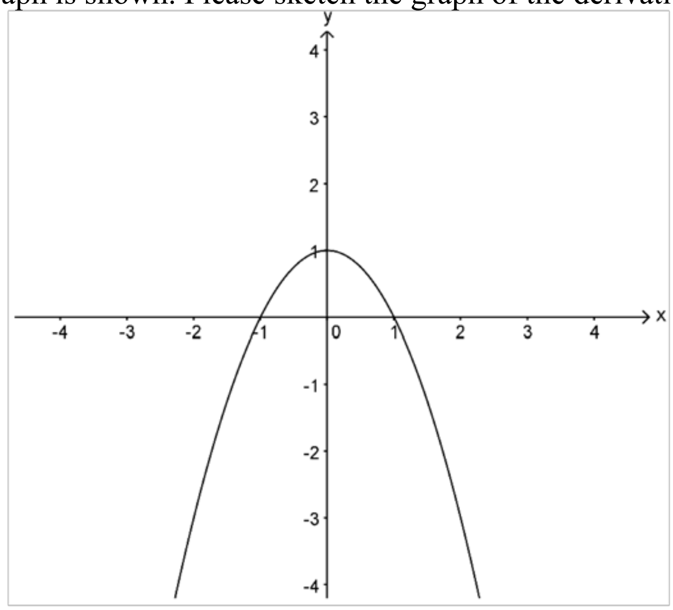

Figure 1. Derivative task.

In assessing students' performance on the MPIC Derivative and Antiderivative tests, the students were given a score of 0 for each incorrect answer and 1 point for each correct answer. Thus, for each of the two tests, the total possible score was 10 points. To illustrate the use of the MPIC, we give an example of one of the derivative tasks (see Figure 1) and the corresponding item in the questionnaire. We consider thinking as visual when individuals prefer to use visual methods and thinking as analytic when individuals prefer not to use visual methods when there is a choice on a specific task. Analytic solutions are generally equations-based. An analytic solution to a task presented graphically typically may involve translation to an equation, computing the integral of the equation, and then using this new equation to draw the antiderivative graph.

We observed that instead of estimating equations precisely, analytic students referred to basic groups of functions such as linear, quadratic, or cubic functions and their derivative graphs associated with odd or even powers of x respectively. The following is the analytic solution given on the questionnaire for the derivative task in Figure 1: *I estimated the equation of the graph (or recognized the equation of the graph). For example: This could be the graph of* $f(x) = -x^2$ *so I computed the derivative as* $f'(x) = -2x$ *and drew the derivative graph using this equation.*

Visual solutions are image-based. They are able to visualize the changing slopes of tangent lines to the function and accordingly are able to construct an entire derivative graph with no need to consider individual parts of equations at critical points or intervals. These individuals are able to determine the shape of derivative graphs based on their visual estimates of slopes. The following is the visual solution given on the questionnaire for the derivative task in Figure 1: *From the graph I estimated the slopes (or the slopes of tangent lines) at various points on the graph of the function and used this to draw the graph of the derivative. For example: The slopes of tangent lines are positive and decreasing as x approaches 0 from the left. The slope is zero at* $x = 0$ *because the graph of the function has a horizontal tangent line at (0, 1). The slopes of tangent lines are negative and decreasing as x approaches positive infinity.*

For the tasks presented algebraically, we consider thinking as visual when students prefer to draw the graph of the given function on paper (or in mind) and estimate the slopes of tangent lines at various points on this graph to draw a possible graph of the derivative or antiderivative. On the other hand, we consider thinking as analytic when students prefer to calculate the derivative or integral, and used this equation to draw a possible graph of the derivative or antiderivative. For instance, one of the algebraic tasks requires sketching a possible graph of the antiderivative, given $f'(x) = 3x^2 + 1$. An analytic solution involves computing the integral as $f(x) = x^3 + x^2 + c$ and drawing the graph of f(x) using this equation, whereas a visual solution involves drawing the graph of $f'(x) = 3x^2 + 1$ on paper (or in mind) and using the y values to estimate the slopes to draw the graph of the antiderivative.

Results

Means and standard deviations for each of the ten measures appear in Table 1. In order to determine between cognitive styles as assessed by the MPIC and the MPI visualizer-analyzer questionnaires and the other variables, Pearson product-moment correlations were computed. The correlations between cognitive styles, abilities, and mathematical performances are presented in Table 2.

Correlational Analysis

There was a significant correlation between the three measures of calculus performance. The correlations between MPI and the other two measures of cognitive style V-G and V-A were non-significant and negative. Of the three measures of cognitive style, V-A significantly correlated with AP and P-G. There was a significant but small correlation between V-A and P-G. The MPI had non-significant negative correlations with the three performance measures. The correlations between the three measures of cognitive style and the measures of spatial and logical reasoning abilities were either negative or non-significantly low.

Among the spatial ability measures, only FB had a significant correlation with P-A. CC and CR had the lowest correlations with the performance measures. FB, PF, NS, and DR significantly correlated with AP and P-G. The correlation between CC and P-G was significant, but CR was correlated neither with AP nor with P-G. It can be seen from the correlations of cognitive ability tests, except CR and FB, the four measures of spatial ability significantly correlated with each other. The two measures of logical reasoning ability significantly correlated with each other. DR correlated three of the four measures of spatial ability, CC, FB, and PF, whereas NS only correlated with FB, suggesting that FB was the only spatial ability measure correlating with both measures of logical reasoning ability.

Table 1
Means and Standard Deviations of Measures

Measure	Label	M	SD	n
1. AP Calculus Exam	AP	2.69	1.55	150
2. MPIC Derivative & Antiderivative Tests - Graphic	P-G	0.46	0.26	150
3. MPIC Derivative & Antiderivative Tests - Algebraic	P-A	0.24	0.18	150
4. Cube Comparisons Test	CC	9.99	4.70	150
5. Card Rotations Test	CR	59.01	15.35	150
6. Form Board Test	FB	7.74	5.68	150
7. Paper Folding Test	PF	6.52	2.23	150
8. Nonsense Syllogisms Test	NS	2.55	4.50	150
9. Diagramming Relationships Test	DR	8.69	3.74	150
10. MPIC Visualizer-Analyzer Questionnaire - Graphic	V-G	1.09	0.67	150
11. MPIC Visualizer-Analyzer Questionnaire - Algebraic	V-A	0.60	0.56	150
12. MPI Visualizer-Analyzer Questionnaire	MPI	0.62	0.18	150

Table 2
Correlation Matrix for Twelve Measures

Measure	1	2	3	4	5	6	7	8	9	10	11	12
1. AP	—											
2. P-G	.62*	—										
3. P-A	.42*	.54*	—									
4. CC	.23	.28*	.20	—								
5. CR	.16	.24	.04	.50*	—							
6. FB	.38*	.40*	.28*	.45*	.23	—						
7. PF	.33*	.33*	.15	.36*	.35*	.47*	—					
8. NS	.30*	.40*	.17	.20	.14	.27	.10	—				
9. DR	.36*	.40*	.23	.34*	.18	.37*	.30*	.41*	—			
10. V-G	.31*	.51*	.18	.09	.08	.19	.17	.18	.16	—		
11. V-A	.11	.28*	.11	.02	.01	.15	.08	.12	.11	.40*	—	
12. MPI	−.08	−.08	−.05	.07	.04	.12	.09	−.08	.11	−.09	.06	—

*$p < .05$.

Conclusions

This study contributes to the existing research by examining the relationships between cognitive styles, cognitive abilities, and mathematical performances in calculus. The correlational matrix revealed that spatial orientation ability, measured by Card Rotation and Cube Comparisons tests, did not correlate with calculus performance. Unlike spatial visualization and logical reasoning ability, spatial orientation seems to be unrelated to calculus performance although visualizing mathematical objects from different perspectives is crucial to understanding calculus. The significant correlation between spatial visualization and calculus performance could be partially attributed to the tasks that require sketching derivative or antiderivative graphs; however, this trend is also evident in consideration of correlations with AP test scores. The MPIC and MPI visualizer-analyzer questionnaires did not correlate with spatial ability and logical reasoning ability measures, suggesting that cognitive abilities do not influence students' preference for visual or analytic thinking, and vice versa. This is consistent with previous research (Hegarty & Kozhevnikov, 1999; Lean & Clements, 1981; Moses, 1977; Suwarsono, 1982). Krutetskii (1976) also observed that gifted students do not possess strong spatial abilities and might prefer not to use visual methods.

A factor analysis on the twelve variables in Tables 1 and 2 provides interesting results. Using the varimax rotation, eleven of these variables load onto four easily interpretable factors: a calculus performance factor with AP (0.582), P-G (0.721), P-A (0.612); spatial ability factor with CC (.717), CR (.639), PF (.523), FB (.495); logical reasoning factor with DR (.76), NS (.416), and cognitive style factor with V-G (.729), V-A (.522), P-G (.426). A modified version of Mathematical Processing Instrument ([MPI], Suwarsono, 1982) was used to measure the calculus students' visual preference. The MPI did not did not load on any of the four factors and did not correlate significantly with any measure. On the other hand, the MPIC test and questionnaire regarding derivative and antiderivative tasks presented graphically loaded substantially on the cognitive style factor and correlated significantly with calculus performance measures, suggesting that the MPIC is measuring an important component of cognition. Our results are consistent with those of Galindo-Morales (1994), who reported that visuality as assessed by the MPI was not related to calculus performance. However, when calculus derivative and antiderivative tasks were used to measure mathematical performance and visuality, significant correlations can be found. Moreover, most calculus students have acquired a deep conceptual understanding of mathematics and might have characteristics that distinguish them from others (Ferrini-Mundy, 1987).

Our work with AP calculus students has generated new information about ability, style and mathematical performance in calculus. We believe that analyses of data obtained with the MPIC have produced results worthy of continued study, and that the MPIC has the potential to be a measure of performance and preference for visual thinking in calculus.

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