Comparing The Mathematical Thinking Experiences Of Students At Faculty Of Education And Faculty Of Arts And Sciences

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ABSTRACT
The aim of this study is to compare the mathematical thinking experiences of fourth grade students at faculty of education and faculty of arts and sciences in the stages of specializing, generalizing, conjecturing, and proving. The study was conducted with 72 fourth grade students in the spring term of the academic year of 2015-2016. While 36 were from the elementary mathematics teaching programme at faculty of education; 36 of students were from the department of mathematics at faculty of arts and sciences. The data were collected via the study worksheets and unstructured observations that were performed during the application and they were analyzed via content analysis method. Findings acquired from the worksheets and observations that were performed during the application show that students at faculty of education are more successful in the stages of mathematical thinking than students at faculty of arts and sciences.
Keywords: Mathematics education, mathematical thinking, faculty of education, faculty of arts and sciences.

INTRODUCTION
Mathematics is one of the most important tools of improving thinking (Keskin, Akbaba Dağ, & Altun, 2013) and requires a specific form of thinking. This form of thinking is referred to as "Mathematical Thinking (MT)". MT is not essentially different from daily and scientific thinking, but a form of daily thinking developed with a specific method (Yıldırım, 2004). Since MT is one of the most significant objectives of mathematics education (Baki, 2008; Stacey, 2006), it is of great importance for people. Because people need to think in order to continue their lives (Sternberg, 1996).

It is noted in the literature that MD consists of following components: *specializing* (Alkan & Bukova Güzel, 2005; Hacısalihoğlu, Mirasyedioğlu, & Akpınar, 2003; Liu, 2003; Piggott, 2004; Stacey, Burton, & Mason, 1985)*, abstraction* (Alkan & Bukova Güzel, 2005; Tall, 2002), *synthesizing* (Tall, 2002), *generalizing* (Alkan & Bukova Güzel, 2005; Hacısalihoğlu et al., 2003; Liu, 2003; Piggott, 2004; Stacey et al., 1985; Tall, 2002), *conjecturing* (Alkan & Bukova Güzel, 2005; Hacısalihoğlu et al., 2003; Liu, 2003; Piggott, 2004; Stacey et al., 1985), *modelling* (Tall, 2002), *problem solving* (Piggott, 2004; Tall, 2002), *proving* (Alkan & Bukova Güzel, 2005; Hacısalihoğlu et al., 2003; Liu, 2003; Stacey et al., 1985; Tall, 2002; Yıldırım, 2004), *analogy* (Liu, 2003), *induction* (Liu, 2003; Yıldırım, 2004), *deduction* (Liu, 2003; Yıldırım, 2004)*, and reasoning* (Umay, 2003). When these components are examined, it seems that specializing, generalizing, conjecturing, and proving stand out among other components. Since it would not be possible to assess all components in a single study, it was decided to investigate only the components which stand out among others. Components included in the study are described briefly below:

Specializing is the main component of MT (Mason, Burton, & Stacey, 2010). Specializing can be defined as the act of examining special conditions when faced with a problem situation (Burton, 1984). Working on such special conditions is of great importance in terms of providing a foundation for conjecturing and generalizing (Çelik, 2016). In specializing, concrete examples of abstract problems are considered (Nickerson, 2010).

The word generalizing is defined as "mind's act of thinking in general or the transition from special to general" in the Dictionary of Turkish Language Association (Turkish Language Association [TLA], 2011). Generalizing is one of the main activities of mathematics education (Baki, 2008) and the second main component after specializing (Hashemi, Abu, & Kashefi, 2013). The generalizing process involves revealing patterns between certain examples and conjecturing about larger set/sets which involve these examples as well (Çelik, 2016).

In the Dictionary of Turkish Language Association, the word conjecture is defined as "the theoretical thought or hypothesis which is not yet verified with experiments, but expected to be verified" (TLA, 2011). Conjecturing is the process of sensing that something might be true, estimating, and researching whether it is true (Çelik, 2016).

This process automatically occurs in a circular manner when performing the specializing phase and the generalizing phase (Arslan & Yıldız, 2010; Mason et al., 2010).

Proving is the process of revealing the accuracy of something by showing evidences (TLA, 2011). Proving is the last stage of the activity in which ideas are concluded during problem solving (Tall, 1991). For mathematicians, proving involves considering new conditions, focusing on important bits, taking relations into account, making predictions, formulating definitions when necessary, and forming valid arguments (Hanna & de Villiers, 2012).

From these descriptions, it can be said that "MT is a process in which these four components follow each other" (Alkan & Bukova Güzel, 2005; Arslan & Yıldız, 2010; Hacısalihoğlu et al., 2003; Keskin et al., 2013). In some studies in the literature (Hacısalihoğlu et al., 2003; Hendersen, 2002; Piggott, 2004; Tall, 2002), it is noted that MT skill can be improved with activities related to problem solving. The importance of MT and problem solving is highlighted in updated mathematics teaching programs as well (Ministry of National Education [MoNE], 2013a, 2013b). In this regard, this study investigates MT processes of students attending different faculties focusing on activities related to problem solving. Therefore, this study aims to reveal differences between 4th year mathematics students attending the Faculty of Education (FE) and the Faculty of Arts and Sciences (FAS) in terms of MT processes. Determination of differences between students attending different faculties in terms of MT will shed light to how teaching and learning activities should be carried out, guide faculty members in determining course content, and examine improvement of FE and FAS students in MT processes.

METHOD

The study utilizes the qualitative research approach. Qualitative research is a method which examines the study problem in an interpretative approach based on a holistic point of view (Karataş, 2015).

Study Group

The study group consists of 36 fourth year students attending the elementary mathematics teaching program of the FE in Giresun University and 36 fourth year students attending the mathematics department of the FAS in the same university in 2015-2016 academic year. This study utilizes the maximum diversity sampling to determine common or different aspects in a variety of situations, thus describe the problem in a wider framework (Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz, & Demirel, 2009). All students in the study group participated in the research on a voluntary basis. Among FE students, 10 were male and 26 were female, whereas among FAS students, 17 were male and 19 were female. Also, most students in both faculties were regular high school graduates and attended the science department in high school.

Data Collection Tools

The data were collected via three worksheets developed by the researcher and through unstructured observations. Questions in the worksheets were prepared utilizing works of Baki (2008) and Watson and Mason (1998). Each worksheet consists of two activities and 9 questions in total. It was concluded from opinions of three academics, experts in mathematics education, that questions in the worksheets were aimed at specializing, generalizing, conjecturing, and proving phases. The questions were encoded as W_aS_b, where indicates the worksheet and b indicates the question number. The first worksheet (W_1) contained questions related to unit squares and the second worksheet (W_2) and the third worksheet (W_3) contained questions related to unit cubes. The 1st and 5th questions in the worksheets were related to *specializing*; the 2nd and the 6th questions were related to *generalizing*; the 3rd, 7th, and the 9th questions were related to *conjecturing*; and the 4th and 8th questions were related to *proving*. In order to test the feasibility of the worksheets and determine the time required to answer questions in the worksheets, a pilot study was performed with 3rd year prospective mathematics teachers. During the pilot application, it was realized that the number of cubes in the second worksheet was wrong and made the necessary correction to give the worksheet its final form. One of the most important data collection tools in qualitative research is observation (Yıldırım & Şimşek, 2008). For this reason, unstructured observations were used in this study in order to observe behaviors of FE and FAS students in classroom environment and describe these behaviors in detail.

Implementation of Data Collection Tools

The students were given three hours to answer the questions in the worksheets during the actual implementation. The students worked in groups of two. The researcher participated in the implementation without hiding his identity and guided students. The researcher tried to collect data by asking Watson and Mason's (1998) MT encouraging questions without leading students to any direction.

Data Analysis

The data obtained from the worksheets were analyzed using the content analysis method. Firstly, an answer key containing possible answers from the students was created. Then, answers given by the students were tabulated according to questions. Then, the data in the tables were read by the researcher multiple times and draft codes were created for each question in the worksheets. Answers with the same meaning were placed under the same code. Another researcher was asked for help to ensure the reliability of the encoding and answers from the students were encoded separately by two researchers. The following formula was used to calculate the consistency of codes prepared by two researchers: "Reliability = [Agreement / (Agreement + Disagreement)]" (Miles & Huberman, 1994). Using this formula, the consistency between two researchers was found to be 92.3%. Two researchers discussed on codes on which they did not agree, reached an agreement on codes and the common codes were presented to the reader in tables. The observation data and answers given by students to questions asked during the observation process were used in order to interpret answers given to questions in the worksheets.

Limitations

Questions in the worksheets were aimed at specializing, generalizing, conjecturing, and proving phases of MT. Also, instead of all subjects in mathematics, these questions were related to unit squares and unit cubes. Finally, this study was limited to 36 fourth year students attending the elementary mathematics teaching program of the FE in Giresun University and 36 fourth year students attending the mathematics department of the FAS in the same university.

FINDINGS

This section involves answers given by students to questions aimed at specializing, generalizing, conjecturing, and proving phases and findings obtained from observations.

Specializing

In the 1st and 5th questions in the worksheets, students were asked to draw 3 and 4 unit squares side-by-side and calculate the number of adjacent edges in the final shape (W_1S_1); find the perimeter of shapes composed of 3 and 4 unit squares drawn side-by-side (W_1S_5); draw 3 and 4 unit cubes side-by-side and calculate the number of adjacent edges in the final shape (W_2S_1); find the surface area of shapes composed of 3 and 4 unit cubes drawn side-by-side (W_2S_5); draw 8 and 10 unit cubes in a way that the number of junction points will be 3 and 4 (W_3S_1); calculate the number of adjacent surfaces in the final shapes consisting of 8 and 10 unit cubes in a way that the number of junction points will be 3 and 4 (W_3S_5). Codes created for the specializing phase and student answers related to these codes are given below:

Code 1: Drawing All Systematic Shapes Correctly: This code was related to drawing both shapes which were systematic and had a certain pattern correctly, therefore associated with W_1S_1, W_2S_1, and W_3S_1. The success rate related to this code was 88.9%, since some groups were not able to draw any of the shapes. Examples from answers given by the students to W_2S_1 and W_3S_1 are given below:

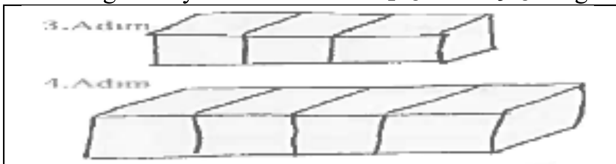

Figure 1a: Shapes drawn by FE students for W_2S_1

Figure 1b: Shapes drawn by FAS students for W_3S_1

Code 2: Correctly Finding what is Asked for Special Conditions: This code was related to answering questions taking two special conditions into account and following a systematic path, therefore associated with W_1S_1, W_2S_1, W_3S_1, W_1S_5, W_2S_5, and W_3S_5. It was found that the students correctly found all of what was asked from them. Examples from answers given by the groups to W_2S_5 and W_3S_5 are given below:

Küp Sayısı	Şeklin Yüzey Alanı
1	6
2	10
3	14
4	18

Figure 2a: One of the answers given to W_2S_5 by FE students

Küp Sayısı	Bitişik Yüzey Sayısı
4	4
6	7
8	10
10	13

Figure 2b: One of the answers given to W_3S_5 by FAS students

The frequency of codes created for the specializing phase is shown in Table 1:

Table 1: The frequency of codes related to the specializing phase

	Faculties	W_1S_1	W_1S_5	W_2S_1	W_2S_5	W_3S_1	W_3S_5	Total
Code 1	FE	18	0	15	0	16	0	49
	FAS	16	0	16	0	15	0	47
Code 2	FE	18	18	18	18	18	18	108
	FAS	18	18	18	18	18	18	108
Unanswered	FE	0	0	3	0	2	0	5
	FAS	2	0	2	0	3	0	7

Table 1 shows that all necessary operations related to the specializing phase were correctly performed by the students and the majority of students drew shapes perfectly. It was seen during the observations in the classroom environment that the groups answered questions related to the specializing phase in a short amount of time.

Generalizing

In the 2nd and 6th questions in the worksheets, students were asked to mathematically describe patterns in the number of adjacent edges in unit squares drawn side-by-side (W_1S_2); the perimeter of shapes composed of unit squares drawn side-by-side (W_1S_6); the number of adjacent surfaces in shapes composed of unit cubes drawn side-by-side (W_2S_2); the surface area of shapes composed of unit cubes drawn side-by-side (W_2S_6); the number of junction points in shapes composed of unit cubes drawn side-by-side (W_3S_2); the number of adjacent surfaces in shapes composed of unit cubes drawn side-by-side (W_3S_6). The codes created in relation to the generalizing phase are briefly explained and examples from student answers are given below:

Code 1: Making Verbal Generalizations: This code was related to the students' ability to verbally describe relations between numbers or variables and relevant questions were answered correctly by some of the students. Two examples from answers given by the students to W_1S_2 and W_2S_2 are given below:

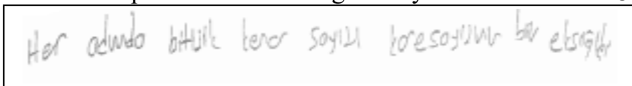

Figure 3a: One of the correct verbal generalizations made for W_1S_2 by FE students

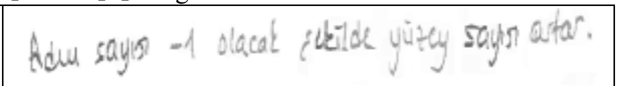

Figure 3b: One of the incorrect verbal generalizations made for W_2S_2 by FAS students

Code 2: Making Mathematical Generalizations: This code was related to the students' ability to mathematically describe relations between numbers or variables and relevant questions were answered correctly and incorrectly by the students. Correct and incorrect answers given to W_3S_2 are given below:

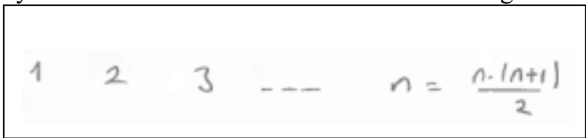

Figure 4a: One of the incorrect mathematical generalizations made for W_3S_2 by FE students

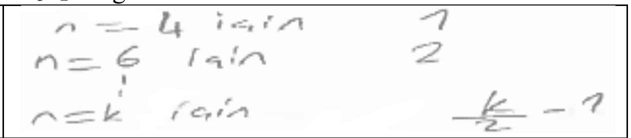

Figure 4b: One of the correct mathematical generalizations made for W_3S_2 by FAS students

Code 3: Making Verbal and Mathematical Generalizations: This code was related to the students' ability to verbally and mathematically describe relations between numbers or variables and relevant questions were answered correctly by all groups. Some examples from answers given by FE and FAS students to W_2S_2 and W_3S_2 are given below:

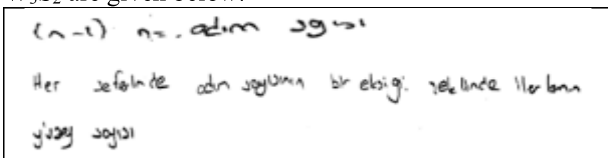

Figure 5a: One of the verbal and mathematical generalizations made for W_2S_2 by FE students

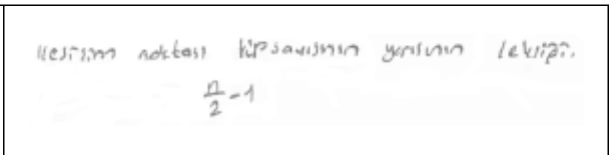

Figure 5b: One of the verbal and mathematical generalizations made for W_3S_2 by FAS students

The frequency of codes created for the generalizing phase is shown in Table 2:

Table 2: The frequency of codes related to the generalizing phase

	Faculties	W_1S_2	W_1S_6	W_2S_2	W_2S_6	W_3S_2	W_3S_6	Total
Code 1	FE	4	1	1	0	1	3	10
	FAS	4	2	4	0	2	2	14
Code 2	FE	13	16	16	16	13	12	86
	FAS	13	10	13	11	14	11	72
Code 3	FE	1	1	1	2	4	3	12
	FAS	1	2	1	2	2	2	10
Unanswered	FE	0	0	0	0	0	0	0
	FAS	0	4	0	5	0	3	12

Table 2 shows that FE and FAS students mostly preferred to express relations between numbers or variables mathematically. It was seen in observations made in the classroom environment that the students did not have difficulties in terms of making mathematical generalizations. However, some students were not able to find the patterns asked in W_3S_2 and W_3S_6 and asked for help.

Conjecturing

In the 3rd, 7th, and 9th questions in the worksheets, students were asked to mathematically describe relations between numbers of adjacent edges in unit squares drawn side-by-side (W_1S_3); perimeters of shapes composed of unit squares drawn side-by-side (W_1S_7); numbers of adjacent surfaces in cubes drawn side-by-side (W_2S_3); surface areas of shapes composed of unit cubes drawn side-by-side (W_2S_7); numbers of junction points in unit cubes drawn side-by-side (W_3S_3); numbers of adjacent surfaces in shapes composed of unit cubes drawn side-by-side (W_3S_7); numbers of adjacent edges in unit squares drawn side-by-side and perimeters of shapes (W_1S_9); numbers of adjacent surfaces and surface areas of shapes composed of drawn side-by-side (W_2S_9); numbers of conjunction points and numbers of adjacent surfaces of shapes composed of unit cubes drawn side-by-side (W_3S_9). Codes created for the conjecturing phase and answers given by the groups related to these codes are shown below:

Code 1: Making Verbal Conjectures: This code was related to expression of conjectures verbally and it was found that students made correct and incorrect conjectures. Examples from correct and incorrect verbal conjectures made by the students for W_1S_9 are given below:

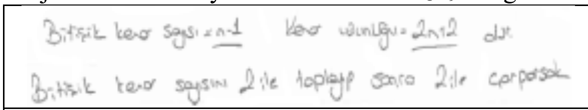

Figure 6a: One of the correct verbal conjectures made for W_1S_9 by FE students

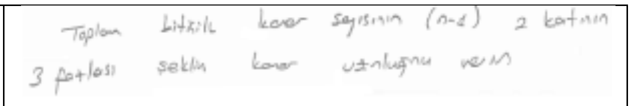

Figure 6b: One of the incorrect verbal conjectures made for W_1S_9 by FAS students

Code 2: Making Mathematical Conjectures: This code was related to describing conjectures mathematically. It was found that the students made mathematically correct or incorrect conjectures related to this code. Some of the answers given to W_2S_9 are shown below:

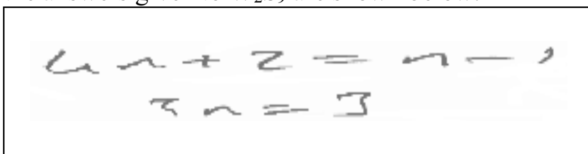

Figure 7a: One of the incorrect mathematical conjectures made for W_2S_9 by FE students

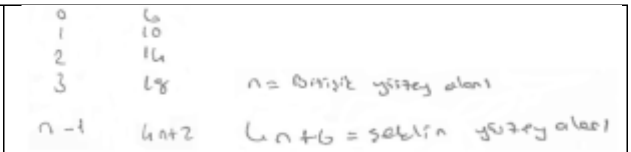

Figure 7b: One of the correct mathematical conjectures made for W_2S_9 by FAS students

Code 3: Making Verbal and Mathematical Conjectures: This code was related to the students' ability to verbally and mathematically describe relations between numbers or variables and relevant questions were answered incorrectly by some students. Two examples from answers given to W_1S_7 and W_3S_7 are given below:

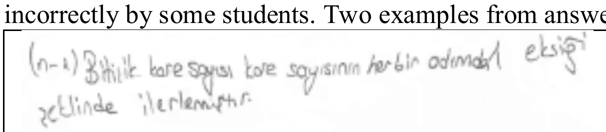

Figure 8a: One of the correct verbal and mathematical conjectures made for W_1S_7 by FE students

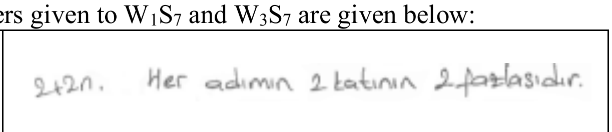

Figure 8b: One of the incorrect verbal and mathematical conjectures made for W_3S_7 by FAS students

The frequency of codes created for the conjecturing phase is shown in Table 3:

Table 3: The frequency of codes related to the conjecturing phase

	Faculties	W_1S_3	W_1S_7	W_1S_9	W_2S_3	W_2S_7	W_2S_9	W_3S_3	W_3S_7	W_3S_9	Total
Code 1	FE	8	8	8	11	11	7	11	11	6	81
	FAS	5	4	6	4	5	5	6	5	1	41
Code 2	FE	4	5	4	5	6	6	3	5	5	43
	FAS	12	13	5	13	12	5	12	12	4	88
Code 3	FE	5	5	2	2	1	1	3	2	0	21
	FAS	0	1	1	1	1	5	0	1	4	14
Unanswered	FE	1	0	4	0	0	4	1	0	7	17
	FAS	1	0	6	0	0	3	0	0	9	19

Table 3 shows that the students made verbal, mathematical or verbal and mathematical conjectures as in the generalizing phase. Also, it was observed that FAS students preferred mathematical conjectures, whereas FE students preferred verbal conjectures. Observations showed that most groups had difficulties with the 9th question of each worksheet, which were related to the conjecturing phase. When asked about why they could not answer these questions, the students gave answers such as, "*We are having difficulties with making associations. What should we do*?" or "*They do not have a relation.*"

Proving

4th and 8th questions in the worksheets were related to the proving phase. In this context, the students were asked to calculate and prove the number of adjacent edges at the nth step of a pattern created by drawing unit squares side-by-side and the perimeter of the shape in W_1S_4 and W_1S_8 respectively and calculate and prove the number of adjacent surfaces at the nth step of a pattern created by drawing unit cubes side-by-side and the surface area of the shape in W_2S_4 and W_2S_8 respectively. Also, the students were asked "What would be the number of conjunction points in at the nth step of a pattern created by drawing unit cubes side-by-side?" in W_3S_4 and "What would be the number of adjacent surfaces at the nth step of a pattern created by drawing unit cubes side-by-side?" in W_3S_8. Codes created for the proving phase and student answers related to these codes are given below:

Code 1: Proving Algebraically: This code was more significant for FE students and related to proving a mathematical expression by induction. The students proved mathematical expressions either completely correctly, partially correctly or completely incorrectly. Also, the number of students who proved mathematical expressions completely correctly was quite low. Examples from answers given to W_2S_4 by the students are shown below:

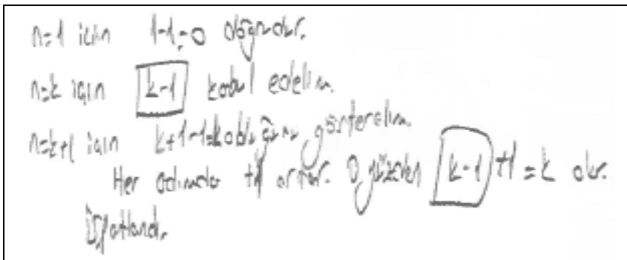

Figure 9a: One of the algebraically correct proofs for W_2S_4 by FE students

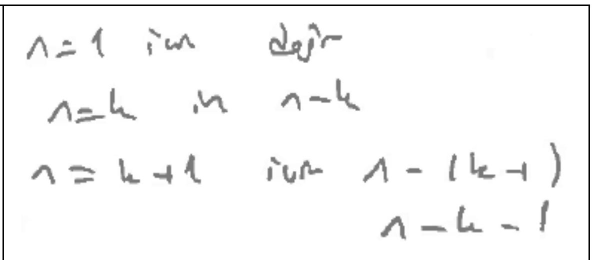

Figure 9b: One of the algebraically incorrect proofs for W_2S_4 by FE students

Code 2: Proving Arithmetically: This code was more significant for FAS students and related to proving a mathematical expression by giving values to variables. The answer given by one of the groups to W_3S_8 is shown below:

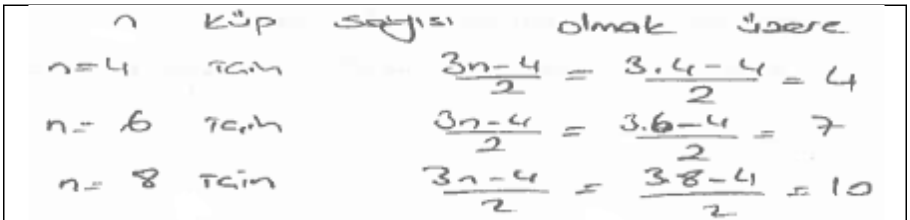

Figure 10: One of the arithmetic proofs for W_3S_8 by FAS students

The frequency of codes created for the proving phase is shown in Table 4:

Table 4: The frequency of codes related to the proving phase

	Faculties	W_1S_4	W_1S_8	W_2S_4	W_2S_8	W_3S_4	W_3S_8	Total
Code 1	FE	18	18	18	18	16	13	101
	FAS	3	2	2	3	2	1	13
Code 2	FE	0	0	0	0	0	0	0
	FAS	15	16	16	15	13	17	92
Unanswered	FE	0	0	0	0	2	5	7
	FAS	0	0	0	0	3	0	3

Table 4 indicates that the students algebraically and arithmetically proved their conjectures. Observations showed that most FAS students attempted to prove mathematical expressions arithmetically in the 4th and 8th questions by giving values to variables and explained this as "*Since we do not know which proving method to use, we tried to prove through trial and error.*" or "*We took the more convenient road and tried to prove expressions by giving values, since we have insufficiencies in terms of proving by induction.*" It was found that all FE students used algebraic proving with induction in the 4th and 8th questions and the students explained this as "*Our teachers emphasized how to prove expressions with induction especially in the general mathematics and the abstract mathematics classes*." Also, it was observed that most groups performed algebraic proving incorrectly or could not complete the proof correctly.

DISCUSSION

It was found that FE and FAS students answered a considerable portion of questions related to the specializing phase perfectly. The fact that the students did not experience any problems when answering questions related to the specializing phase shows that operational knowledge is given importance in mathematics courses. This is mentioned in some studies in the literature as well (Arslan & Yıldız, 2010; Keskin et al., 2013; Uğurel & Moralı, 2010). In terms of generalizing, it was seen that FE and FAS students mostly expressed relations between numbers or variables mathematically. In addition, it was observed that the students did not have difficulties in expressing these relations with mathematical symbols. The finding that the students did not have problems with making mathematical generalizations is not consistent with studies conducted by Arslan and Yıldız (2010), Keskin et al. (2013), Özmantar, Bingölbali, and Akkoç (2008) and Tall (2008). It was observed that FAS students preferred mathematical conjectures, whereas FE students preferred verbal conjectures, which shows that FAS students were more successful in terms of expressing relations between numbers or variables mathematically compared to FE students. The fact that students have difficulties related to conjecturing is mentioned in the literature as well (Arslan & Yıldız, 2010; Keskin et al., 2013). However, FE and FAS students are expected to conjecture, test conjectures, and express conjectures with mathematical symbols and notations. In this context, it should be remembered that creating an environment for students using in-class activities where they can make conjectures is important for the improvement of their MT skills.

Although FAS students developed a formula through trial and error, they were not able to produce valid arguments related to proving the accuracy of these formulas. Said answers were mostly numerical, but not in the form of algebraic representations. The fact that FAS students attempted to prove expressions experimentally by giving values to variables shows that they performed specializing instead of proving. Students were found to attempt to prove mathematical expressions experimentally and this attempt was found to be their dominant strategy in some studies in the literature (Almeida, 2001; Arslan & Yıldız, 2010; Çelik, Güler, Bülbül, & Özmen, 2015; Özer & Arıkan, 2002). FE students, on the other hand, developed valid formulas, but they produced partially correct arguments related to the accuracy of these formulas. All these answers utilized induction and were in the form of algebraic representations. An important part of FE students found the answer for "n=k+1" incorrectly when proving with induction. It was reported in some studies in the literature (Baker, 1996; Movshovitz-Hadar, 1993; Stylianides, Stylianides, & Philippou, 2007) that students had difficulties related to proving with induction. In conclusion, it was understood that FE and FAS students had insufficiencies in terms of proving.

Finally, FAS students correctly answered 95.7%, 86.1%, 81.5%, and 6.5% of questions related to specializing, generalizing, conjecturing, and proving phases respectively. Finally, FAS students correctly answered 96.9%, 95.4%, 82.7%, and 25% of questions related to specializing, generalizing, conjecturing, and proving phases respectively. It seems that the success rate of students from both faculties decreased toward the proving phase, FAS students in particular.

CONCLUSIONS and RECOMMENDATIONS

It was found that FE and FAS students answered a considerable portion of questions related to the specializing phase perfectly and showed a high success rate in the specializing phase. Therefore, it is understood that FE and FAS students were competent in specializing. In this context, it is recommended that conceptual questions are given importance as well as operational questions in primary, middle, secondary, and undergraduate levels. In terms of generalizing, it was seen that FE and FAS students mostly expressed relations between numbers or variables mathematically. A similar result was found for FAS students in the conjecturing phase as well. FE students, on the other hand, mostly used verbal conjectures in the conjecturing phase. This shows that FE students were not as successful as FAS students in terms of making mathematical conjectures. For this reason, it is necessary for faculty members to express relations between numbers and variables using more mathematical symbols and notations and allow students to use the daily and mathematical language in an efficient manner. The fact that some questions were left unanswered, answered incorrectly or partially correctly in the proving phase shows that FE and FAS students had more difficulties in the proving phase compared to other components of MT. In order to improve students' skills related to proving, faculty members should mention the importance of MT phases and proving methods and allow students to work on different proofs of problems. Finally, it was found that the success rate of FE and FAS students decreased when transitioning from one phase of MT to another. It is recommended that proving and proving methods are emphasized more in FE and FAS and worksheets related to proving are added to undergraduate level textbooks.

REFERENCES

Alkan, H., & Bukova Güzel, E. (2005). Development of mathematical thinking in the student teachers. *Gazi University Journal of Gazi Educational Faculty, 25*(3), 221-236.

Almeida, D. (2001). Pupils' proof potential. *International Journal of Mathematical Education in Science and Technology, 32*(1), 53-60.

Arslan, S., & Yıldız, C. (2010). Reflections from the experiences of 11th graders during the stages of mathematical thinking. *Education and Science, 35*(156), 17-31.

Baker, J. D. (1996) *Students' difficulties with proof by mathematical induction.* The Annual Meeting of American Educational Research Association, New York.

Baki, A. (2008). *Kuramdan uygulamaya matematik eğitimi* (3. Baskı). Trabzon: Derya Kitapevi.

Burton, L. (1984). Mathematical thinking: The struggle for meaning. *Journal of Researching Mathematics Education, 15*(1), 35-49.

Büyüköztürk, Ş., Kılıç-Çakmak., E., Akgün, Ö., E., Karadeniz, Ş., & Demirel, F. (2009). *Bilimsel araştırma yöntemleri* (4. Baskı). Ankara: Pegem Akademi Yayıncılık.

Çelik, D. (2016). Matematiksel düşünme. In E. Bingölbali, S. Arslan, & İ. Ö. Zembat (Eds.), *Matematik eğitiminde teoriler* (s. 17-42). Ankara: Pegem Akademi Yayıncılık.

Çelik, D., Güler, M., Bülbül, B. Ö., & Özmen, Z. M. (2015). Reflections from a learning setting designed to investigate mathematical thinking. *International Journal of Educational Studies in Mathematics, 2*(1), 11-23.

Hacısalihoğlu, H., Mirasyedioğlu, Ş., & Akpınar, A. (2003). *Matematik öğretimi: Matematikte yapılandırıcı öğrenme ve öğretme*. Ankara: Asil Yayın Dağıtım.

Hanna, G., & de Villiers, M. (2012). Aspects of proof in mathematics education. In Hanna, G., & de Villiers, M. (Eds.). *Proof and proving in mathematics education: The 19th ICMI Study* (pp. 1-10). New York: Springer.

Hashemi, N., Abu, M. S., & Kashefi, H. (2013, October). *The importance of generalization in teachers lesson plan for integral concept.* The 2nd International Seminar on Quality and Affordable Education 2013, KSL Hotel & Resort, Johor, Malaysia.

Henderson, P. (2002). *Materials development in support of mathematical thinking.* Retrieved from: http://portal.acm.org/citation.cfm?id=783001.

Karataş, Z. (2015). Sosyal bilimlerde nitel araştırma yöntemleri. *Manevi Temelli Sosyal Hizmet Araştırmaları Dergisi, 1*(1), 62-80.

Keskin, M., Akbaba Dağ, S., & Altun, M. (2013). Comparison of 8th and 11th grade students behaviours at mathematical thinking. *Journal of Educational Sciences, 1,* 33-50.

Liu, P. H. (2003). Do teachers need to incorporate the history of mathematics in their teaching? *The Mathematics Teacher, 96*(6), 416-421.

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathematically* (2nd Ed.). London, England: Pearson Education Limited.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded source book* (2nd Ed.). Thousand Oaks, CA: Sage Publications.

Ministry of National Education [MoNE]. (2013a). *Middle school mathematics curriculum (5, 6, 7, and 8th grades).* Ankara: MoNE Board of Education.

Ministry of National Education [MoNE]. (2013b). *Secondary school mathematics curriculum (9, 10, 11, and 12th grades).* Ankara: MoNE Board of Education.

Movshovitz-Hadar, N. (1993). The false coin problem, mathematical induction and knowledge fragility. *Journal of Mathematical Behavior, 12*, 253-268.
Nickerson, R. S. (2010). *Mathematical reasoning: Patterns, problems, conjectures, and proofs*. New York, NY: Psychology Press, Taylor and Francis Group.
Özer, Ö., & Arıkan, A. (2002, Eylül). *Lise matematik derslerinde öğrencilerin ispat yapma düzeyleri.* V. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, Orta Doğu Teknik Üniversitesi, Ankara.
Özmantar, M. F., Bingölbali, E., & Akkoç, H. (2008). *Matematiksel kavram yanılgıları ve çözüm önerileri*. Ankara: Pegem Akademi Yayıncılık.
Piggott, J. (2004, January). *Developing a framework for mathematical enrichment.* Paper presented at the symposium on Critical Thinking in Teaching and Learning. University of West Indies, St. Augustine, Trinidad.
Stacey, K. (2006). What is mathematical thinking and why is it important? In *Progress report of the APEC project: Collaborative studies on innovations for teaching and learning mathematics in different cultures (II)-Lesson study focusing on mathematical thinking*, Tokyo: CRICED, University of Tsukuba.
Stacey, K., Burton, L., & Mason, J. (1985). *Thinking mathematically*. England: Addison-Wesley Publishers.
Sternberg, R. J. (1996). What is mathematical thinking. In R. J. Sternberg & T. Ben-Zeev (Eds.). *The nature of mathematical thinking* (pp. 303-318). Mahwah, NJ: Lawrence Erlbaum.
Stylianides, G. J., Stylianides, A. J., & Philippou, G. N. (2007). Preservice teachers' knowledge of proof by mathematical induction. *Journal of Mathematics Teacher Education, 10*(3), 145-166.
Tall, D. (1991). The psychology of advanced mathematical thinking. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 3-21). Dordrecht, The Netherlands: Kluwer.
Tall, D. (2002). *Advanced mathematical thinking*. USA: Kluwer Academic Publishers.
Tall, D. (2008). *The historical and individual development of mathematical thinking: Ideas that are set-before and met-before.* Colóquio de História e Tecnologia no Ensino de Matemática (HTEM), UFRJ, Brazil.
Dictionary of Turkish Language Association [TLA]. (2011). *Turkish dictionary.* Ankara: Turkish Language Association Publications.
Uğurel, I., & Moralı, S. (2010). A close view on the discussion in relation to a activity about proving a theorem in a high school mathematics lesson via students' discourse. *Buca Faculty of Education Journal, 28,* 135-154.
Umay, A. (2003). Mathematical reasoning ability. *Hacettepe University Journal of Education, 24,* 234-243.
Watson, A., & Mason, J. (1998). *Questions and prompts for mathematical thinking.* Association of Teachers of Mathematics.
Yıldırım, A., & Şimşek, H. (2008). *Sosyal bilimlerde nitel araştırma yöntemleri*. Ankara: Seçkin Yayıncılık.
Yıldırım, C. (2004). *Matematiksel düşünme* (4. Baskı). İstanbul: Remzi Kitabevi.