LEARNING TRAJECTORIES AS A TOOL FOR MATHEMATICS LESSON PLANNING

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In this paper, I examine the utility of a mathematics learning trajectory as a tool to support teachers' attention to students' mathematical thinking. I present findings from one second grade teacher's use of a learning trajectory as she planned a sequence of three mathematics lessons. Findings suggest learning trajectories support teachers in choosing appropriate tasks and learning goals, and in anticipating students' likely approaches and difficulties. Learning trajectories, as representations of student thinking, provide teachers with a means of evaluating evidence of student learning of intended goals and afford them with a range of instructional moves based on their students' current conceptions.

Keywords: Learning Trajectories; Instructional Activities and Practices

Attention to student thinking has been identified as a critical tool to initiate changes in teachers' knowledge for teaching and improvements in classroom instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Franke, Carpenter, Levi, & Fennema, 2001; Kazemi & Franke, 2004; Sherin & van Es, 2009). Moreover, an emerging hypothesis in the field is that the construct of a *learning trajectory* (LT) has the potential to support teachers in making sense of and use student thinking to improve teaching and learning. The authors of the Common Core State Standards (2010) emphasized the use of research-based LTs in the development of the new standards and committed to using research and evidence of student learning to inform future revisions. Daro, Mosher, and Corcoran (2011) state that LTs serve "as a basis for informing teachers about the (sometimes wide) range of student understanding they are likely to encounter, and the kinds of pedagogical responses that are likely to help students move along" (p. 12). However, little is known about how teachers come to learn about LTs and appropriate them into their instruction. In this study, I identify the ways in which one elementary teacher used a LT to support attention to her students' mathematical thinking during instruction. In particular, the teacher's use of the LT as she planned her math lessons, identified learning goals, and anticipated likely student responses will be highlighted.

Background

Learning trajectories utilize research on student learning to describe probable pathways of learning over time. Researchers that have studied the implications of LTs for teachers have found that LTs provide a framework for making instructional decisions (Bardsley, 2006; Sztajn, Wilson, Confrey, & Edgington, 2011b; Wilson, 2009) and afford teachers with a means to focus on their students' mathematical thinking (Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Edgington, Sztajn, & Wilson, 2011; McKool, 2009; Mojica, 2010). As teachers increasingly attend to student thinking in lesson planning and instruction, research must consider the role of LTs in supporting teachers' complex work. Research has yet to address how teachers adjust their lesson planning and instruction when they have information about the progression of more sophisticated levels of thinking inherent in LTs, or how teachers use evidence of student thinking to inform future instruction in light of LTs. This study contributes to the research on teachers' uses of LTs to support attention to student thinking in planning for mathematics instruction.

For the purposes of this paper, I am reporting one teacher's use of a LT in lesson planning through three different processes: identification of learning goals, choice of instructional tasks, and anticipation of students' work. Often considered a core routine of teaching, lesson planning refers to the time teachers spend preparing for instruction before students enter the classroom. Grossman and colleagues (2005) refer to this as the "preactive" aspect of practice, where teachers focus on lesson planning, unit planning, or even planning for classroom management.

Studies on what teachers attend to in planning their lessons indicated that teachers focus on ideas such as content, activities or tasks, materials, textbooks, routines, as well as students' needs and backgrounds (Fernandez & Cannon, 2005; McCutcheon, 1980; Yinger, 1980; Zahorik, 1975). In his 1975 study of teacher planning, Zahorik showed that teachers attended to content more often than objectives, followed by activities. In a study of 12 elementary school teachers, McCutcheon (1980) found that teachers used their textbook as a main source for activities and depended heavily upon suggestions from the teachers' guide. In a later study, Brown (1988) examined the lesson planning practices of 12 middle school teachers in various content areas. She found that teachers relied heavily on curriculum materials, building their lessons off of objectives expressly stated in the curriculum resources.

More recently, Superfine (2008) studied three teachers' lesson planning with respect to a specific mathematics curriculum. Her study revealed two planning problems: difficulty anticipating student work, misconceptions, and potential errors for a given task, and understanding the treatment of the content in the curriculum. She concluded that the conceptions teachers hold with respect to the teaching and learning of mathematics as well as years of experience mediated their management of the planning problems.

Conceptual Framework

In light of reform efforts to improve the teaching and learning of mathematics, one may question what *should* be the focus of planning when instruction attends to students' mathematical thinking. Teachers must consider how to construct lessons that address specific learning goals and allow teachers to gather evidence of their students' understanding towards the chosen goals. Moreover, as student learning progresses over time, teachers must be able to consider how to build students' current conceptions to reach intended learning goals.

The conceptual framework for this study draws upon the work of Hiebert, Morris, Berk, and Jansen (2007) as well as that of Stein, Engle, Smith, and Hughes (2008). Hiebert et al. (2007) proposed a framework for competencies necessary to analyze teaching with the goal of improving on instruction. Stein et al. (2008) presented five practices to support productive mathematical discourse structured around students' responses to mathematical tasks. These two frameworks were chosen because of their emphasis on student thinking as a central feature. During lesson planning, teachers not only *choose intended learning goals*, but they decompose learning goals into smaller sub-concepts that comprise larger goals (Hiebert et al., 2007). In *considering mathematical tasks* proposed in a lesson, teachers use their own content knowledge as well as their knowledge of how students are likely to approach the task to *anticipate students' responses* and likely areas of difficulty. In this way, teachers can consider how students' responses, both correct and incorrect, can lead to the intended learning goals (Stein et al., 2008). By comparing evidence of student learning to the intended learning goals, teachers can determine what aspects of their instruction helped or hindered their students' understandings (Hiebert et al., 2007). Once instruction has been evaluated, careful planning is important so that teachers consider new learning goals and instructional tasks that build on students' current conceptions and move students to more complex mathematical understanding.

Attending to student thinking can support teachers as they engage in lesson planning. This attention allows teachers to acknowledge their students' current conceptions and design lessons that build on prior knowledge. Furthermore, as teachers consider evidence of students' thinking, they can more explicitly connect students' conceptions to important mathematical ideas. As representations of student thinking, learning trajectories are tools which help advance teachers ability to make sense of this evidence and use it to develop instruction that addresses their students' existing conceptions and moves learning forward.

Method

This study seeks to understand how teachers use the construct of a LT to support attention to students' mathematical thinking and addresses the following research question: *In what ways do teachers use LTs during lesson planning to choose learning goals and instructional tasks, and anticipate students' approaches to intended instructional tasks?* A qualitative approach is appropriate in order to understand participants' created meaning of their use of one particular LT in mathematics instruction. Specifically, case studies allow the researcher to uncover and examine significant interactions that are characteristic of the phenomenon under study as well as provide concrete and contextual knowledge as evident in the end product (Merriam, 1998).

Context

Learning Trajectory-Based Instruction (LTBI) is a research project with a strong professional development component for elementary school teachers (Sztajn, Wilson, Confrey, & Edgington, 2011a). The project was motivated by the adoption of the Common Core State Standards (2010) and current research on learning trajectories in mathematics education (Battista, 2004; Clements & Sarama, 2009; Confrey et al., 2009), with the goals of examining the ways in which teachers learn about learning trajectories and use them in their classrooms to define the concept of learning trajectory-based instruction.

As the context for the professional development, teachers learned about one LT: the equipartitioning LT (EPLT). Based on a synthesis of research and clinical interviews, Confrey and her colleagues developed the EPLT that describes how children use their informal knowledge of fair sharing as a resource to build an understanding of partitive division that unifies ratio reasoning and fractions (Confrey, in press).

The EPLT begins with experiences of fairly sharing collections of items or single wholes. In equipartitioning, students must learn to coordinate three criteria: (1) creating equal sized groups or parts, (2) creating the correct number of groups or parts, and (3) exhaust the entire collection or whole. As students enact strategies to complete these tasks, they gain proficiency in mathematical reasoning practices such as justification and naming (e.g., as a count, fraction, or ratio) and begin to develop understandings of fundamental mathematical properties that later influence the ways that they fairly share multiple wholes (Confrey, Maloney, Wilson, & Nguyen, 2010). The trajectory describes how these strategies, practices, and properties ultimately unify as a generalization of partitive division that relates ratio reasoning and fractions. Important to the trajectory are not only the levels of sophistication of reasoning but parameters associated with the tasks, including the number of wholes and number of sharers. Beginning with equipartitioning collections, the next task parameters address equipartitioning single wholes (rectangles and circles), building on primitive splits such as halves and powers of two, to eventually include arbitrary integer splits. The upper levels of the trajectory address tasks that involve multiple wholes and multiple sharers when the number of wholes is both less than and greater than the number of sharers.

Participants

LTBI is a four-year project and, in its first year, involved 22 kindergarten through fifth grade teachers from one elementary school in a mid-sized urban school district in the Southeastern United States. Project participants were offered the opportunity to continue working with the research team in some respect in the second year of the project. The second grade team, consisting of five teachers, expressed an interest in developing a set of equipartitioning lessons based on the EPLT. The findings presented here are from the analysis of one particular case, Bianca. Bianca is a Hispanic female and had been teaching second grade for five years. For her, LTs represented “a continuum of learning where there are key stopping points and also major misconceptions.”

Data Sources and Analysis

The primary sources of data for this study are transcripts from grade level planning meetings, pre-lesson questionnaires, classroom observations of teachers’ instruction, and transcripts of teacher interviews. Each data cycle began with a grade level lesson planning meeting, followed by individual classroom observations, and concluded with individual post-lesson interviews. Prior to each lesson, each participant completed a pre-lesson questionnaire to obtain information about the teachers’ learning goals and any adaptations they may have made to the lesson plan. Observations took place in each participant’s classroom during her regularly scheduled math instructional time and were video recorded. Following each lesson, a semi-structured interview was conducted with the participant to discuss the teacher’s perceptions of what learning took place as well as evidence of that learning, and how the teacher used that evidence to inform future learning goals.

Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). (2012). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Kalamazoo, MI: Western Michigan University.

Data were analyzed using ATLAS.ti (2012) qualitative data analysis software. Evidence from the grade level meetings and pre-lesson questionnaires were used to examine the ways in which teachers used the EPLT to select learning goals and tasks, and anticipate students' responses. Evidence from post-lesson interviews and grade level meetings were considered to determine the ways in which the EPLT was used to reflect on the impact of instruction on student learning, evaluate evidence of student learning, and to inform future instruction. Using a constant comparative method to build and refine categories (Strauss & Corbin, 1998), open coding and pre-determined codes were utilized. The findings reported here focus on the use of the EPLT to choose learning goals, select tasks, and anticipate likely student responses for three sequenced lessons.

Results

Lesson #1: Sharing a Collection for 2, 4, and 3 Friends

During the first grade-level planning meeting, Bianca initiated the discussion of ideas for their first equipartitioning lesson by suggesting they use an activity they created and tried out the previous year where students engaged in fairly sharing a collection of 24 counters among 2, 4, and then 3 friends, justify their work, and name the resulting shares. In considering her own students, Bianca stated that she was interested in knowing if her students knew the three criteria for equipartitioning and thought that sharing collections would be a reasonable place to start since it is low on the trajectory. She stated, "I mean, for me my objective is to see do they know the three criteria. Which you can see, but you can't fully see. Because they may, with a two split, they may or may not do that right, even if they don't know all three [criteria]." After observing her students' work in the first few weeks of school, she used the nature of the task parameters of the EPLT as a justification for adapting the lesson to increasing the size of the collection from 24 to 36 counters: "that's why I went to the higher number because I know that, you know, that's going to increase the level of difficulty."

Bianca anticipated how her students would engage with the first lesson, expecting them to use dealing strategies along with number facts and doubles facts to help them determine fair shares of collections. She used the EPLT to also anticipate obstacles her students might have by saying, "I think one difficulty will definitely be naming the shares. I know that it is a more difficult task on the learning trajectory and they haven't had many experiences doing so."

After the first lesson, Bianca confirmed the difficulties her students had with naming a share and stated, "I feel as if the naming is the hardest part…Because when we teach fractions explicitly, I feel like they get to the wholes and they get to the actual sharing of things. But I feel as if we'd be doing our kids a disservice if we didn't hit on what they are most needing. Which I, from my class, I definitely think the naming thing. I definitely would not recommend sharing collections with higher numbers. I know that shouldn't be your next step, because that is what I thought."

Lesson #2: Sharing a Rectangle and a Collection for 2 Friends

Bianca used what she knew about her students' understanding and the structure of the LT to consider possible follow-up activities. She recognized that a next instructional step could be to change the task parameters but still focus on naming. Based on her teaching experience, she hypothesized that naming would be easier with a whole so in the second grade-level planning meeting, she suggested starting with sharing a rectangle for two and naming the resulting share to help scaffold students' ability to name the resulting share from equipartitioning a collection for two. She also recognized from the first lesson that students readily made connections to doubles facts, so that could potentially scaffold students' ability to name 2-splits. "What if we went, this is, I'm just throwing this out there, this could be, you know. But what if we went to wholes and just worked on halving to see if a name came out of that? And then we went back to doubles with collections and see, saw if the, you know if the vernacular, if the vocabulary came out with a whole, if they would transfer it then to collections."

The group agreed to begin the lesson by having students share two different sized rectangles fairly for two people and discuss naming the share as "one half," or "one out of two pieces" with respect to the different sized rectangles. Then, students would work to fairly share small collections as represented in drawings of arrays of 6, 8, and 10 counters and name the resulting share for each collection. Bianca identified "naming a share as 'half'" for both rectangular wholes and for collections as the goal for the lesson. She anticipated that the structure of the arrays in the second activity would help her students make a connection between "half" of the rectangle and "half" of the collection of counters. After the second lesson, Bianca considered that a possible follow up activity could be to increase the size of the collection to 12 or 14, but still share for two people, or move to sharing small collections or single wholes for four or three people and continue to focus on naming the resulting share. She attended to the interactions between the proficiency levels of the EPLT (*strategies* for sharing a collection or whole and *naming* the resulting share) and the task parameters (changing the size of the collection or the number of people sharing) to consider follow up activities for her students.

Lesson #3: Sharing a Rectangle for 2, 4, and 8 Friends

During the third lesson planning meeting, the teachers agreed to use a task they called "the wrapping paper task" where they used the context of fairly sharing wrapping paper to wrap holiday gifts. Bianca again attended to the task parameters as a way to address naming with her students. Her specific learning goals were for students to "share a whole fairly for 2, 4, and 8 people. Students will focus on how they might name the share in relation to the whole, for example each person got 'one of 8 pieces.'" She also considered that because her focus was on naming, which is higher in the trajectory, that keeping the task parameters lower would allow her students to focus more easily on the name rather than on the strategy for equipartitioning. When asked why she chose 2, 4 and 8, she stated, "Yeah, so I wanted to keep with repeated halving just knowing the trajectory. You know, I know that that's easier and since naming is a little bit harder, I didn't want the sharing to be as diff—too difficult for them … I wanted them to be able to feel successful sharing so that they could focus on what do we call what we've just shared."

Bianca anticipated that her students would make connections from the previous lesson to sharing the wrapping paper for two people. Because she purposefully chose powers of two, she also predicted that her students would use a repeated halving strategy: "I hope that a few of them notice the repeated halving and give their own language and explanation as we go from 2 to 4 to 8." Bianca also used the EPLT to expect different mathematical names such as "one out of four, or one part of the four whole parts, or one part out of eight parts, etc."

After the third lesson, Bianca again considered the task parameters in relation to her students' learning. She believed they were successful with equipartitioning a rectangle and naming the share for 2, 4, and 8 people and considering changing the task parameter from rectangles to circles for a follow up lesson:

> Well, I think—I mean I would like to see—I would probably do something the same, maybe with circles. And still focus on naming because we've kind of gotten there. But I know we're taking it down a—I would still do two, four and eight, but let's do circles and see can we still name them, but are our shares—but, like, at the side be like, "okay, what's going to happen when we get a circle? Can we share it?" Because here, they were successful sharing it and so they could be successful naming it, all right so now we're just going to take a circle and we're going to try to share it fairly.

She recognized that in changing the task parameter to a circle, she would first need to investigate if her students could use successful strategies to equipartition a circle and then move higher up the trajectory to address naming the resulting shares.

Discussion

Overall, Bianca coordinated the proficiency levels and task parameters of the EPLT to design tasks that focused on the learning goals that she chose for her students related to naming. She was able to bring together the existing curriculum with aspects of the trajectory, such as relating doubles facts with sharing for two people and naming fractional parts of halves and fourths. Bianca was also able to use the structure of the EPLT to consider possible follow up activities based on her students' learning. In considering the planning problems identified by Superfine (2008), Bianca offers evidence that the structure of LTs can

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support teachers in anticipating student work and in making connections between mathematical content and existing curricula.

For each lesson, Bianca was very clear about the specific learning goals she chose and used information from the trajectory to anticipate how her students would engage with tasks she selected. In light of the LT, Bianca was able to anticipate a variety of students' approaches and difficulties and consider how these related to her students' current conceptions and the intended learning goals. The LT was a tool to gauge the appropriateness of the tasks based on the understandings her students exhibited with the intention of moving students towards more sophisticated conceptions.

The tasks and goals she chose were in service of the larger, long-term goal of understanding the relationship between equipartitioning collections and wholes to naming the resulting share using a fractional name in relation to the collection or whole. Moreover, Bianca used open tasks that provided evidence of her students' thinking with respect to multiple proficiency levels of the EPLT, supporting the engagement of students with various conceptions of development along the trajectory (Sztajn, Confrey, Wilson, & Edgington, in press). Her ability to coordinate the levels of the EPLT with the task parameters supported her in considering follow-up activities based on the learning she observed from her students.

Because LTs describe concepts from less formal to more sophisticated ideas, LTs can aid teachers in selecting appropriate learning goals and provide information about what sub-goals are associated with larger conceptual goals. Hiebert and colleagues (2007) contended that learning goals are the basis for gauging the effectiveness of particular instructional activities and for measuring evidence of student learning. LTs afford teachers with information about likely strategies, misconceptions and important milestones that teachers can then anticipate as they plan instructional activities. Anticipating students' approaches to a task prior to instruction allows teachers to begin to think about how students' work relates to the intended mathematical goals (Stein et al., 2008). The knowledge of student learning inherent in LTs provides teachers with more detail as they compare evidence of student learning to learning goals, and gives them a repertoire of instructional moves based on the understandings their students' exhibit. Bianca was able to use the LT to better understand her students' mathematical thinking and target her lesson planning to her students' needs. Considering the fact that the Common Core State Standards were developed using LTs, it is important for the field to continue to explore the utility of LTs as a tool to aid teachers in attending to their students' mathematical thinking not only in lesson planning, but also during classroom instruction.

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