A GENERAL THEORY OF INSTRUCTION TO ASSIST THE PROFESSIONAL DEVELOPMENT OF BEGINNING MATHEMATICS TEACHERS

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In this paper we discuss how a general theory of instruction called Concept-Focused Instruction (CFI) can assist in the professional development of prospective teachers. CFI provides prospective teachers with a foundation and logical decision-making process for selecting, designing, and teaching mathematics. First, we provide a theoretical and practical context for having a theory of instruction. This leads into a description of the theory. The background and three core principles are provided, in particular in the context of the university methods course. The paper concludes with an overview of the findings and areas for future development.

Keywords: Teacher Education–Preservice; Teacher Beliefs; Teacher Knowledge

Introduction

Researchers in mathematics education have shown considerable interest in the professional development of prospective mathematics teachers. Ball and her colleagues (Ball, Lubiennski, & Mewborn, 2001) have contributed for over a decade to this knowledge base by contemplating the specific knowledge, skills, and dispositions that prospective mathematics teachers need to learn while they are in their university program. For example, prospective mathematics teachers need to learn how to elicit student talk; how to lead a discussion by eliciting and asking good questions; how to interpret students' thinking so they can learn to develop and support student learning. In a more general sense, Darling-Hammond and her colleagues have suggested the importance of framing the university experience more towards the sensitive nature of learning and the effects of teaching. In particular, Darling-Hammond (1998) states there is a need to prepare prospective mathematics teachers "with greater understanding of complex situations rather than seeking to control them with simplistic formulas or cookie-cutter routines" (p. 10).

According to Sowder (2007), "successful professional development programs remains the greatest challenge we educators face… our goal should be to prepare them [prospective teachers] for future learning, in part because at the university we can focus only on learning-for-practice (and not enough of that), and we know they have much more to learn *in* practice while teaching" (p. 213). Prospective teachers need opportunities to understand and use effectively and creatively fundamental mathematical content and concepts; teach content through the perspectives and methods of inquiry and problem solving; integrate education theory with actual teaching practice; and integrate mathematics teaching experiences with research on how people learn mathematics.

Hiebert, Morris, Berk, and Jansen (2007) address this challenge, of preparing the beginning teacher, by proposing a framework composed of four competencies: (a) setting learning goals for students, (b) assessing whether the goals are being achieved during the lesson, (c) specifying hypotheses for why the lesson did or did not work well, and (d) using the hypotheses to revise the lesson. They claim that teaching prospective teachers the knowledge, skills, and dispositions in each of these areas will allow the future teachers to have a deliberate, systematic path to analyzing cause-effect relationships between teaching and learning; and therefore, becoming an effective teacher over time.

The mathematics methods course typically lays the foundation for developing the knowledge, skills, and dispositions for mathematics teaching. The challenge there, as established by mathematics educators, is the prospective teachers often interpret the university-based course as disconnected to what actually happens in classroom experiences in schools. Frequently the preservice teachers expect they will learn *how* to teach mathematics but instead feel like they are presented with a menagerie of instructional theories (Mewborn, 1999, 2000). In addition, the seemingly discrete instructional techniques and approaches presented in the methods class create the impression that the mathematics methods course is theoretical

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and impractical. This perception is then reinforced during field experiences where classroom management issues and the cooperating teachers' views seem incompatible with information presented by the mathematics methods instructor Ebby (2000). Over three years ago, this disconnect between the methods course and the student teaching classroom experience, caused us to consider an alternative approach for the mathematics methods university course. Our alternative, related to what Heibert and colleagues describe above, was to frame the course using a theory of instruction.

According to Jerome Bruner (1966), a theory of instruction "sets forth rules concerning the most effective way of achieving knowledge or skill…a theory of instruction, in short, is concerned with how what one wishes to teach can best be learned, with improving rather than describing learning" (p. 40). Further, Jerome Bruner says a viable theory of instruction (1) identifies the experiences that are compatible with the way students learn, (2) explains the structure of the knowledge within a discipline, (3) identifies the most effective instructional sequences, and (4) addresses appropriate pacing and motivational strategies. Others have described a similar set of principles, each focusing on the application of knowledge and guidance on how to help students learn (Reigeluth, 1999). In summary, instructional theories are created as a set of principles and guidelines. They are not rigid sets of rules that must be followed at all cost but are guidelines that help the practitioner.

One could argue that every teacher employs a theory of instruction when they select, design, and teach mathematical content, albeit typically an implicit theory of instruction. If prospective teachers could begin their professional development with an explicit theory of instruction that meets the above criteria as defined by Bruner, the new teacher could leave the university experience with a means to logically select, design, and teach mathematics. As Bruner (1966) states, the viable theory of instruction must identify with the experiences that are compatible with the way students learn mathematics, explains the structure of mathematical knowledge, identifies an effective instructional sequence, and addresses appropriate pacing and motivational strategies. If prospective teachers experience this during their professional development at the university, would they be more likely to connect their university coursework experiences with the public classroom?

In this paper, we wish to share the evolution of a general theory of instruction that was developed for prospective mathematics teachers. The theory of instruction, called Concept-Focused Instruction (CFI), has been used the last three years in the university mathematics methods course. Each year, the theory and how it is being implemented in the methods course has been refined. Preliminary findings indicate the prospective teachers from 2011-2012 demonstrate (1) an improved understanding of mathematical concepts, (2) lesson plans and teaching practices that are learner-centered, and (3) more in-depth reflections and conversations about student learning and understanding in the classroom.

In the next section of this paper, the theory of instruction is shared by first describing the process that led the authors of this paper to consider this alternative approach for teaching the methods course. Concept-Focused Instruction (CFI) is then defined, including an overview of the progress that led up to the current version. Afterwards, a brief overview of the research findings showing how CFI has had a positive impact with the training of prospective mathematics teachers is described. The paper ends with a discussion of current thoughts, challenges and questions.

Concept-Focused Instruction

Background and Context

Over four years ago the authors of this paper began a conversation about the relationship between the methods course experiences and the student teaching, or internship, experience. This conversation was consistent with the established research by Mewborn (1999), that is, the prospective teachers did not employ much of the knowledge and skills learned in the course. The prospective teachers seemed to view the knowledge they gained in professional education classes as rules or prescriptions to apply to classrooms, and because these rules were not consistent with what they experienced, they saw it as a disconnect between the university coursework and classroom experience. The views of the cooperating teacher became the student teachers' dominant practice.

This encouraged us to look at the traditional teacher-training model, in particular for the mathematics methods course. The course typically addresses the instructional blocks framed in one of these three elements (Figure 1).

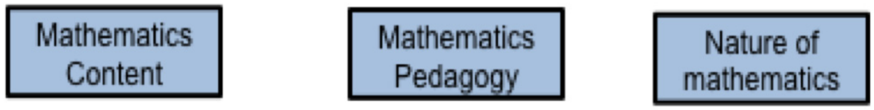

Figure 1: The traditional mathematics methods course elements

What we found is that the prospective teachers seemed to interpret these elements as separate and distinct. Mathematics content was what was learned in their mathematics coursework. The nature of mathematics was considered a philosophical idea that did not relate to what they were there to learn, and pedagogy was the sole purpose of the methods course. Because mathematics educators have established that the teacher's mathematical knowledge for teaching is directly related to how a teacher teaches and how well students learn (Hill, Rowan, & Ball, 2005) as well as their beliefs about the nature of mathematics (Borko & Putman, 1996; Philipp, 2007), our initial attempt was to find ways to connect the course elements.

Even when we purposefully tried to connect these elements, our preservice teachers did not connect the blocks. At the conclusion, preservice teachers still interpreted the set of information as discrete elements and believed the sole purpose of this course was to teach them *how* to teach mathematics. Then during the student teaching experience, when the prospective teacher would try to apply the knowledge learned in the methods course, the issue of classroom management would become prevalent. This resulted in their not thinking at all about the methods course material. Instead the teachers found themselves in a figurative sea of educational ideas, lesson plans and activities which in turn made the task of planning and implementing unfamiliar, yet recommended practices, daunting. They resorted to the educational practices of the cooperating teacher, which were often incompatible with the methods course knowledge, skills, and dispositions. In our conversations, the preservice teachers would recall the knowledge discussed in the methods about teaching and student learning, but did not have the mindset to access and apply this information in their practice. This finding led us to reconsider again the methods course elements and towards the idea of finding a clear and decisive format that prospective teachers could make sense of and use to make effective instructional decisions that focused on student learning. In other words, we aimed to identify an alternative approach for teaching prospective teachers about the nature of mathematics, mathematics content, and pedagogy so they would begin to (1) see the connections between mathematics and mathematical knowledge, (2) understand how students learn mathematics, and (3) teach for conceptual understanding.

Reframing the Methods Course Elements

Instead of looking at the three blocks of information as separate, we began to focus our attention on finding a common idea across the blocks. The common idea was conceptual understanding. For the nature of mathematics, in the methods course, the aim was to develop a view of mathematics being a dynamic, conceptualizing process. In terms of mathematical content, the emphasis was on conceptually understanding the mathematics the prospective teachers may teach. Finally, in the pedagogy block, a key element was how to teach and assess conceptual understanding. It was hypothesized that focusing on conceptual understanding would allow us to naturally address the nature of mathematics, pedagogy, and content topics.

To place an emphasis on conceptual understanding, we first had to clarify this idea for a prospective teacher. We established all mathematical concepts have three attributes: a macroscopic, model, and symbolic attribute (Hitt, 2005; Hitt & Townsend, 2007). The macroscopic attribute defines the context for the mathematical concept, in other words, a situation that enables individuals to visualize an application of the concept. The model attribute is the tangible representation of the macroscopic attribute. Finally, the symbolic attribute embodies the definitions and formulas associated with the concept. Figure 2 below shows this relationship.

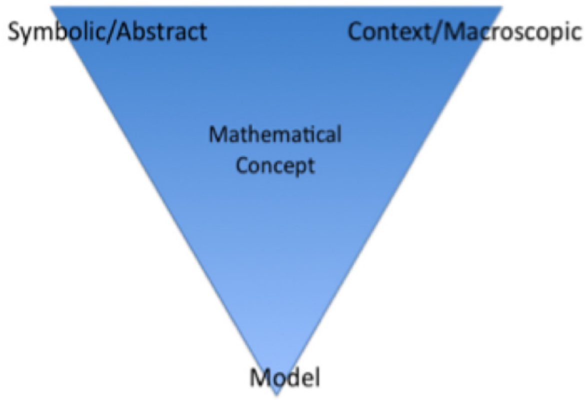

Figure 2: Attributes of mathematical concepts

For example, when thinking about adding fractions, it connects two concepts: fractions and addition. The macroscopic attribute could be a word problem that applies these concepts (Bob and Sue ordered 2 identical-sized pizzas, one cheese and one pepperoni. Bob ate ¾ of a pizza and Sue ate 1/8 of a pizza. How much pizza did they eat together?). The model would be a drawing of the pizzas, and the symbolic would include the vocabulary and algorithms for adding and finding common denominators. See Figure 3.

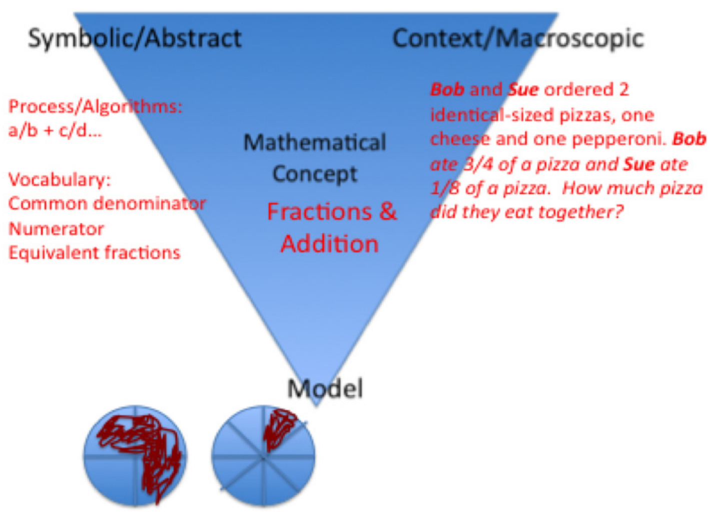

Figure 3: Three attributes describing the concepts Fractions and Addition

The methods course shifted from addressing the nature of mathematics, pedagogy, and content to a focus on identifying and analyzing macroscopic, model, and symbolic attributes of mathematical concepts first as a learner, then as a teacher. Within 2 years, we could establish Concept-Focused Instruction (CFI) as a theory of instruction. Readers may wish to refer to earlier publications that explain and justify CFI as a theory of instruction (Forrest & Hitt, 2010). In this paper, the focus will be on explaining how the theory of instruction has provided a systematic and deliberate framework for training prospective teachers, and specifically how it has been a better format for the university mathematics methods course.

Concept-Focused Instruction (CFI) Principles

Concept-Focused Instruction (CFI) is based on three core principles. The core principles are (1) mathematics is a conceptualizing process; (2) when individuals can explicitly reflect on the three attributes of mathematical concepts (macroscopic, model and symbolic) and can relate them to one another, they achieve conceptual understanding; and (3) in order to teach conceptually, teachers need to provide instruction that addresses the three attributes of mathematical concepts beginning with the macroscopic and model relationship (macroscopic, model and symbolic). These three related core principles frame the methods course design and delivery. Our hypothesis is that CFI enables prospective teachers to make better sense of teaching and learning mathematics as well as apply fundamental perspectives and methods that support education theory and the research on how people learn mathematics. It is not a novel idea to train prospective teachers by focusing on mathematics and developing understanding. Sowder (2007) describes successful implementations of such programs, for example, Cognitively Guided Instruction (CGI). CFI extends this line of work.

The focus of the next section is on the core principles of Concept-Focused Instruction (CFI). Each principle clarifies further the notion of conceptual understanding. Each principle can be interpreted as a basic hierarchical process building up and clarifying further the thoughts around conceptual understanding.

Core principle #1. Mathematics is a conceptualizing process. It has been shown by researchers who have studied mathematics teachers' epistemological beliefs that teachers who view mathematics as a dynamic conceptualizing process will be more inclined to use approaches for teaching mathematics. Additionally, students who understand mathematics as a conceptualizing process will likely have a more accurate perspective on what it means to understand and learn mathematics.

In the methods course, prospective teachers begin as mathematicians analyzing and solving a collection of mathematical tasks. The outcome is that the prospective teachers experience mathematics as a conceptualizing process. Discussion focuses on the process and thinking they used to solved the tasks. The instructor role models the tenets of Concept-Focused Instruction (CFI), highlighting each attribute of the concept to allow the prospective teachers to derive at Core Principle #2.

Core principle #2. When individuals can explicitly reflect on the three attributes of mathematical concepts (macroscopic, model and symbolic) and can relate them to one another, they achieve conceptual understanding. This principle helps us address two key ideas for teaching and learning mathematics. First, it provides a fairly simple explanation for mathematical understanding. The prospective teachers analyze a variety of mathematical topics in terms of the macroscopic, model, and symbolic attributes. That is they make and/or find possible macroscopic observations that allow a mental picture of the concept to be formed, create models that represent the phenomena, and then state the symbolic terms and formulas that are applicable. As a result, not only do the prospective teachers develop a better understanding of the mathematical content they are expected to teach. Through this process they begin to conceptualize and integrate complex mathematical ideas into their thinking. They also start to realize and discuss reasons why individuals may have a superficial and restricted understanding of mathematical content. Finally, they realize how critical the model attribute is when discussing mathematics. This sets up the third principle, which focuses on teaching mathematics.

Core principle #3. In order to effectively teach mathematical concepts, teachers need to provide instruction that addresses the macroscopic, model and symbolic attributes of concepts beginning with the macroscopic and model relationship. Specifically teachers provide a macroscopic experience that allows an opportunity to visualize the mathematics being taught. The students then create models to represent the macroscopic experience, and the teacher is then instructed to use the students' models to diagnose whether the students have a working model. Once a working model has been established, students are prepared to learn the symbolic attributes.

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During the phase of instruction where the teacher is diagnosing the student models, the prospective teachers are trained to ask critical questions such as "How does your model explain this situation?" "Show me where your model addresses this particular idea?" etc. In addition, prospective teachers are instructed that once students have generated models, and while instruction is focused on students learning the mathematics, the instruction will be highlighting either the macroscopic attribute of the mathematics, that is the visual, or instruction will be highlighting the symbolic attribute. Highlighting refers to what is being made loud and clear during the instruction.

Looking back again at our diagram of the three attributes of a concept (the figure is repeated below for convenience), one can see how the model attribute becomes the pivotal attribute. Once students produce a model of the macroscopic experience, depending on whether this model is a workable model, the prospective teacher thinks about emphasizing either the macroscopic-model relationship or the model-symbolic relationship. This allows the prospective teacher to focus his/her instruction on a specific outcome.

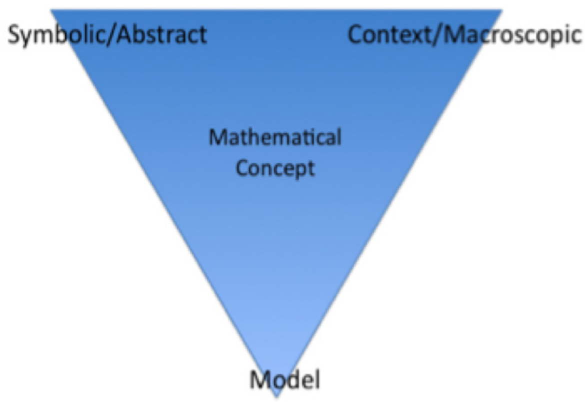

Figure 4: Attributes of mathematical concepts

In summary, using the three core principles of Concept-Focused Instruction (CFI) to frame the university methods course has resulted in our prospective teachers having better understanding and practice in the teaching and learning of mathematics. Without explicitly teaching the knowledge, skills, and dispositions related to the nature of mathematics, mathematics pedagogy and mathematical content, our preservice teachers have naturally acquired these skills in a manner that makes sense to them. They now comprehend how the nature of mathematics, pedagogy, and content are related when it comes to teaching students mathematics.

Concept-Focused Instruction is beginning to show promise as a means for developing prospective mathematics teachers, but it is still a work in progress. Each year, empirical evidence is gathered to create a case documenting each of the prospective teachers experience in the university mathematics methods course through the internship. Each case includes documentation showing the prospective teachers' mathematical content knowledge, beliefs about teaching and learning, the prospective teachers' views of mathematics and mathematical knowledge, and use of inquiry and mathematical processes in planning and delivery. In addition, documents ranging from lesson plans, supervised observations and evaluations are part of each prospective teacher's case.

Conclusion

Concept-Focused Instruction (CFI) is a theory of instruction that has been successfully used to develop prospective teachers' understanding of mathematics, as well as their knowledge, skill, and dispositions for teaching and learning mathematics. However, the authors are just beginning to establish a comprehensive

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research design to test the specific impact of CFI on preservice teachers. For example, the authors have identified one issue that they feel warrants further investigation. It is unclear ***how*** Concept-Focused Instruction (CFI) shapes prospective teachers' perspectives on mathematics or their ability to teach lessons effectively. In order to determine when and how preservice teachers integrate the CFI core principles into their thinking, it is critical to collect and analyze more qualitative data from artifacts such as journal reflections, class assignments and digitally recorded teaching presentations.

In closing, we want to summarize (1) the purpose of integrating Concept-Focused Instruction (CFI) into mathematics methods instruction, and (2) clarify how CFI fits within the current practices in the field of preservice mathematics teacher education. First, a theory of instruction, specifically CFI, can help prospective teachers simplify and visualize the connections between how mathematics is done, the way that students intuitively learn mathematics and effective approaches for teaching mathematics. When prospective teachers are just beginning their professional development, they tend to see these ideas as separate because as a student of mathematics, they have no explicit experiences with connecting these elements. We have found that once the above connections make sense to the prospective teacher, they are better able to understand and discuss many of the current ideas found in the mathematics education literature. In addition, we have realized until the connections make sense to the preservice teacher, the preservice teacher will fail to integrate the more constructivist ideas in their instruction.

Second, Concept-Focused Instruction (CFI) is not a replacement for instructional approaches discussed in methods courses, such as problem-based or inquiry-based instruction. CFI merely provides a foundation for prospective teachers to build upon. Once this foundation is established, the instructional approaches discussed in the method course make more sense to them. Concept-Focused Instruction merely provides preservice teachers with a basic understanding of how to think about mathematics learning and teaching, which then assists them in understanding and then designing and implementing learner-centered instruction.

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