A TENSION IN TEACHING "MATH FOR TEACHERS": MANAGING COGNITIVE AND AFFECTIVE GOALS

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This paper presents partial results of a completed study which investigated the experience of teaching mathematics content courses to preservice elementary teachers. Interviews with ten mathematics instructors who teach these courses revealed several major tensions, including one that arises as instructors strive to set priorities and balance their affective and cognitive goals for their students. An analysis of three of the instructors' experiences of this particular tension will provide insight into the factors that contribute to it and how it is managed.

Keywords: Teacher Education–Preservice; Affect, Emotion, Beliefs, and Attitudes; Mathematical Knowledge for Teaching

Background

Concern over the mathematics preparation of elementary school teachers has led to increasing calls for prospective teachers to take specialized mathematics content courses, i.e. Math for Teachers (MFT) courses, during their undergraduate programs (Greenburg & Walsh, 2008; Conference Board, 2010). These courses are usually taught by instructors in mathematics departments, and some recent studies have begun to call into question whether these instructors are equipped to meet the needs of MFT students, particularly with respect to affect (Hart & Swars, 2009).

Although there has been some research done into teaching styles of post-secondary mathematics instructors generally (Strickland, 2008), and into the difficulties they face in implementing reform approaches (Wagner, Speer, & Rossa, 2007), there seems to be little information about mathematics instructors in the context of teaching MFT courses. The original study upon which this paper is based sought to address this gap in the literature. Interviews with ten mathematics instructors who teach MFT courses at various post-secondary institutions in British Columbia were analyzed in order to answer questions, including: What are the major tensions they experience? What factors contribute to these tensions and how are they managed?

Given space limitations, this paper will discuss only one of six major tensions revealed in the full study, specifically, the tension related to instructors' efforts to balance their affective and cognitive goals for their students.

Supporting Literature

The research reported in this paper is informed by prior research into the cognitive and affective needs of prospective elementary school teachers (with respect to mathematics), as well as literature on tensions.

Cognitive and Affective Needs

With respect to the students in MFT courses, there is evidence to support concerns that they have poor understanding of the elementary school mathematics topics; Ball's (1990) study of 252 preservice teachers revealed "understandings that tended to be rule-bound and thin" (p. 449). Regarding their beliefs, the elementary preservice teachers in the group "tended to see mathematics as a body of rules and facts, a set of procedures to be followed step by step, and they considered rules as explanations" (p. 464). Preservice elementary teachers often suffer from mathematics anxiety (Hembree, 1990), and some are only enrolled to "fulfill a requirement rather [*sic*] to learn more mathematics" (Kessel & Ma, 2001, p. 477).

Although it is clear that MFT students have much mathematics to learn, and often come in with negative attitudes and beliefs, the literature does not provide specific advice on whether cognitive skills or

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affect should take precedence in teacher preparation. In fact, there is considerable literature engaged in debate over this issue. While some researchers make a case for the priority of strong mathematics knowledge, pointing out that such knowledge can both boost confidence and make teacher practice (i.e., the implementation of teachers' pedagogical beliefs) more effective (Schwartz & Riedesel, 1994; Goulding, Rowland, & Barber, 2002), a large number advocate for an emphasis on teachers' beliefs in specialized mathematics content (and methods) courses (Kessel & Ma, 2001; Liljedahl, Rolka, & Roesken, 2007), observing that beliefs will affect both students' learning in preservice mathematics courses and their later teaching. Still others promote the view that students' knowledge *and* beliefs need to be challenged in teacher education programs (Borko et al., 1992). This debate in the literature is reflected in the tension experienced by the MFT instructors described in this study.

Tensions

Tensions, often expressed as "dilemmas," have been recognized as an integral part of teaching practice, dating back at least to the early 1980s. In their seminal work, Berlak and Berlak (1981) examined the complex and sometimes contradictory behaviors of teachers in responding to the curriculum within socio-cultural contexts. Their use of the language of dilemmas was taken further by Lampert (1985), who emphasized the personal and practical aspects of dilemmas.

Lampert (1985) observes that tensions in teaching are often "managed" rather than resolved. She characterizes teachers as "dilemma managers" who find ways to cope with conflict between equally undesirable (or desirable but incompatible) options without necessarily coming to a resolution. Faced with a teaching dilemma, the teacher must take action, finding a way to respond to the particular situation, even while the "argument with oneself" (p. 182) that characterizes the dilemma remains. For Lampert, the ongoing internal struggles presented by the tensions arise from and contribute to the developing identity of the teacher, and as such have value in themselves. Furthermore, she comments: "Our understanding of the work of teaching might be enhanced if we explored what teachers do when they choose to endure and make use of conflict" (p. 194).

More recently, Berry (2007) focused on "tensions" in a self-study that examined her own efforts to improve her practice as she made the transition from *teacher* to *teacher educator*, finding that the notion "captured well the feelings of internal turmoil experienced by teacher educators as they found themselves pulled in different directions by competing pedagogical demands in their work and the difficulties they experienced as they learnt to recognize and manage these demands" (p. 119). In the study presented here, the instructors are similarly experiencing a transition between teaching future *users* of mathematics and teaching future *teachers* of mathematics, a situation which influences the tensions they experience.

Methodology

Data for this study were gathered through interviews with ten participants, five male and five female, all instructors in mathematics departments at post-secondary institutions who teach MFT courses. Theoretical sampling (Creswell, 2008) was used to achieve variety in type of post-secondary institution represented, as well as varying degrees of experience in teaching MFT. The ten instructors represented nine different institutions, and their experience teaching the MFT course ranged from novice to 20 years.

The one-hour long interviews were semi-structured, beginning with a set of core questions but allowing for variations and additional questions to be asked as needed. Such an open-ended ("clinical") approach is advocated by Ginsburg (1981) in situations where discovery or identification/description of a phenomenon is the objective. The questions sought to elicit the instructors' conceptions of the MFT course by asking them to examine their goals, describe the approaches they take, compare the teaching of MFT with teaching of other mathematics courses, and reflect on the challenges and the successes they experience.

The interviews were audio-recorded, transcribed, and analyzed using constant comparative analysis (Creswell, 2008). An iterative coding process (Charmaz, 2006) was employed in order to allow concept codes and themes to be identified. Very few new codes emerged after the tenth interview, suggesting

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saturation of the data. Specific concept codes, including "priorities," "wishes," "doubts," "barriers," and "resistance," helped to locate instances of instructor tensions in the transcripts.

Analysis of the tensions was further facilitated by techniques of discourse analysis (Rowland, 1995) and considerations of positioning (Harré & van Langenhove, 1999). The former helped to locate hesitation, uncertainties and inconsistencies, while the latter supported understanding of contexts and factors contributing to the tensions identified.

Results

One of the major tensions that emerged through the coding and thematic analysis involved instructors' struggles with managing their cognitive and affective goals for their students in the MFT course. Each of the instructors experienced this tension differently, however, due to space limitations, only the cases of Bob, Maria and Alice (all pseudonyms) will be described here. These three cases will suffice to exemplify the diversity and scope of the views expressed.

Bob

Bob's experience of this tension emerged in the contrast between his comments at the beginning of his interview with respect to his primary goals in his MFT course and his later reflections on the ultimate outcomes for his students.

Bob reported that his main emphasis is on building deep conceptual understanding in his students, although affective considerations are also very important to him. His course "focuses on a very sound fundamental ability to appreciate [mathematics], in a theoretical way, why things work," along with having "a secondary by-product of what you do in the classroom is to get the students to enjoy it." The cognitive and affective are closely related for Bob. From his responses to prompts about dealing with students' anxiety, he expresses the view that his students' anxieties are at least in part caused by, and at the same time the cause of, their lack of arithmetic skills. It is his hope that helping his students learn about the structure of mathematics will solidify their understanding, giving them confidence, competence, and enjoyment.

Later in the interview, commenting on what his students leave the course with, Bob observed that his students "have improved most in their technical abilities," along with having gained some problem-solving skills, although these need to continue to be developed. But he is ultimately disappointed, both in his hopes to build deep theoretical understanding, and in his hopes to increase his students' appreciation for, and love of, mathematics.

> *Bob*: In terms of appreciating some of the more subtle aspects of the theory, I think that's another thing that they could do better, if they had better basic arithmetic skills, coming in. So ... yeah, in terms of what I produce, I guess, in terms of the other goal, for love of math? Unfortunately, the course is so packed, that in some ways, I think they do get a little bit beaten by the end, and they're just tired.

He does see some success in improving their technical skills, although their depth of understanding still falls short, but admits with regret that he is less than successful (by his own standards) in terms of affective aims. He is trying to cover too much, to the extent that his students are overwhelmed.

A closer look at this passage, with particular attention to pronoun use, offers some further insights. In the first sentence, he ostensibly places the responsibility on the students, "*they* could do better, if *they* had better basic arithmetic skills." However, as Bob is aware, the prerequisites for the course are not controlled by the students, or by him, but are set by the larger community. Whether it is the fault of this community or the students themselves, he sees the lack of student skills coming into the course as an impediment to his ability to realize his goals for his students.

He then switches to consider what he ("I") produces. Having already mentioned that students have increased their technical abilities, he moves to his "secondary" goal, improving affect. The results here are "unfortunate"; he describes his students as "beaten" and "tired"—not at all what he desired. The phrase, "the course is so packed," is telling. It is offered as an explanation for the students' states of exhaustion;

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there is too much material in too little time. Bob's use of the passive voice here suggests that he is not in control of the course content; with it he positions himself as unable to remedy this "unfortunate" situation. The course, as he believes he is expected to deliver it by his institution, demands too much of the students.

Bob is not a new instructor of the MFT course, and so has likely lived with this problem for some time. He is stuck in this dilemma. On one side he has students who are unprepared for the level of mathematics he believes they need in order to "appreciate" the mathematics (both in a cognitive and in an affective sense). On the other side, he has a prescribed curriculum he is expected to "cover." He feels a strong responsibility as a mathematics instructor, seeing himself as being charged with "delivering the content" (Bob's words). From Bob's perspective, the situation could be improved if the students were stronger coming in, but this is not within his immediate power to change. So he manages the tension by adhering to his practice in his other mathematics classes—he focuses on the content, despite his dissatisfaction with the outcomes.

Maria

Similar to Bob, Maria expressed a strong intention to improve students' mathematical understanding, emphasizing cognitive goals. However, Maria's use of the past tense in describing these goals in her interview, even though she was teaching the course at the time, suggested she was having second thoughts about her priorities.

Maria was a first-time instructor of the course at the time of the interview, and was surprised by the needs of her students, not only their weak mathematics skills, but their mathematics anxiety and the barrier to learning it presented.

> *Maria*: So my goal was, primarily, sort of more content, and I [...] knew that there would be some issues of, let's describe it as "math phobia" or anxiety, with math. I just [was] still surprised to see it so strong at this level, that it overrides their learning, that it blocks their learning! That's what I discovered, and it surprised me that it would be this strong.

She went into the course expecting that she would be teaching mathematics and would need to deal with math anxiety, but at a certain point she realized that, at least for some of her students, the affective issues would need to be addressed before they could learn the mathematics. Maria commented that she believed she had lost about a third of her students, and was not sure how to get them back on track.

> *Maria*: For this group of students at this point, content? Forget it. I need an attitude change. I need [their] perception of math to change. And I can't reach it anymore. It was very high, you know, it was a good high in the beginning of the course, because of what I did, free, sort of, problem-solving, open discussion, everybody let's just ... [there was a] fuzzy, cozy atmosphere. But the topic does get difficult, yeah?

Maria seemed to feel that she had missed an opportunity. For this particular group of students, she did not believe it would be possible for them to progress without an attitude change, and this change was not possible to attain "anymore." She spoke nostalgically about a time at the beginning of her course when her approach was different: there was "open discussion," "free" problem solving, and a friendly atmosphere. She seemed to take responsibility for these initial positive feelings; it was good because of "what [she] did," but something changed; her approach changed, and in this excerpt the reason offered for the change was the "topic," i.e., the mathematics, which gets more difficult as the course progresses.

Like Bob, during her interview Maria expressed a sense of obligation to complete the prescribed mathematics content for the course, a disposition that appeared to be in tension with her goals to both promote deep conceptual understanding and address her students' affective needs. Her desire to "cover the content" influenced her choice of teaching methods, leading her to reduce in-class activities, such as open discussions of readings and problem-solving sessions, methods that she described as effective, but not time-efficient. At the same time, she reported that it troubled her that she was leaving students behind, students who would continue to suffer from negative attitudes to math and continue to have weak skills.

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An additional consideration for Maria that contributes to this tension is a perception that the MFT course has the potential, if not the responsibility, to act as a filter. Early in her interview, comments with respect to the importance of deep content knowledge for mathematics teachers (not cited here) revealed a strong commitment to ensuring that she does her part in the preparation of future elementary teachers; i.e., if the mathematics skills of the prospective teachers are too weak, they should not be permitted to go on to become teachers.

Maria was far from resigned to living with this tension. At the time of the interview she was still seeking to understand her students better and find methods that would be more effective for them, to find a way to change their attitudes so that the mathematics could be learned.

Alice

In contrast to Bob and Maria, Alice was less concerned about building mathematics knowledge and much more concerned about affect. This also created tension for her, although this was not evident early in her interview. Alice's emphasis was on helping her students see the "fun" of mathematics. She described the many ways she tries to address her students' anxieties and to build their confidence, including striving for a very relaxed classroom atmosphere where questions are encouraged and student interaction and exploration of concepts is the norm.

The tension between her affective and cognitive goals for her students did not emerge until she considered whether her students will be prepared to go on to be teachers of mathematics.

> *Alice*: That's a very good question. That, that's a very deep question. Because we don't teach so much math in that class, you know. We don't drill them on whether they can do those fractions. We kind of believe they have the elementary math, that's how we let them in [...]. But how much above it should they be? You see they always say that you should be significantly above what you want to teach, because then you have the big picture, you see the troubles and all that. I don't know that much about that. [...] Many at least will not be afraid to go for it. But I still think there are people who will be afraid. I still think I let people go in there being afraid.

In this passage, she begins with the admission that improving students' mathematics proficiency is not a major objective in her course. This is followed by a justification that students are presumed to come into the course with sufficient mathematics skills, however the hedge "we kind of believe" and other comments in her interview suggest that she realizes those skills are often lacking. She then considers that even if they could do the arithmetic, perhaps that would not be enough, that teachers of mathematics should have a *deeper* understanding of the subject. She even provides reasons for why this deeper understanding might be helpful, but then quickly dismisses this as education theory, something she is not an expert in. She looks to her goal of improving attitudes next, to see if "at least" her students will no longer be afraid of mathematics, but sadly admits that even in this respect, some of her students are not ready.

A careful parsing of this passage reveals some of the different forces contributing to the tensions that Alice operates under. As she thinks aloud, her pronoun use changes from "we" to "they" to "I." "We" likely represents her institution as she describes what does not happen in the course: there is not much math and no skill drill. Even if she disagrees, the objectives for the course are set by her institution. In the phrase "they always say...," the "they" seems to point to education experts, or at least to those who have an informed opinion, but she disassociates herself from this group, switching to the pronoun "I," and denying any expertise in deciding what students need. Ultimately, responsibility for the content and objectives of the course is deferred to others, her institution and/or the community.

Alice seems to believe that the goals for improving students' attitudes and diminishing their anxiety are important, and this is consistent with the reported aims of her institution. As a result of this local orientation, she does not express the same concern as Bob and Maria with respect to "covering" the course content. But there is still a tension here as she contemplates what she achieves with the course, and what future teachers might need both in terms of mathematics proficiency, which she does not address to a great extent, and attitudes towards mathematics, which she tries to address, but feels she does not entirely succeed in. She deals with this tension by deferring authority for deciding these priorities to others at her

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institution and within the teacher education system, but remains concerned with the implications for future teachers.

Discussion

Bob, Maria and Alice experience the tensions differently, but all struggle with finding the balance between building students' mathematics proficiency/understanding and fostering positive attitudes, within the parameters set for the course. Bob hopes to improve affect through building cognitive skills (one of the views reflected in the literature), but his affective aims are sabotaged by an emphasis on content that he sees as too much for his students to absorb given their skills coming into the course. He opts in favor of covering the course content, fulfilling his perceived obligation as a post-secondary mathematics instructor, even though this means the students leave the course far less excited about mathematics than he would like. Maria's comments revealed a growing awareness that her cognitive aims cannot be attained, at least for some of her students, until affective barriers have been removed (reflecting the other side of the affective/cognitive debate). She, too, sticks to the course curriculum, even though students are left behind, largely to try to ensure that students who do not have a certain level of understanding will not become teachers before they are ready. For Alice, whose emphasis is already primarily on the affective, there is an uneasiness that what her course provides may not be enough to meet either one of her students' affective or cognitive needs, at least for some of her students.

Both Bob and Maria seem to manage this tension between cognitive and affective aims by staying true to the course syllabus and "covering the material," even though they are unhappy with the consequences. There are indications within the broader study that this commitment to the prescribed course content is a prevalent norm amongst post-secondary mathematics instructors. This may not be surprising given that the traditional calculus-stream mathematics courses that they generally teach tend to be sequential in nature with topics in one course building on knowledge of earlier course content. It is unclear whether instructors are consciously aware of this norm or have considered its appropriateness in the context of MFT courses.

Exceptionally, Alice does not appear to adopt this norm in her MFT course, at least in part because the mandate from her institution is a focus on affective goals. Yet this does not free her from the experience of tension. She manages her situation by deferring to the authority of others at her institution, but she is left uneasy with the mathematics proficiency of some of her students in the context of their role as future teachers, while not being wholly satisfied that her affective goals are being reached. Her comments point to additional factors that contribute to this tension, including the often weak mathematics skills of students coming into the MFT course, also observed by Bob, and the sense that one of the roles of the course is to act as a filter to prevent those with poor mathematics skills from becoming elementary school teachers, also expressed by Maria.

Both of these concerns point to larger problems within the system of teacher preparation, problems with defining the level of mathematics proficiency elementary teachers need, and with clearly defining the role of MFT courses in their preparation.

Conclusion

This tension is not easily resolved. As illustrated by the case of Alice, it is certainly not simply a matter of refocusing priorities on affective rather than cognitive goals—both are important in the development of future teachers of mathematics. Rather than attempt to resolve the tension, in the spirit of Lampert (1985), we consider instead what can be learned from it.

The study by Hart and Swars (2009) suggests that approaches of MFT instructors may negatively impact student affect. This study counters that even when instructors are concerned about students' attitudes and beliefs, their ability to respond to the affective needs may be constrained by normative commitments to course syllabi, beliefs about the level of mathematics proficiency needed by future teachers, and understandings of the role of the MFT course. Maria's comments about reducing in-class activities in order to get through the material suggest that these factors may also be barriers to instructors'

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adoption of more reform-oriented approaches. Furthermore, a perceived mismatch between students' prior mathematics preparation and course expectations is also implicated.

The question then becomes, how can mathematics instructors best be supported by the mathematics education community as they strive to manage both cognitive and affective aims for MFT courses? This study suggests that identifying norms in post-secondary mathematics instruction that may differ from those in teacher education may bring to light preconceptions that inhibit instructors' transition from teaching future *users* of mathematics to teaching future *teachers*. Furthermore, ongoing research into the knowledge, beliefs and attitudes needed by teachers of mathematics would assist in clearer articulation of goals for MFT courses and a better understanding of their place in the process of teacher preparation. Finally, although the debate between the priority of affective versus cognitive goals in the literature is exemplified within the cases of these instructors, it is worthy of note that the research literature does not play a direct role in informing these instructors' efforts to deal with their tensions. This is even more evident in the larger study. Although the literature to date offers no clear resolution, closer contact with the mathematics education community might expose these instructors to new strategies or alternate perspectives, placing them in a better position to manage their tensions.

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