EVALUATING TEACHERS' DECISIONS IN POSING A PROOF PROBLEM

Karl W. Kosko
Kent State University
kwkosko@gmail.com

Patricio Herbst[1]
University of Michigan
pgherbst@umich.edu

When Geometry teachers pose proof problems to students, it is the teacher who provides the givens and the statement to be proven; we hypothesize that teachers of geometry recognize this to be the norm. This study examined teachers' decision-making in regards to the posing of a proof problem, and whether recognition of this norm accounted for the decision made. Results of a multinomial regression indicated that the more participants recognized that norm of posing proof problems, the less likely they were to select an action that breached the norm.

Keywords: Instructional Activities and Practices; Reasoning and Proof; Teacher Beliefs; Research Methods

Background and Objectives

During the early 1970s, teachers' decision-making became a focus of educational research through parallel investigations led by Alan Bishop, Lee Shulman, and Richard Shavelson (Borko, Roberts, & Shavelson, 2008). Each initiative viewed the examination of teacher decision-making as a means to better understand teaching. For Shulman (1986), this research on teachers' decision-making exemplified how research on teaching had brought attention to teachers' cognition to a field that had up to then only considered teachers' characteristics and behaviors. Accordingly, much of this early research posited individual resources such as beliefs, goals, knowledge, or schemas as resources for decision-making (Schoenfeld, 2010). But another of the paradigms for the study of teaching that Shulman (1986) described, the classroom ecology paradigm spearheaded by Doyle, had undertaken to improve the study of teaching by attending carefully to its activity structures. Contributing to this approach, Herbst and Chazan (2011) have addressed teachers' decision-making by proposing that teachers draw upon resources of a different kind to justify their pedagogical moves. "Combined with the personal assets (including knowledge, skills, and beliefs) that an individual teacher brings with them to that position and that role, [instructional norms and professional obligations] can help explain teacher action and decision-making" (p. 417). As described by Herbst and Chazan, professional obligations are resources of the profession that regulate the position of a mathematics teacher while instructional norms are resources embedded within the various activity structures in which the teacher plays a role. Thus, in this perspective, the justification of a teacher's decision depends not solely on the individual teacher's personal resources but also on their recognition of those norms and obligations (a recognition that could be tacit). Yet, what remains unclear is the degree to which an individual teacher's resources and their recognition of instructional norms account for the decision that a teacher makes in the moment. The purpose of the current study is to examine this phenomenon in a specific decision-making context.

Personal Resources for Decision-Making

In their review of early literature on teachers' decision-making, Shavelson and Stern (1981) suggest that teachers "make judgments and decisions, and carry them out on the basis of their psychological model of reality" (p. 461), which, in turn, is composed of various beliefs such as those concerning pedagogy and the subject matter. Shulman and Elstein (1975) also suggested that personal resources of the teacher influence judgments. Shulman (1987) later restated this relationship in terms of particular types of knowledge professional teachers hold, and how such knowledge influences teachers' decision-making. In the past several years, Deborah Ball and colleagues have expanded this idea to describe their conception of Mathematical Knowledge for Teaching (MKT) (Ball, Thames, & Phelps, 2008). Like Shulman (1987), Ball et al. (2008) suggest that teachers' in-the-moment decisions require "…coordination between the mathematics at stake and the instructional options and purposes at play" (p. 401).

The literature suggests that one clear resource teachers use in making pedagogical decisions appears to be their pedagogical content knowledge (PCK) (e.g., Ogletree, 2007). Yet, Bishop's (1976) account of decision making made a strong argument for the importance of teaching experience in the development of the schemas that may be associated with decision making. Osam and Balbay (2004) provide additional evidence for this, finding that surveyed novice teachers were more concerned with technical details of a lesson in their decisions while experienced teachers were more concerned with the way students behaved during the lesson.

Another potential resource that may influence teachers' decisions is their degree of autonomy to make decisions about their instructional practices. Behm and Lloyd (2007) observed that while different student teachers were provided different resources, the degree of autonomy afforded those student teachers was a critical indicator of what they were able to do with the materials at hand. Examining decision-making in science classrooms, Gess-Newsome and Lederman (1995) found that teachers' autonomy was a highly influential factor in the types of instructional decisions made. With these considerations in mind, we consider teacher autonomy, along with PCK and teaching experience, to be critical personal resources of teachers in their decision-making.

A Professional Resource for Decision-Making

Aho et al. (2010) note that more than teachers' own personal resources influence their decision-making. Rather, "teaching is influenced by the surrounding society, culture and traditions" (p. 400). Teachers interviewed by Aho et al. noted that some pedagogical decisions they made were agreed upon with school colleagues. Further, these types of collective decisions over time work their way into the routines of the teacher. We argue that while such routines are operationalized by individual teachers, their genesis are social in origin and therefore may be more characteristic of actions normative of a group than of particular individuals: In this case we are interested in the obligations that bind a professional group and the norms of the activities in which they play a role. Herbst et al. (2009) provide an example of one such type of norm. Observing similarities in how proof was facilitated across different teachers' classrooms, Herbst et al. note that "these similarities can be expressed by a common system of implicit norms regulating the events on the surface" (p. 266). Such norms appear to influence teacher decisions in the classrooms particularly shaping the division of labor, or who does what, when the situation is one of doing proofs.

Situational norms and professional obligations on the one hand and individual resources of teacher autonomy, experience, and knowledge are thus two kinds of constructs that might account for the decisions teachers make (e.g., Ball et al., 2008; Bishop, 1976; Gess-Newsome & Lederman, 1995). Given these various resources, it is prudent to investigate the degree to which they influence teachers' decision-making. We focus on the instructional situation of doing proofs (Herbst et al., 2009), and on a particular norm of doing proofs (*when posing proof problems, the teacher provides students with the given information and the statement to be proved*). With this situation-specific focus, we sought to answer the following research question:

> *To what degree do teachers' recognition of an instructional norm account for their decision-making in posing a proof problem, and to what degree do the individual resources of PCK, teaching experience, and perceived teaching autonomy contribute to their decision.*

Methods

Sample and Measures

Data were collected from 55 secondary mathematics teachers (grades 8 to 12) in a Midwestern state. The sample included 43.6% male and 56.4% female teachers. Participants were sampled from a wide range of districts, both urban and rural, and of varying levels of socio-economic status. For example, some participating teachers taught in schools with 4% of the population eligible for free and reduced lunch, while others came from schools where 59% of students were eligible. Of the 55 sampled participants, 44 (80%) completed all assessments that we include in the current analysis, and represent our effective sample.

Participants were invited to complete a series of assessments on an online platform (LessonSketch.org), of which we include data from four of the assessments. LessonSketch allowed for the incorporation of multimedia survey instruments in which participants viewed and answered questions concerning representations of teaching, of which was particularly useful in assessing teachers' situation-specific decision-making.

Dependent Variable

We assessed participants' decision-making based on their multiple-choice responses to a representation of teaching. Participants were presented with a cartoon-based, two-frame teaching representation, preceded with a brief overview of the lesson as one taking place in a high school geometry class in which the teacher was going to assign a proof problem. The representation depicted the teacher drawing a diagram and reviewing with students that to write a proof they would need a set of givens and a statement to prove. Participants were then presented with four potential actions that could follow and were asked to select which they would be most likely to do next following the scenario. Each action was a single-frame depiction representing either compliance or breach with the normative action: *when posing a proof problem, the teacher provides the givens and prove statement to students*. Participants were asked "which action would you be most likely to take in the teaching scenario?" and then to "please explain your reasoning for choosing this action."

Choice A depicted a breach of the norm where the teacher instructs students they will have a discussion to decide, as a class, what the givens and the prove statement will be. Choice B is another breach of the norm where the teacher asks students to work individually, decide what the givens and prove statement are, and then do the proof. The students would later compare their proofs to their peers'. Choice C is a breach where the teacher provides the prove statement, but instructs the class that they will discuss, as a class, what givens they will need to do the proof successfully. Choice D is compliant with the norm where the teacher provides both the givens and "prove" statement, and then asks an initial question for class discussion on how to do the proof. Responses were well distributed with 22.7% selecting Choice A, 18.2% selecting Choice B, 31.8% selecting Choice C, and 27.3% selecting Choice D.

Independent Variables

We included four independent variables in our analysis. The independent variable of interest (*normativity*) was a score representing participants' endorsement of the norm: *when posing a proof problem, the teacher provides the givens and prove statement to students*. The variable, described in detail below, was designed as an indicator of participants' recognition of an instructional norm. Specifically, scores for *normativity* were interpreted to assess the degree to which individual participants recognized the identified norm in the situation 'doing proofs' in Geometry instruction.

We assessed this recognition with a 10-item survey that presented participants with explicit statements regarding the norm of focus. A sample item and available responses is presented in Figure 1. Items were written to assess participants' view of how appropriate it was for the professional group of Geometry teachers to provide students with the givens and prove statement in posing proof problems. Interpretation of the items was validated through cognitive interviews prior to collection of the current data, with results suggesting that items were interpreted as intended. Additionally, we calculated an alpha coefficient of .89, suggesting the items had sufficient reliability as well as validity. Participant responses were averaged into a composite score, *normativity* ($M = 3.46$, $SD = 1.08$), for inclusion in the present analysis. Higher scores represented a greater recognition of the norm, and vice versa.

When starting a proof problem, how appropriate is it for teachers to have students decide upon the 'prove' statement (conclusion)?

1	2	3	4	5	6
Very Inappropriate	Inappropriate	Somewhat Inappropriate	Somewhat Appropriate	Appropriate	Very Appropriate

Figure 1: Sample item assessing participants' recognition of the norm

The next independent variable included was a measure of participants' perceived autonomy in their mathematics teaching (*autonomy*). As noted in our literature review, teachers' autonomy has the potential to regulate the effect of teacher beliefs, and therefore represented a useful factor for investigation. Items measuring *autonomy* were adapted from multiple sources to focus both on the content of mathematics and the role of teacher (Deci & Ryan, 2011; Kosko & Wilkins, in review; NCES, 1998; Reeve et al., 2003). Teachers were asked to rate their agreement with statements such as the following: *I am discouraged from teaching mathematics in the way I would like to* (reverse-coded sample item). Available responses were on a 6-point Likert-scale (1–Strongly Disagree; 2–Disagree; 3–Somewhat Disagree; 4–Somewhat Agree; 5–Agree; 6–Strongly Agree). Items showed sufficient reliability ($\alpha = .89$) and responses were averaged into the composite score *autonomy* ($M = 4.62$, $SD = .81$).

The third variable included for analysis was years of teaching experience (*Years*). Teachers in the sample taught an average of 13 years ($SD = 7.30$). While Bishop (1976) noted that it was schemas developed through experience that influenced teachers' decision-making, we used *Years* as an indicator of having more sophisticated forms of such schemas.

The final independent variable included was a measure of pedagogical content knowledge in geometry (*PCKG*). The assessment included 10 items covering various Geometry topics and addressing teachers' *knowledge of content and teaching* and *knowledge of content and students* (see Ball et al., 2008 for a detailed description of these domains of mathematical knowledge for teaching). Items were validated through cognitive interviews before collection of the present data. Item analysis of present data showed biserial correlations of .30 or higher, and a Cronbach's alpha coefficient of .70, suggesting the construct had sufficient reliability. Scores were based on percentage of items answered correctly, with a possible range of 0 to 1 ($M = 0.46$, $SD = .23$). Herbst and Kosko (2012) provide more details on the development of this instrument.

Analysis and Results

We used multinomial logistic regression (MLR) to examine participants' decision-making. Specifically, participants were asked to select one of four potential actions following a depicted teaching scenario. While one of these actions was considered compliant with the norm and the other three breaches of the norm, we did not consider one action as necessarily better than any other. Further, the participant choices could not be ordered in any natural way. Therefore, the responses represent nominal data suitable for an MLR. MLR is a form of logistic regression which uses one category (one of the choices available) as a reference outcome, and creates separate logistic regression comparisons between the reference outcome and each other classification (see Hosmer & Lemeshow, 2000 for a detailed description).

The model examined in the current analysis is presented in the equation below. The outcome of reference is the normative action, Choice D, and is designated by 0 in the equation. Each alternative choice (breaches of the norm in Choices A, B, and C) are represented in variable m, such that we have three distinct regression equations; one for each comparison. So, we evaluated the degree to which each independent variable contributes to participants choosing Action A rather than Action D, Action B instead of Action D, and Action C instead of Action D.

$$g_m(x) = ln\left[\frac{P(Y = m \mid x)}{P(Y = 0 \mid x)}\right]$$
$$= \beta_{m0} + \beta_{m1}(normativity)_1 + \beta_{m2}(autonomy)_2 + \beta_{m3}(Years)_3 + \beta_{m4}(PCKG)_4$$

Customary in performing MLR is an initial checking of model fit, both for the model as a whole as well as for particular variables within it. While the model represented in the above equation had overall model fit ($\chi^2 = 39.34$ (df = 12), $p < .01$), the variable PCKG was found to not have a statistically significant relationship with participants' choices ($\chi^2 = 3.91$ (df = 3), $p = .271$). This initial finding suggests that there was little relationship between participants' *PCKG* scores and their chosen action following the scenario and, therefore, *PCKG* should be considered for removal in the analysis to provide a more parsimonious model. The standard errors associated with *PCKG* were also high (above 2.0), suggesting potential collinearity. Also, a separate MLR with only *PCKG* in the model still suggested no statistical relationship with choice of action. However, an examination of the descriptive statistics suggest that while participants selecting the normative action tended to have higher *PCKG* scores, there was a large degree of variance in these scores, further justifying the removal of *PCKG*. The new model, which includes *normativity*, *autonomy*, and *Years* as predictors, was found to have good overall fit ($\chi^2 = 35.43$ (df = 9), $p < .001$), with no need for further simplification of the model.

Results from the MLR analysis are presented in Table 1, with coefficients represented in logits. In each model comparison, *normativity* scores were found to be a statistically significant predictor of choice at the .10 level, when accounting for participants' perceived teaching autonomy and years of teaching experience. Using the conversion:

$$\frac{e^{\beta_{m0}+\beta_{m1}(normativity)_3+\beta_{m2}(autonomy)_2+\beta_{m3}(Years)_3}}{1+e^{(normativity)_3+\beta_{m2}(autonomy)_2+\beta_{m3}(Years)_3}}$$

we can determine the probability that a participant in our sample would select a particular choice rather than the normative action represented in Choice D. For example, a participant with an average *autonomy* score ($M = 4.62$) and years of teaching ($M = 13$) for the sample, a low *normativity* score of 1.00 would suggest such a participant is 99.9% more likely to select Choice A over Choice D. However, if a similar participant had a high *normativity* score of 6.00, there is a practically zero probability that they would select Choice A over Choice D. These and similar calculations are illustrated, for convenience, in Figure 2.

Table 1: Results from Multinomial Logistic Regression.

Comparison		β (logits)	S.E.	Wald Statistic
Choice A | Choice D	Intercept	12.75	5.27	5.13**
	normativity	-5.35	1.83	8.56**
	autonomy	1.59	1.00	2.55
	Years	-.29	.11	6.52**
Choice B | Choice D	Intercept	-4.17	4.90	.72
	normativity	-1.51	.79	3.68*
	autonomy	2.21	.98	5.15**
	Years	-.10	.08	1.71
Choice C | Choice D	Intercept	2.20	3.13	.49
	normativity	-1.08	.60	3.20*
	autonomy	.57	.57	1.00
	Years	-.04	.06	.48

*$p < .10$. **$p < .05$.

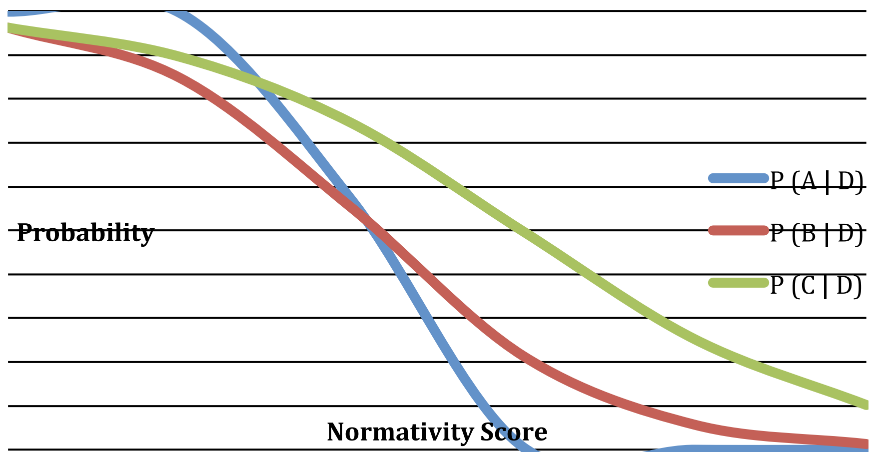

Figure 2: Effect of normativity score on probability of selecting an option other than choice D, with average *autonomy* and *Years* for the sample

These findings indicate that, for each comparison, the degree to which participants recognized the norm was a consistent determiner of how likely they were to select Choice D or an alternative. Additionally, it appears that the more participants recognized the norm, the more likely they were to select the normative action, Choice D, instead of an action that included a breach of the norm. Further, while perceived autonomy and years of teaching experience did influence whether participants would choose one action over another for some comparisons, *normativity* consistently did so and generally at larger magnitudes.

Discussion and Conclusion

The findings from our analysis are preliminary, in that they represent the decision-making regarding the teaching norm of focus for only one particular teaching scenario. Yet, examination of participants' choices suggests that participants who recognize the norm tend to act according to that norm. Additionally, participants' perceived teaching autonomy influenced decision-making in a manner that contrasted *normativity*. Specifically, a higher perception of autonomy was shown to increase the likelihood a participant chose Action B over the normative action, while a higher *normativity* score decreased the likelihood those participants would choose Action B over the normative action (see Table 1). This statistical conflict between *autonomy* and *normativity* is representative of what Pepitone (1989) described as the conflict between rights and obligations. Pepitone noted that "the reaction to the violation of an obligation may be tempered by an internalized right that is in opposition to the obligation, perhaps the very same right claimed and exercised by the 'violator'" (p. 14). Applied to the context of this study, participants' 'violation' or breach of the norm through selecting Action B may have been tempered by their sense of autonomy, which in turn can represent any number of internalized beliefs about mathematics teaching and learning.

The conflict between *autonomy* and *normativity* discovered in the present analysis suggests that for the particular scenario examined, *normativity* wins the conflict. While *autonomy* was shown to have a larger logit size for P(Action B | Action D), *normativity* consistently predicted the decision-making patterns for all actions relative to Action D (the normative action). While this pattern may vary given differing scenarios and options for decisions, the main claim from our analysis suggests that participants' recognition of situational norms in teaching are an important influence in their pedagogical decision-making. Therefore, if we wish to better understand teachers' decision-making, more attention should be given to the characteristics of the situations in which teachers act, as well as to the resources of individual teachers.

Endnote

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