PROMPTING MATHEMATICAL KNOWLEDGE FOR TEACHING THROUGH PARENT-TEACHER LEARNING COMMUNITIES

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Parents, K–8 teachers, and 4th–8th grade children participated as equals in math-focused learning communities through the Math and Parent Partners (MAPPS) program. Pre/post testing and qualitative interviews revealed that the learning communities served as a platform for improvement in mathematical knowledge for teaching of participating teachers. Moreover, teachers learned about parents' knowledge and strategies, a construct analogous to Knowledge of Content and Students that we describe as "Knowledge of Content and Parents."

Keywords: Mathematical Knowledge for Teaching; Teacher Knowledge; Informal Education

Background and Research Questions

Student achievement lags in many economically disadvantaged schools. Two factors associated with this achievement gap include inadequate teacher knowledge and low parental involvement (Hill, Rowan, & Ball, 2005; Jackson & Remillard, 2005). A school district in the Southeast partnered with the local university to boost student achievement in Title I schools through Math and Parent Partners (MAPPS) parent-teacher learning communities in mathematics (MAPPS, 2009). We asked,

> Does parental involvement in a standards-based mathematics program such as MAPPS carried on at Title I K–8 schools improve student understanding and achievement in mathematics? Secondarily we asked, how might this improvement occur? In particular, do parents and teachers in MAPPS develop mathematical knowledge for teaching?

Students were found to improve standardized test scores significantly over a three-year period (Knapp, Jefferson, & Landers, in press). However, this paper focuses on factors that may have prompted the student improvement. In particular, we describe *teachers'* development in mathematical knowledge for teaching as they participated in MAPPS learning communities.

Theoretical Framework and Literature Review

Hill, Rowan, and Ball (2005) reported a study in which teachers' *mathematical knowledge for teaching* (MKT) was linked to student achievement in first and third grade. Moreover, they found that teachers in economically disadvantaged schools tended to possess lower MKT. The framework of mathematical knowledge for teaching (MKT) relates to the knowledge and habits of mind needed to teach mathematics well (Ball, Thames, & Phelps, 2008). In the framework, MKT includes six constructs of which we focused on the following four in investigating the Math and Parent Partners learning communities. Common content knowledge (CCK) is basic, lay-person knowledge of the mathematical content. Specialized content knowledge (SCK) is the way the mathematics arises in classrooms, such as for building representations. Knowledge of content and students (KCS) indicates a teacher's knowledge about how students think in mathematical contexts. Knowledge of content and teaching (KCT) indicates a teacher's knowledge of advantageous representations or teaching sequences. MKT encompasses both content knowledge (CCK & SCK) and pedagogical content knowledge (KCS & KCT).

Studies have additionally shown that parent involvement in their children's education is linked with children's academic outcomes (D'Agostino, Hedges, Wong, & Borman, 2000; Epstein, 1994; Kellaghan, Sloane, Alvarez, & Bloom, 1993). As Henderson and Mapp (2002) stated, "The evidence is consistent, positive and convincing: families have a major influence on their children's achievement. When schools, families, and community groups work together to support learning, children tend to do better in school,

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stay in school longer, and like school more" (p. 7). Low-income parents may be untapped resources for the mathematical achievement of their children. Henderson, Mapp, Johnson, and Davies (2007) asserted that districts serious about closing the achievement gap would have to address the school culture gap that expects parents to remain relatively uninvolved in their children's mathematics learning. Although parental involvement may be linked to student achievement, parents are often not accessed as resources for helping children learn mathematics in standards-based school environments (Jackson & Remillard, 2005; Perissini, 1998). In this paper, we describe a study of a parental involvement program that engaged parents and teachers in mathematics learning communities.

Participants and Context

The Math and Parent Partners (MAPPS) program equips families to act as mathematical resources for their children and for schools. MAPPS curriculum was developed with National Science Foundation funding to engage K–8 parents in exploring with peers the concepts and skills behind the mathematics that their children are learning in schools (see http://mapps.math.arizona.edu/). Currently, the MAPPS program serves sites in six states and the Virgin Islands. One MAPPS site, located in the Southeast and the focus of this article, worked toward improving the mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) of both parents and teachers in Title I schools within its school district. All parents, teachers, paraprofessionals, and children from selected schools were invited to participate. The local university partnered with MAPPS and the school district to offer Mini-courses for parents and teachers, while young children participated in related mathematical activities and games. Children in 4th-8th grade accompanied their parents in the Mini-course classes. Mini-course sessions convened two hours per week for eight weeks. Over the course of three years, eight separate 8-week Mini-courses, centered on the National Council of Teachers of Mathematics' (NCTM) (2000) content and process standards, were offered. These Mini-courses were hosted by the University's Office of Continuing Education, and instructors were graduate students in mathematics education who were also practicing teachers.

8-week Mini-course Title	NCTM Content Standard Addressed
Thinking About Numbers (offered two times)	Number & Operations
Thinking About Fractions, Decimals, and Percents (offered 3x)	Number & Operations
Thinking in Patterns (offered once)	Algebra
Geometry for Parents (offered once)	Geometry and Measurement
Data for Parents (offered once)	Data Analysis & Probability

Figure 1: Math for parents Mini-course curriculum

In all, 115 children, 59 parents, and 33 teachers from primarily four Title I elementary schools attended at least one Mini-course on a regular basis. Nearly twice that many participants attended sporadically. Approximately 75% of attendees were single parents, and those that attended the Mini-courses did so with one to three children. Most of the parents had graduated from high school with some technical training, and they typically held low-income jobs. Attendees were approximately 40% Caucasian, 40% African-American, and 20% Hispanic. Teachers who attended faithfully received stipends and professional learning units.

MAPPS Mini-courses engage parents in doing mathematics using hands-on materials, working in small groups to solve problems, and presenting their solutions to the whole group as outlined by the NCTM process standards (NCTM, 2000). Both content knowledge and pedagogical content knowledge are intertwined into the instruction for parents (Ball, Thames, & Phelps, 2008), with pedagogical considerations made relevant by Mini-course instructors depending on grade levels of participating children.

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To illustrate the intervention and details of the MAPPS program, we describe a learning activity from the two-hour Week Eight session from the Fractions, Decimals, & Percents Mini-course (see Figure 1). For the task, participants were to have shaded a given percentage of various grids for homework from the previous session. The first grid, a bar divided into fifths, required 60% to be shaded (see Figure 2). Participants had to figure out what percent each fifth represented for the entire grid to equal 100%, and they discussed their findings at the beginning of the session.

Shade 60% of this grid.

Figure 2: MAPPS homework task

A father and his 6th grade daughter found that each rectangular fifth must be 20%. The father held up his hand to demonstrate his fingers as the rectangle saying, "Each finger is 20, so we shaded three of them to make 60, see (pointing to his fingers) 20, 40, 60." Later in the session, parents, teachers, and children made percent strips that they then compared to the fraction and decimal strips made during previous sessions. At the end of each task, group members reported their various solutions and strategies to the entire class. Sometimes the *children* presented unique strategies allowing parents and teachers to learn from the children, and visa versa.

Data Analysis

To assess the impact of the MAPPS Mini-courses, parents and teachers took pre/post tests on mathematical knowledge for teaching (Hill, Schilling, & Ball, 2004) and pre/post attitude surveys (Tapia, 1996). Pre/post tests and surveys were administered before and after each 8-week Mini-course. A focus group of parents, teachers, and children also participated in 95 pre/post interviews. Interviews lasted approximately 15 minutes, and questions were such as these: (1) Have you learned anything about mathematics that you did not know before? Explain. (2) Have you learned anything in MAPPS that helped you help your child or students with math? Explain. Interviews were coded for evidence of improved student understanding, achievement, and factors that might affect that improvement, such as the elements of mathematical knowledge for teaching: CCK, SCK, KCT, SCK (see Table 1). After coding the interviews and pre/post surveys, we tallied the 59 codes to identify the salient areas of participant growth as well as factors prompting that growth. We looked for clusters in the data each year, producing primary and secondary results for each year. At the end of the study, we compressed codes and identified themes based on the primary and secondary codes. Themes arising from the coding process included strengthened teacher content knowledge, improved teacher Knowledge of Content and Teaching, and benefits of the learning community.

Table 1: Teachers' Results from 34 Teacher Interviews

Code	Freq	Description of Result
Primary:		
Knowledge of Content and Teaching (KCT)	56	
Content Knowledge SCK(16) CCK(6) *GLM(6)	28	Primarily SCK for teachers *GLM-General learning of mathematics reported that could not be identified as CCK or SCK
Enjoyment of/Valuing MAPPS	40	High value placed on program.
Secondary:		
Learning Community	23	The learning community was valued.
Broader impact of program	10	Program impacted non-MAPPS students.
Student learning/achievement	8	

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Results and Discussion

We present interview data from several teachers to amplify our coding process and themes that emerged. Examples of codes are in bold. Teacher A, a primary teacher, shared the following:

AK: Well can you just talk to me about how the program went for you?

Teacher A: Well, coming in as a teacher, it really helped me see, uh…just a little more in depth look at the math. Because math isn't one of my strengths, I will tell you, it's not one of my strengths. So I came out to actually deal with like ideas, like fractions, and really at my own pace, kind of look at what is it, what is a fraction.

AK: …Can you give me a specific one [example] that maybe you understood superficially and you could teach it, but now you understand it in a different way?...

Teacher A: One of my favorite ones was when we talked about fractions. She gave us strips and she said, "Okay, fold this strip into one third; fold it into one eighth, …twelve. Fold your strip into twelfths. Eighths and thirds." And first, when you wanted to make the equal sections you kind of thought well this is going to be my stopping point. For that line, which is what we put on our strips as the stopping point. It's the space between that makes that one third. Just that space. Even with measurement, when they looked at the ruler, you know they have all the little increments between 1 and 2, but she kind of let me see that that space. It's not as important as what the numbers are, it's that space, the little spaces between the numbers that, I hadn't really [considered].

In this Mini-course, Teacher A developed **Specialized Content Knowledge (SCK)** when she learned what was mathematically significant about a fractional increment using fraction strips. She came to understand the content more deeply as it arises in a classroom. Moreover, she and other teachers stated that after attending the MAPPS Mini-course on fractions, decimals and percents, that the MAPPS materials became their handbook for teaching the unit on fractions in their classrooms. After attending MAPPS, this teacher and numerous others made the decision to pursue graduate degrees in Elementary Mathematics Education (**Higher Education**).

Another teacher who attended MAPPS described the benefit of the learning community for herself, her parents, and her students.

AK: Is there anything in MAPPS that has helped you to better explain math to kids?

Teacher B:…when we did fractions, decimals, and percents I had so many kids that would not understand that, and so I would literally I would have one child in my room on this side of me and their parent would be right here. And then I would have another one from my room on this side and we would literally work through exactly what we were doing in class with adding and subtracting decimals. We would take the unit cube, and it would be the one whole or the units, rods, and

AK: flats.

Teacher B: flats. And that helps too with the kids when they go home and say, "Well our teacher told us that 5 and 50 are the same." And the parents are going, "No," you know.

This episode exemplifies the parent-teacher interaction enshrined in the **learning community**. The teacher could see first-hand the disconnect between school learning and home learning. It became evident to her that children appeared to understand the concept in class, but they were unable to verbalize their misconceptions adequately to their parents who did not know about the base-10 block representation for decimals. She realized that students were making confusing comments such as, "Our teacher told us that 5 and 50 are the same," to the parents. The experience allowed the teacher to develop **Knowledge of Content and Students (KCS)** about student misconceptions, in this case not understanding that 5 rods were the same as 50 unit cubes. The MAPPS session amplified the student and parent misunderstandings for the teacher, and at the same time, helped the parent to develop **Specialized Content Knowledge (SCK)** about how base-10 blocks can represent decimal operations.

Teacher B additionally expressed that the MAPPS instructors modeled good explanations for her and helped her be reflective. Other teachers stated that MAPPS helped them learn connections between

mathematical topics such as fractions, decimals, and percents. Paraprofessionals and substitute teachers who participated also reported that MAPPS equipped them to assist with instruction in mathematics. Finally, the MAPPS environment helped teachers add rigor to their teaching practice. Teacher A stated, "I think I wasn't really going in depth as much as I could." She learned to facilitate conceptual understanding at a deeper level than had been afforded through teaching fractions by rote.

The qualitative result that teachers improved their content knowledge was substantiated by MKT test results. Significant changes were noted when the first Mini-course to the last Mini-course scores were compared (n=20; p=.052). The content knowledge tests were designed such that a well-prepared elementary teacher would get 50% of the questions correct (Hill, Schilling, & Ball, 2004). Although the test scores improved significantly, the average scores did not rise above this 50% benchmark. This data suggests that teachers involved in the program were in need of further instruction in mathematics for teaching and highlights the importance of the result that the MAPPS parent-teacher learning communities built teachers' **confidence** as mathematics learners and emboldened some to attend graduate school in mathematics education.

The next aspect of mathematical knowledge for teaching that developed for both parents and teachers during MAPPS was Knowledge of Content and Teaching (KCT). Teachers reported learning to model problems with tasks and manipulatives, instead of relying as heavily on direct instruction and drill. We considered this shift to be KCT because teachers were expanding their repertoire of effective examples and teaching sequences that they over and over reported taking back to their classrooms, sometimes as an entire grade level (**broader impact**). An interview displayed a teacher learning about the instructional advantage of a dynamic representation to help her teach subtraction with regrouping.

Teacher B: The other thing that I used that they showed us talked about the virtual manipulatives, was the website where you can, there are the little unit cubes, the rods, and the flats…

AK: The base ten blocks?

Teacher B: Yeah and you can drag them over and show and the kids can go up to the active board and manipulate those around and they just loved that because I needed a tool when I was teaching the kids even just when we were learning subtraction with regrouping.

Teacher B went on to explain that her students were confused when using the concrete base-10 blocks and that her static drawings were inadequate. However, after attending MAPPS, she engaged students in a MAPPS task using the virtual base-10 blocks. She said, "But I pulled that website up, and I could just move it right around. And it was just so convenient, and it was easy for them to see because it was color coded too where my little drawings were crude…" Thus, this teacher developed **KCT** related to choosing effective examples and representations.

We found that although some teachers had access to manipulatives, they were unaware of how concrete manipulatives could undergird young children's understanding of mathematics content. A special education teacher said, "Since MAPPS, I've done a lot more work with manipulatives. I make a point to go to the manipulatives quickly and then the abstract." He additionally said, "To approach it [content] rather than drill it and kill it that [MAPPS activity] was a problem solving model that I wouldn't help them with except you know I would get them over humps and stuff. But it was a way to get them to think and to realize, 'Oh I get it, I get it,' and the light bulb [would] click on. You could see it happening, and it was really good." This teacher began using MAPPS activities exclusively for Saturday school instruction. This school's standardized mathematics test scores rose from 64.3% passing in 2008 to 81.3% passing in 2011.

Aside from improved, purposeful manipulative and task use the classroom, Knowledge of Content and Teaching in general improved, as evidenced by teacher statements such as this: "It [MAPPS] gave me ways to bridge that gap between what I know and really making it something that they know." Teacher learning continued to develop after MAPPS, as teachers reported adapting materials to other grade levels and mathematics content areas from year to year.

Teacher C: I think I learned as much from it as the parents did.

RL: And that is what it does for you.

Teacher C: I don't think I learned as much as I did *until I brought it back into my classroom* [emphasis added].

Thus, although teacher learning did occur during MAPPS, there seemed to be a delayed amount of learning that took place. Teachers adapted MAPPS tasks to their own grade levels and state standards, and in the process of enacting the tasks in their own classrooms, or collaborating about the tasks with their colleagues, they furthered their Knowledge of Content and Teaching.

In addition to strengthening teachers' mathematical knowledge for teaching, the learning community afforded by MAPPS strengthened parent-teacher relationships as well. Bonding formed because teachers got to know parents in a different way than in the negatively-connoted position of power, telling parents what to do or not to do in regards to their children. Teachers and parents enjoyed a level playing field in which all were learning for the desired end of helping children (**Enjoyment of/Valuing MAPPS**). One teacher said, "When they saw me get excited about something, they were like, 'Wow, she didn't know this. We're learning this together.'"

Moreover, parents appreciated teachers' extra effort to help children learn, and teachers came to view parents as dedicated individuals, invested in the academic success of their children. The light-hearted nature of the Mini-courses drew families and teachers back for not only more mathematics learning, but relationships fueled by a desire to learn mathematics. Even 4th-8th grade children participated as equal learners of mathematics content, often presenting solutions or strategies that parents and teachers learned from. The casual, non-threatening learning environment served as a relationship-building environment. Finally, teachers helped each other make the tasks and rigor relevant to their respective classrooms. The MAPPS learning community forged a Parent-Teacher-Child triangle of knowledge and respect (see Figure 3).

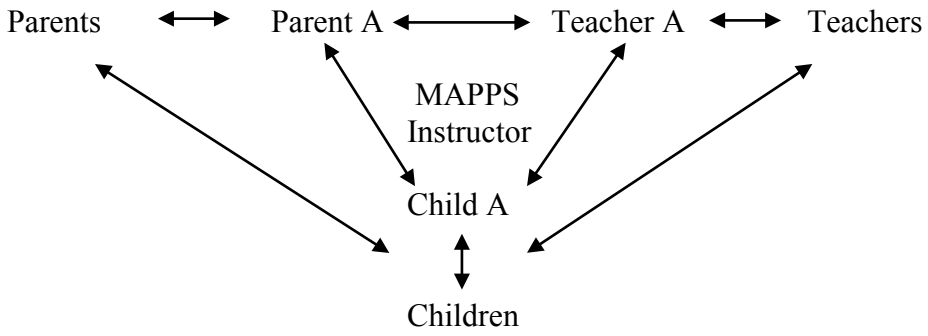

Figure 3: MAPPS learning community

A notable benefit of the learning community was *teachers learning about parents'* knowledge and strategies, a construct we call "Knowledge of Content and Parents." An interview evidenced this.

RL: Tell me, how did you feel about working in groups with other parents and other teachers?
Teacher D: It enlightened me a lot. I didn't know that they didn't know so much of the vocabulary. I had no clue. They really had no clue of how to talk about manipulatives and use them. They didn't know what that meant when their children came home and discussed it.

Just as Teacher B learned that parents didn't understand how to use manipulatives to teach decimal operations when the child said, "5 and 50 are the same," this teacher learned that parents did not possess the *vocabulary* related to the manipulatives. Another teacher explained that she learned about parents by listening to them talk with their children about mathematics. The MAPPS environment allowed teachers to see *why* parents struggled to assist their children with mathematics. Moreover, teachers learned about parent content knowledge and strategies as parents and teachers collaborated to solve problems. Teachers also listened to parents present their strategies, such as when the parent presented his strategy of finding 60% of the fraction grid (see Figure 2). We believe that this *content-focused teacher learning about*

parents' knowledge and strategies, or *Knowledge of Content and Parents*, is analogous to Knowledge of Content and Students and is also an aspect of Schulman's (1987) category of teacher knowledge, "knowledge of educational contexts" (p. 8).

Through the clinical experience of the MAPPS learning community, teachers gained Mathematical Knowledge for Teaching and Knowledge of Content and Parents. As such, teachers learned what misconceptions parents had, that parents did not know about many manipulatives commonly used in the classroom, and that parents lacked vocabulary needed to connect manipulatives to conceptual understanding of content. Furthermore, teachers learned about parents' content knowledge, problem-solving strategies, explanations to their children, and desire to help their children. Thus, the parent-teacher mathematics learning community provides a unique professional development environment for teachers. Teachers learn both mathematics for teaching and how to access parental involvement in a way that enhances student learning. This study implies that Knowledge of Content and Parents can and should be taught through parent-teacher mathematics learning communities. Further research is needed on the nature of Knowledge of Content and Parents and its relationship to student achievement in mathematics.

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