MATHEMATICS TEACHERS INVESTIGATING REASONING AND SENSE MAKING IN THEIR TEACHING

Lindsay M. Keazer
Michigan State University
keazer@msu.edu

The purpose of this study was to generate an understanding of the experiences of mathematics teachers examining recommendations for Reasoning and Sense Making (NCTM, 2009) and investigating them in their practice. Narrative inquiry incorporates the voices of teachers and illustrates the phenomenon studied through narratives of participants' experiences. This paper presents the findings through four analogies that convey abbreviated narratives of teachers' experiences enacting recommendations for Reasoning and Sense Making.

Keywords: Standards; Teacher Education/Professional Development; High School Education

Recent recommendations for improving the nature of teaching and learning mathematics across the United States can be traced back to 1980 with the National Council of Teachers of Mathematics (NCTM) publication of *An Agenda for Action* (NCTM, 1980). In subsequent years, NCTM published a series of standards documents (1989, 1991, 1995, 2000) to clarify new goals and curricular recommendations for mathematics education. When evaluating the state of mathematics classrooms, discourse within the field often focuses on the deficits, making broad generalizations pointing to a gap between the state of mathematics classrooms throughout the nation and the classroom environments promoted by these recommendations. Hiebert (1999), for example, declared that "the same method of teaching persists, even in the face of pressures to change," (p. 11). Similarly, the Conference Board of the Mathematical Sciences (1975) asserted that "teachers are essentially teaching the same way they were taught in school," (p. 77) referring to the lack of impact of the earlier "new math" reform movement of the 1960s.

A contributor to the gap between curricular recommendations and classroom practice is the complexity of learning to teach mathematics differently. The changes proposed by reform efforts such as the NCTM standards have the underlying assumption "that teachers will change their world view of mathematics, mathematics teaching, and mathematics learning" (Shaw & Jakubowski, 1991, p. 13). Even when such changes are desired or instigated by the teacher, many have described difficulties they encountered as they attempted them in their own teaching (e.g., Ball, 2000; Cady, 2006; Chazan, 2000; Heaton, 2000). In short, making changes to one's teaching is a complex process.

To add to the conversation surrounding teachers' responses to NCTM recommendations, this study sought to develop an understanding of the experiences of mathematics teachers attempting to enact recommendations for mathematics teaching from *Focus in High School Mathematics: Reasoning and Sense Making* (NCTM, 2009). This document proposed that "reasoning and sense making are the foundations of the NCTM Process Standards" (NCTM, 2009, p. 5), and should be incorporated into "every mathematics classroom every day."

A collaboration of seven mathematics teachers was formed by recruiting teachers interested in investigating their practice and incorporating recommendations into their teaching. The purpose of this study was to learn about the experiences of mathematics teachers as they investigated NCTM recommendations for *Reasoning and Sense Making* (NCTM, 2009) and attempted to make changes in their practice through informal teacher action research. Particularly I focused on five aspects of the experience: conceptions of reasoning and sense making, actions that the teacher took in their teaching, challenges, opportunities, and the teacher's interpretations of the results of their actions. Teacher action research was conceptualized as a self-critical inquiry into one's practice with the goal of improving practice as well as developing a better understanding of that practice (Carr & Kemmis, 1986). Stenhouse (1975) promoted applying curricular recommendations to the formation of one's action research inquiry, suggesting that

Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). (2012). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Kalamazoo, MI: Western Michigan University.

"the crucial point is that the proposal is not to be regarded as an unqualified recommendation but rather as a provisional specification claiming no more than to be worth putting to the test of practice" (p. 142).

Teachers represented six high schools and ranged from 0 to 11 years of teaching experience (mean of 3.5). They agreed upon the theme of *Reasoning and Sense Making* as the focus of their work together. We met regularly throughout the school year, a total of nine times. Teachers initially read and discussed *Reasoning and Sense Making* and began to focus their action research inquiries in individual ways by selecting specific actions to take in their practice to incorporate their interpretation of the recommendations. Teachers learned informally about the methods of action research through PowerPoint presentations, reading excerpts of methods handbooks, and narrative examples of teacher action research. I created a library of practitioner readings that were related to their goals, from which they selected additional readings. Meetings served as a time for them to discuss readings and share their goals, challenges, and successes.

Data Analysis

During this study a variety of data sources, or *field texts* (Clandinin & Connelly, 2000), were collected to generate an understanding of teacher's experiences. See Figure 1 for an illustration of the data sources that inform this analysis.

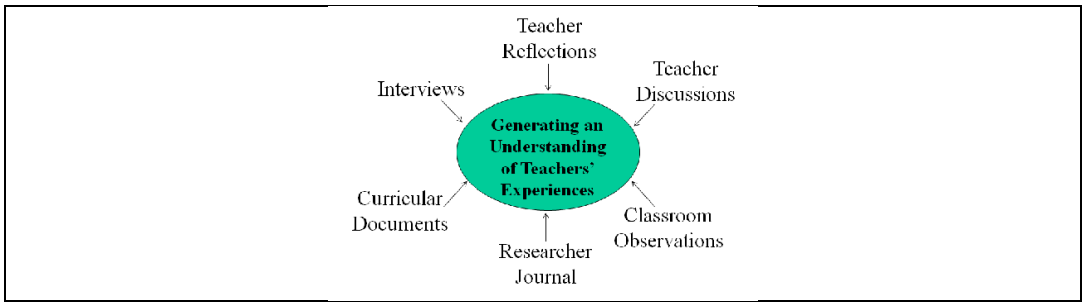

Figure 1: Data sources that inform the research question

This study used narrative inquiry to investigate the ways teachers incorporated recommendations into their practice. Narrative inquiry was selected to allow the voices of teachers to be heard and expand our understandings of "what the experience is like." Clandinin and Connelly (2000) describe narrative inquiry as "a way of understanding experience. It is a collaboration between researcher and participants, over time, in a place or series of places, and in social interaction with milieus" (p. 20). Narrative researchers illustrate a phenomenon studied through creating unified, coherent narratives that convey the meaning of their experiences working together. The method of analysis is emplotment and narrative configuration (Polkinghorne, 1995), in which data snapshots are pieced together to develop a plot. This requires a synthesis of the data rather than separating it into its constituent parts.

As data was collected, I continuously reviewed it. After all data was collected, I organized the data pieces pertaining to each teacher into chronological order in spreadsheets. I coded data pieces according to the aspects of the experience previously identified: conceptions of reasoning and sense making, actions that the teacher took in their teaching, challenges, opportunities, and the teacher's interpretations of the results of their actions, plus the additional category of contextual information. Coded data for each teacher was then reorganized into condensed spreadsheets according to category. I continuously reviewed these consolidated spreadsheets until recurring ideas and connections developed to synthesize the information into the "plot" of the narrative. Then the process of writing of *interim texts* (Clandinin & Connelly, 2000) or smaller drafts of the research text, was an important element of the emplotment and narrative configuration. Through repeatedly experimenting with the writing process by writing interim texts, and then sharing those texts with the teachers, I eventually produced the final research texts. More details about the analysis will be shared in the presentation.

Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). (2012). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Kalamazoo, MI: Western Michigan University.

Findings

The analysis revealed the complexity of each teacher's experience. Teachers varied in their past experiences as a mathematics teacher, and in the awareness they held of the ways they influenced student's opportunities to reason and make sense of mathematics. Teachers also varied in the actions they took to adapt their teaching in response to the recommendations. Their action research foci varied from improving their questioning strategies, curriculum, role in discussion, prompting students' justification, prompting writing about mathematics, and incorporating student creativity into the doing of mathematics. As I developed narratives of the teachers' experiences, it became apparent that within the different journeys, subtle similarities existed. I compared the plotlines of narratives that held similarities to clarify my understanding, and I examined the differences across groups. As I read and reread my data, I tested different categorization schemes in my own sense making process to understand the ways I grouped teachers' narratives. Two aspects of their experiences emerged that provided a way to categorize their narratives. The first aspect was their level of *awareness*—both at the time they entered our collaboration and the development of their awareness over time—of the ways that they influenced students' opportunities to engage in reasoning and sense making. The second aspect was their evolution, or development in any direction, of the ways they acted on this awareness by developing *strategies* to promote students reasoning and sense making.

These two aspects, teachers' *awareness* and their *strategies*, were intertwined within each teacher's experience. Particularly, teachers' *awareness* of the ways they impacted students' engagement in reasoning, and their *strategies* for fostering students' reasoning, evolved in response to each other over time. Evolution in teachers' awareness was only recognizable when teachers self-reported new things they had come to realize about their teaching. Evolution in teachers' strategies was more easily identifiable, through teachers sharing new strategies they were developing and through my own observations during classroom visits. As I made sense of differences in teachers' journeys, I generated four analogies to represent their journeys. There isn't sufficient space to present the narratives of teachers' experiences here, but I will offer a glimpse through the four analogies: a linear function, a piecewise function, a step function, and a scatterplot. The independent variable in this mathematical relationship is the time spent studying one's teaching practice. The dependent variable is the evolution of strategies to support students' engagement in reasoning and sense making. While these analogies attempt to illustrate teachers' experiences over the seven months we collaborated, this was a brief stretch amidst their longer journey as a mathematics teacher. In the proceeding sections, I introduce each analogy and provide illustrations from the data of the teachers that they represent.

A Linear Journey

Teachers Peter and Alexis both entered the collaboration having already problematized many aspects of mathematics teaching that *Reasoning and Sense Making* sought to change. Both talked openly about the problematic consequences of teaching mathematics through providing a list of procedures, consequences they had seen firsthand. Peter used humor to tell stories illustrating the negative effects of students' reliance on procedures or the teacher's authority, instead of reasoning and sense making. "I really want my students to start critically thinking. I swear that I could say, 'Your lesson today is to learn that $5 + 8 = 22$.' And they will just write $5 + 8 = 22$, and not even think a thing about what they're actually writing, whether it even makes sense at all" (12/9/10, meeting 3). Peter talked often about how "we're fighting a decade's worth of ingrained math," after seeing indications that his students were well practiced at learning mathematics *without* reasoning. As Peter and Alexis read *Reasoning and Sense Making*, they agreed wholeheartedly with the proposition of the document that teaching mathematics through steps and procedures did not produce positive student learning outcomes.

Along with identifying certain teaching practices as discouraging to students' reasoning and sense making, Peter and Alexis began their action research with a similar awareness of the ways that their role as teacher influenced student's engagement in reasoning and sense making. Both agreed with the philosophy of the recommendations and shared ways they had already made improvements to their teaching that aligned with recommendations. For instance, Alexis shared:

Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). (2012). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Kalamazoo, MI: Western Michigan University.

> I don't teach the $\frac{y_2-y_1}{x_2-x_1}$ formula to find slope. I use t-charts and put six graphs up on the board when I want to start teaching them about slope. … So [the students] figured out when you put it in y=mx+b form, where that [slope] number was coming from. And they realized, you know, if it was negative it went left, positive went right. … And so, we look at all the graphs, and we talk about the change in y over the change in x, and how it goes up and over, and where those numbers came from, and then we just call it change in y over change in x. (11/16/10, meeting 2)

Despite examples of shifts away from a focus on procedures, upon reading *Reasoning and Sense Making* both Peter and Alexis saw themselves as guilty of reliance on practices that did not promote students reasoning and sense making. They both identified room for growth to align their teaching with these recommendations. While their described approaches varied, they each developed their own ways to "transfer the deliverance of my lesson to my students" (Peter, 2/22/11, reflection). Peter focused on removing opportunities for students to rely on his authority instead of their own reasoning, through habits he developed such as "keeping silent," "firing students' questions back at the class," and "going along with wrong ideas." Alexis focused on developing her questioning; restructuring lessons so that students uncovered the mathematical ideas through class discussions facilitated by her questions.

To illustrate the similarities among their journeys and the differences between their journeys and those of others, I draw on the analogy of a linear function (see Figure 2). While Peter and Alexis faced challenges, I conceptualized their evolution of strategies as being fairly linear when compared with that of others. They conceptualized a vision for their teaching and developed their strategies to move continuously towards their goal. Their awareness of their own impact on students' opportunities to engage in reasoning and sense making facilitated a steady progression in the direction of their vision.

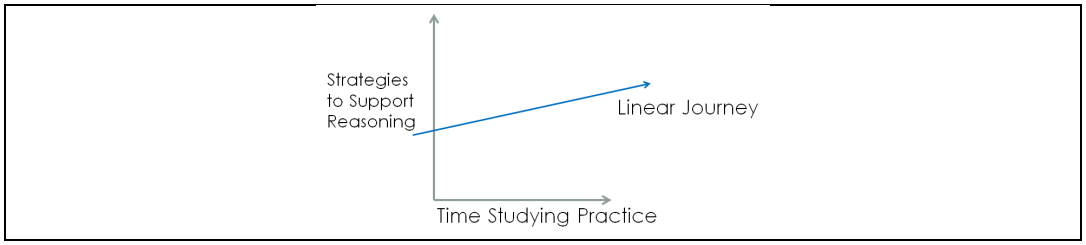

Figure 2: The analogy of the linear function

A Piecewise Journey

Teachers Logan and Melinda both expressed interest in the theme of *Reasoning and Sense Making* as they joined the study, but they did not cite examples of ways that their teaching methods influenced students' opportunities to engage in reasoning and sense making. They hoped to learn more strategies to foster reasoning and sense making as a result of their collaboration in the group.

After reading the recommendations, both Logan and Melinda formulated goals that were related to improving their classroom discussions. Both were interested in changing the structure of lessons to move away from direct instruction by incorporating questions and using student-generated ideas to move a lesson forward. Both identified their initial changes in their teaching as successful based on their students' responses. However, at different points during the school year, each teacher experienced frustration as they encountered students responding to their questions with increasing silence. When their best efforts were met with resistance from students, they became discouraged and wondered if some of their students were not capable of reasoning.

One day after observing Melinda teaching an Algebra lesson, she asked if I would teach the same lesson to the next class walking in. I agreed, and this proved to be a valuable opportunity to foster her thinking about her teaching. After watching me teach her lesson, and noticing the ways her students responded to my questions, she said, "I thought the problem was that my students couldn't reason. But now I see that I was just asking the wrong questions." After that episode, I observed noticeable differences

Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). (2012). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Kalamazoo, MI: Western Michigan University.

in the questioning that Melinda used. Rather than questioning patterns that resembled those described as "funneling" (Wood, 1998), her questioning changed to resemble more closely the pattern described as "focusing" (Wood, 1998). For example, previous questions had directed students towards a particular procedure such as "Which fraction should we use? What if we use this one? Can we cross anything out?" Her new questioning tended to be more open to allow students to determine their own solution methods, such as "How can you find the side length of a square with an area of five?" and "Steve subtracted and then divided. Do we have to do it in that order?" The following year, Melinda continued to e-mail to share ongoing successes she saw as a result of long-term use of her new questioning strategies.

A similar experience happened in Logan's action research. He became discouraged for several months during the spring semester, and began to wonder if the juniors and seniors in his "intro" level Algebra II courses were capable of reasoning. After persuading him to allow me to teach one of his lessons, I attempted to make an "existence proof" that his students *could* reason mathematically. The following is an excerpt of his reflection:

> When watching Lindsay teach my class, I noticed how she was able to get everyone involved. She was calling on students who had not volunteered to share an idea in months. I have made a point to call on each and every student in my class since then. I also do not let students get away with just saying, "I don't know." They were actually saying, "I don't want to think right now," so I have to make them tell me something that they do know. (5/6/11, final reflection)

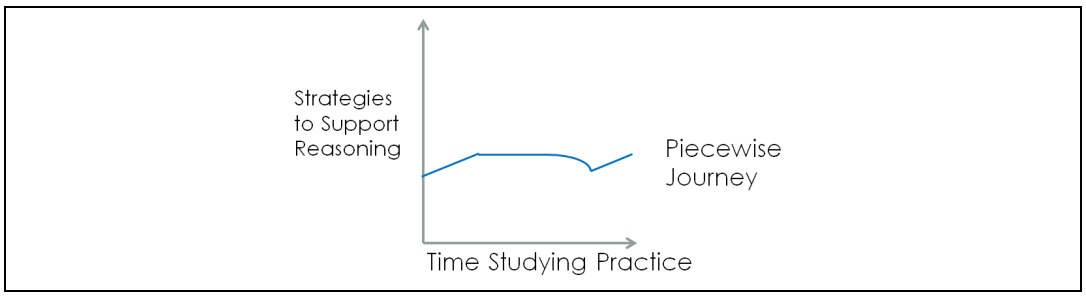

Figure 3: The analogy of the piecewise journey

To illustrate the similarities among Logan and Melinda's journeys, I draw on the analogy of a piecewise function. While Logan and Melinda initially saw short-term improvement in their students' engagement in reasoning and sense making, both also experienced a plateau. They overcome the obstacle when they developed a heightened awareness of ways they impacted students' opportunities to reason. A new awareness of their teaching prompted the development of new actions to support student's reasoning and sense making.

A Step Function Journey

Sarah, a fourth year teacher of high school geometry and algebra, shared that she had not previously considered the importance of fostering reasoning and sense making opportunities until reading these recommendations. The authority of the document convinced her of the importance of developing such practices in students to prepare them for their future. Beginning with suggestions pulled from the document, through trial and reading other practitioner articles, she narrowed the focus to asking more questions and requiring students to justify all ideas. These changes increased the amount of student talk in Sarah's classroom, opening up opportunities for students to "surprise" her with their mathematical ideas. Through studying her teaching, these unexpected incidents became learning opportunities that increased her awareness of how to support students' reasoning and sense making.

> You remember the Algebra class where they wanted to use synthetic division? *(laughing)* I was so caught off guard because I've never thought of using that method [in that context] before in my *life*. I

> was like, "Okay let's go with it." But I was really surprised. And I should've been more calm about it… because then they wanted to know what "my way" was. But it totally caught me off guard. (5/18/11, final interview)

Each new unexpected finding fueled further development of her actions. One thing she learned from her students was the value of allowing them to determine their own solution path:

> Before, I wouldn't let them [solve problems] the way that they wanted to. … I think a lot of times I would just be like, "Well didn't you see this method," instead of just letting them do it their way. I think its okay now just to let them do it a different way, even if it's the hard route. Just let them be, because that's the way they understand. Giving them that freedom. (5/18/11, final interview)

The analogy of a step function illustrates Sarah's experience (see Figure 4). Each step in the function represents actions she tested in her teaching and subsequently learned from, resulting in new knowledge and a heightened awareness of strategies to support students reasoning. The heightened awareness facilitated her in developing her actions further, represented by the next step in the function. Sarah's experience was unique from the others by the pattern of repeated instances of surprise that resulted in new awareness that fueled developments to her action strategies.

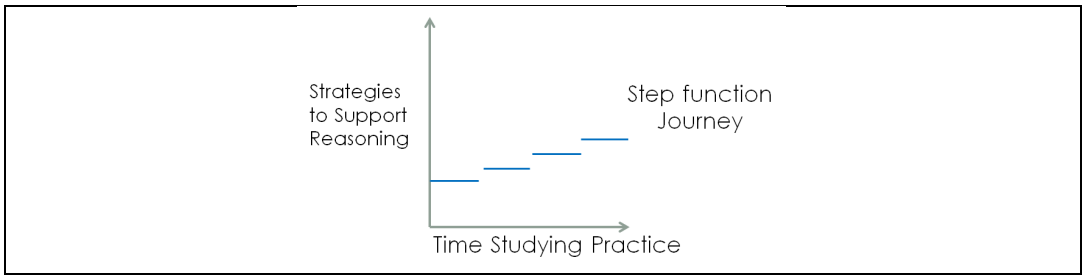

Figure 4: The analogy of the step function

A Scatterplot Journey

Claudia and James were in their first year of teaching, and both juggled many new responsibilities. It took them more time to develop the focus of their actions, and their initial actions changed frequently as they experimented with a variety of different strategies. Claudia reflected on these early months and discussed the challenge of trying to focus her actions:

> With it being my first year and everything, I didn't know what my teaching style was and how I wanted to change or improve it... I kept kind of trying the different things I heard people talking about, thinking, "Is this what I need to work on? Is this something that interests me?" (4/28/11, meeting 9)

Both teachers eventually narrowed their efforts to posing open-response prompts on assessments. This approach to incorporating reasoning was more like an add-on to their teaching than a part of their everyday routine. James explained in a written reflection why he picked a subtle approach:

> I would love to hold classroom discussions and ask questions where students learn from their mistakes, discuss problems with one another, and problem solve when they do not get the correct answer (Eggleton et al., 2001). That type of classroom environment is one that I envision for the future, but I do not believe my classes are ready for such radical changes all at once. To me, writing seems like a natural and subtle way for students to convey their reasoning and sense making. (1/12/11, reflection)

Both teachers also dealt with school-wide pressures to raise students' scores on the state-wide algebra exam. With the many other things vying for their attention, Claudia and James at times would "forget" their focus. Over time Claudia and James recognized the need to incorporate reasoning beyond assessments and into their mathematics lessons. They each tried fostering reasoning through occasional

Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). (2012). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Kalamazoo, MI: Western Michigan University.

student-centered activities. However, limitations in time and resources hindered them from incorporating activities on a daily basis. Each saw room for improvement and made plans to continue their actions in subsequent small steps in the future.

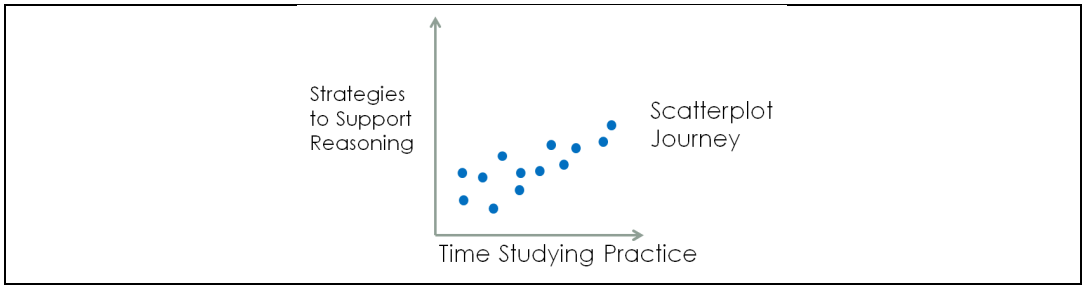

Figure 5: The analogy of the scatterplot

Their journey is illustrated by the analogy of a scatterplot with a positive correlation which became stronger over time. This analogy is distinct from the others as it illustrates the variety of seemingly disconnected actions that Claudia and James tested in their practice but also indicates a progression towards developing more focused and refined strategies.

Conclusion

While each teacher focused their efforts to foster reasoning and sense making in unique ways, the elements they chose to take up and test in their practice were a reflection of those that held meaning for them in the context of their teaching. Common gains among all teachers were a heightened awareness of the ways they impacted students' opportunities to engage in reasoning. Given the trend to focus on the deficit between NCTM recommendations and mathematics classroom practices, this research expands the discourse by illuminating the experiences of teachers attempting changes in their practice. Past research on mathematics teacher change has measured changes in practice along continuums or stages that gauge the degree to which teachers' instructional practices adhere to preconceived change objectives (e.g., Fennema et al., 1996). Alternatively, this study approached teacher change by seeking to understand the complexity of teachers' attempts at change from their perspective. Narrative inquiry offers a valuable perspective to the discourse surrounding mathematics teacher change, validating the knowledge and experiences of teachers and seeking to learn from them.

References

Ball, D. L. (2000). Working on the inside: Using one's own practice as a site for studying teaching and learning. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 365–402). Mahwah, NJ: Lawrence Erlbaum Associates.

Cady, J. (2006). Implementing reform practices in a middle school classroom. *Mathematics Teaching in the Middle School, 11*, 460–466.

Carr, W., & Kemmis, S. (1986). *Becoming critical: Education, knowledge and action research*. London: Falmer.

Chazan, D. (2000). *Beyond formulas in mathematics teaching and learning: Dynamics of the high school algebra classroom*. New York: Teachers College Press.

Clandinin, D. J., & Connelly, F. M. (2000). *Narrative inquiry: Experience and story in qualitative research*. San Francisco: Jossey-Bass.

Conference Board of the Mathematical Sciences. (1975). *Overview and analysis of school mathematics, grades K–12*. Washington, DC: Author.

Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, *27*(4), 403–434.

Heaton, R. M. (2000). *Teaching mathematics to the new standards: Relearning the dance*. New York: Teachers College Press.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education*, *30*(1), 3–19.

National Council of Teachers of Mathematics. (1980). *An agenda for action*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2009). *Focus in high school mathematics: Reasoning and sense making*. Reston, VA: Author.

Polkinghorne, D. E. (1995). Narrative configuration in qualitative analysis. In J. A. Hatch & R. Wisniewski (Eds.), *Life history and narrative* (pp. 5–23). London: The Falmer Press.

Shaw, K. L., & Jakubowski, E. H. (1991). Teachers changing for changing times. *Focus on Learning Problems in Mathematics*, *13*(4), 13–20.

Stenhouse, L. (1975). *An introduction to curriculum research and development*. London: Heinemann.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 167–178). Reston, VA: NCTM.