TEACHING PRACTICES, TECHNOLOGY AND STUDENT LEARNING

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While computing technologies are widely available in secondary schools, these technologies have had only limited impact on changing classroom practices. Partly, this can be attributed to an underdeveloped understanding of the role of the teacher in engaging in classroom practices that can support student learning with technology. In this study, we analyzed the teaching practices that supported students' learning of a conceptually rich and deep topic (the average rate of change) when using an exploratory computer simulation environment. The results illustrate the demands placed on teachers when faced with the multiplicity of student ideas generated by their interactions with the simulation and three aspects of a teaching practice in response to those demands. These findings contribute to evolving frameworks for understanding meaningful and productive technology use in teaching secondary mathematics.

Keywords: Teacher Knowledge; Technology; Modeling; Advanced Mathematical Thinking

Over the past three decades, much research has focused on the potential for computing technology to impact K–16 mathematics education. Graphing calculators, internet access, and (most recently) interactive whiteboards are now widely available in secondary schools and colleges. But the widespread availability of computing technology has had only limited impact in making the kinds of changes to classroom practices envisioned by research. While many factors contribute to the successful adoption of any technology, one crucial factor in any kind of change to classroom practices is the teacher (Godwin & Sutherland, 2004; Ruthven, Deaney, & Hennessy, 2009). An underlying assumption of this study is that our understanding of the role of the teacher in supporting learning with computing technologies is underdeveloped.

The need to understand the relationship between pedagogy and student learning with technology was identified in the early 1990s by Hoyles and Noss (1992) as they observed "the inescapable and perhaps unpalatable fact that simply by interacting with an environment, children are unlikely to come to appreciate the mathematics which lies behind its pedagogical intent" (p. 31); they also noted the sparseness of research that addresses the nature of pedagogies that can support student learning with computer environments. More recently, Ruthven and colleagues have noted that the teaching practices associated with the widespread use of graphing technology have received relatively little attention from researchers (Ruthven et al., 2009). Ruthven et al. argue for the development of teachers' craft knowledge to support their classroom use of technology. This perspective is in contrast to a less situated approach to teachers' knowledge that is characterized by the TPACK (technological pedagogical content knowledge) construct (Mishra & Koehler, 2006; Neiss, 2005).

The larger goal of this study is to contribute to the development of a model of teaching practices that support student learning with exploratory computer simulations. To that end, we investigated the teaching in a pre-college classroom setting where the students used a computer simulation to study of the average rate of change, a traditionally difficult, yet conceptually rich and foundational topic in mathematics. Our study was guided by the following question: what was the nature of the teaching practices that supported students' learning of average rate of change when using an exploratory computer simulation?

Theoretical Background

Much recent work on the relationship between teaching practices and technology has drawn on the TPACK model, often examining the preparation of teachers or the professional development of in-service teachers (e.g., Bowers & Stephen, 2011; Neiss, 2005). However, Graham (2011) and others have criticized the TPACK model for lacking clear theoretical distinctions between the elements of the model, a lack of precision in definitions, and difficulties in discriminating between the proposed constructs of

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"technological content knowledge" and "technological knowledge." The fuzziness at the boundaries of the TPACK model may call into question the existence of the proposed constructs or it may simply point to the need for empirical work on teaching practices that can inform revisions and clarity within the model. Our purpose in this paper is not to critique the TPACK model, but rather to study teaching practices to better understand the role of the teacher when using computer technology, in particular an exploratory computer simulation. As Hoyles and Noss suggested in 1992, such a pedagogy would include introducing a mathematical agenda, a progressive sequence of computer tasks, related paper-and-pencil work and class discussions of computer-based work, and small group activities to bring together computer and non-computer work. Ruthven and colleagues (2009) argue that, when using graphing software, the teacher plays a fundamental role in making the mathematical relationships meaningful for students by supporting the mathematical interpretation of the technology-based representations. Our goal in this study is to contribute to a clearer understanding of the nature of teaching practices with computer technology, particularly as students come to understand the concept of average rate of change.

Over the last twenty years, researchers have documented the difficulties that students encounter in learning to interpret models of changing phenomena (Carlson et al., 2002; Thompson, 1994). In this paper, we draw on a modeling approach to student learning that Kaiser and Sriraman (2006) identify as a "contextual modelling" perspective. This perspective emphasizes the design of activities that motivate students to develop the mathematics needed to make sense of meaningful situations. Much work done within this perspective draws on model eliciting activities developed by Lesh and colleagues (e.g., Lesh & Zawojewski, 2007). Such activities confront the student with the need to develop a model that can be used to describe, explain or predict the behavior of familiar or meaningful situations. Considerably less research has focused on model exploration activities, where students explore the mathematical characteristics of the model. In this paper, we focus on a set of model exploration activities using a computer simulation environment, accompanied by student presentations and teacher-led discussions that focused on the underlying structure of the model, on the strengths of various representations, and on ways of using representations productively. Thus, for this study, we designed an instructional sequence that began with a modeling activity to elicit the construct of average rate of change, followed by model exploration tasks that examined the underlying mathematical structure and its representations. The focus of this study is on the role of the teacher in facilitating student presentations and leading class discussions that support students' understandings of how to represent the average rate of change.

Research Design and Methodology

This study used design-based research as an approach to studying teaching and learning as it occurs within the complexity of a naturalistic classroom setting (Cobb et al., 2003). This approach is intended to generate principles of practice, in this case related to teaching with computer simulations. We draw on the multi-tiered design experiment (Lesh & Kelly, 2000), which provides a framework for collecting and interpreting data at the researcher level, the teacher level and the student level. Central to our analytic approach is the notion that, as researchers, we examine the teacher's actions in the classroom and her interpretations of those actions, which are in turn influenced by the students' interactions with the tasks in the simulation environment. The researchers and the teacher (the third author) collaboratively developed the tasks that were designed to support students in understanding the concept of average rate of change.

Simulation Environment and Task Design

We began the instructional sequence with a model-eliciting activity, using the physical situation of motion along a straight line. Students created graphs using their own bodily motion and a motion detector and wrote verbal descriptions of that motion. This included comparative situations of faster and slower constant speed, changing speed and changing direction. Following the model-eliciting activity, the students engaged in a sequence of model exploration tasks. These tasks were designed to help students to think about the underlying structure of the model of constant and non-constant motion. An important goal of these tasks was to engage students in using informal and formal language to describe the average rate of change and to develop their understanding of the representational systems for describing change. As argued earlier, this brings with it a concomitant role for the teacher in using instructional strategies that will support students in interpreting the mathematical relationships intended in the tasks and instantiated in the computer environment.

The model exploration tasks used SimCalc Mathworlds (Kaput & Roschelle, 1996). This computer simulation environment was designed around the context of one-dimensional motion to explore the relationship among position, velocity and acceleration, the connections between variable rates and accumulation, and an understanding of mean values. The drag-and-drop environment makes use of piecewise linear functions to create position or velocity graphs; these graphs drive the one-dimensional motion of cartoon-like characters in the linked WalkingWorld. The MathWorlds environment reversed and extended the representational space of the model-eliciting activity with the motion detector where bodily motion created a position graph; in the simulation environment, the students created velocity graphs that generated the cybernetic motion of a character. From the simulated motion, the students created position graphs, thus developing an understanding of how the position graph could be constructed by calculating the area between the velocity graph and the *x*-axis. In exploring this linked relationship among the characters' motion, the velocity graph and the position graph, students began to reason about the position of characters solely from information about the velocity of the characters. This model exploration task provided an opportunity for students to develop their abilities to interpret position information from a velocity graph and velocity information from a position graph. Subsequent model exploration tasks introduced the concepts of average velocity, negative velocities, linearly increasing and decreasing velocities and their associated position graphs.

Context and Participants

The sequence of model exploration tasks was part of a larger set of modeling tasks that formed the basis for a six-week course for students who were preparing to enter their university studies. The teacher and the first author collaborated in the development of the entire set of tasks for the course. The teacher had three years of experience teaching secondary and college students; this was her second year teaching the summer course. There were 17 students in the course all of whom volunteered to participate in the study. Three of the participants were female and 14 were male. All participants had completed four years of study of high school mathematics; 11 students had studied calculus in high school and six had not studied any calculus. The model exploration tasks were done individually at a computer; however, the participants were encouraged to discuss their work with each other. Following each task in the sequence, there was a whole-class discussion that usually involved students in presenting the results of the work produced during the model exploration tasks. The class discussion following these tasks focused on the mathematical structure of the model and on the relationships among different representational systems.

Data Collection and Analysis

Consistent with the methodology of multi-tiered design experiments, data for this study were collected at two levels: the level of the teacher and the level of the students. The data sources at the teacher level included videotapes of all class sessions, written field notes and memos, class materials such as worksheets and a record of board work, the teacher's lesson plans and annotations made by the teacher during the lesson. Following each lesson, there was a debriefing session with the teacher, which captured the teacher's reflections on the lesson and any changes to the plans for subsequent lessons. These debriefing sessions were audio-taped and transcribed. The model exploration activities with the simulation world took place over a total of six lessons; each lesson lasted one hour and 50 minutes. Central to our analytic approach is the notion that as researchers we examine the teacher's descriptions, interpretations, and analyses of artifacts of practice that were developed, examined and refined during our collaborative work on the design and teaching of these six lessons. In this paper, we only report on the analysis of the teacher level data, although we acknowledge that this analysis was influenced by the data at the student level.

The analysis of the data took place in three phases. Consistent with the iterative approach of design-based research, the first phase of analysis took place during the six weeks of teaching. In this phase, the research team met with the teacher and regularly engaged in discussion about the model exploration tasks, the progress of the class as a whole, and our observations about students' thinking about average rate of change and their language for expressing their ideas. Analytic memos were written by members of the research team to document their emerging understandings of the teaching practices and observations about student learning.

In the second phase of the analysis, the research team viewed the videotapes and wrote a detailed script of each lesson, identifying the nature of the teacher's activity in each segment of the lesson and its time-stamp and duration. Following the principles of grounded theory (Strauss & Corbin, 1998), preliminary codes were developed to categorize what the teacher did in the classroom. Drawing on this analysis, the research team identified a set of approximately six to eight video segments within each lesson that captured recurrent themes and for which we wanted the teacher's retrospective perspectives and interpretations. These video segments were the basis for video stimulated recall with the teacher and gave further insights into the teaching practices from the perspective of the teacher. This in turn led to further refinement of the coding scheme. In the third phase of the analysis, we coded the videotapes of the six lessons using the revised coding scheme. As we analyzed the teaching practices, we sought confirming and disconfirming evidence in the teacher's lesson plans and annotations during the lesson, and with the teacher's perspective on the lesson from the de-briefing interviews and the post lesson video stimulated recall. This led to the formulation of the results in three broad categories: (1) pressing students for representations; (2) harvesting student ideas; and (3) sorting out and refining student ideas. In this paper, we report on the results in the first category: pressing students for representations.

Results

A representational press occurs when the teacher applies pressure on students for the purpose of furthering the students' emerging understandings of the representations of average rate of change, which in this case occurred within the computer simulation environment and in students' related work. This related work could be any one of the forms of the following representations: language (both written and spoken); table; symbolic (such as function notation and algebraic expressions); iconic or graphical; and enactments (either cybernetically in the simulation world or bodily in the real physical world). We found three categories of representational presses that the teacher engaged in: (1) explicitly inserting a representation into the discussion to support connections to other representations; (2) pressing the students to give interpretations of their representations in terms of the context of the task, while articulating arguments that justify their interpretations; and (3) pressing students to use representations to clarify a situation or question. Due to space limitations, we report here only on the second and third categories.

Interpreting Representations

In this episode, we illustrate how the teacher pressed the students to give interpretations of their graphs in terms of the context of the task while articulating arguments that would justify their interpretations. This episode occurred in the fourth day in the sequence of the six lessons. The teacher led a whole class discussion about the characteristics of three different linear velocity graphs and their corresponding position graphs, which had been the focus of the tasks with the simulation environment. The three velocity graphs are shown in Figure 1 and their corresponding position graphs are shown in Figure 2.

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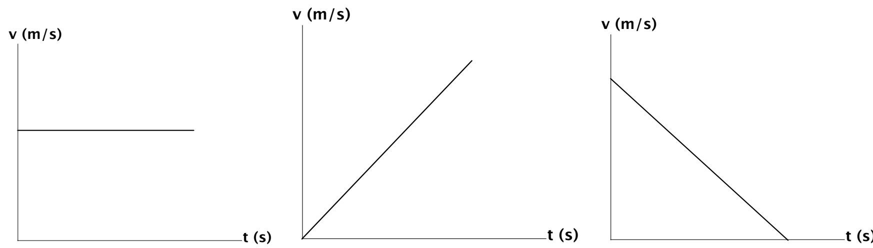
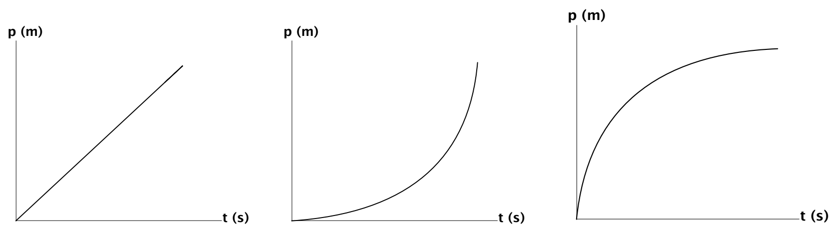


(a) (b) (c)

Figure 1: Comparing three velocity graphs from the simulation environment



(a) (b) (c)

Figure 2: Comparing the corresponding position graphs

During the whole class discussion, the teacher labeled the graphs on the blackboard with the students' verbal interpretations of the graphs. Graph (a) was described as constant velocity and constant speed; graph (b) was described as increasing velocity, increasing speed, and acceleration; and graph (c) was described as decreasing velocity, decreasing speed, and acceleration. The position graphs shown in Figure 2 were interpreted as: (a) linear, increasing position; (b) curved, accelerating, increasing position, "walk slow then fast"; and (c) accelerating, increasing position, "walk fast then slow." In the following excerpt from the class discussion, the teacher focused students' attention on the velocity graph (c) in Figure 1. In this exchange, we see the teacher pressing students (1) for the use of appropriate language to describe the graph, (2) for making connections between cybernetic and physical enactments, and (3) for understanding the meaning of the relationship between a constant or linearly changing velocity graph and its associated position graph.

1 *Tchr:* How would you describe this motion here [graph (c) in Figure 1]?
2 *Chris:* Uhmm, it's deceleration [inaudible]
3 *Tchr:* Okay, so we also have acceleration here, okay, uhmm, because why?
4 [Several students talking]
5 *Tchr:* Because why?
6 *Chris*: Umm, as the…because the velocity is changing
7 *Tchr:* Um, how would you have to walk? If you were trying to match that graph from the third day we did *Hiker* [an earlier activity]? You're holding the motion detector. How would you tell the person to walk?
8 *Quent:* For which one? [Teacher points to graph (c) in Figure 1]
9 *Vic:* You tell him to walk away from the censor;
10 *Quent:* Real fast
11 *Tchr:* Real fast
12 *Vic:* And then slowing down
13 *Tchr:* And then slow… Okay.

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This episode began with the teacher pressing the students to interpret the decreasing velocity graph from the simulation environment and to verbalize deceleration as changing velocity. In turn 7, the teacher pressed for a description of this changing velocity in terms of enacted physical motion. She invited the students to describe an enactment of the motion in terms of a device (the motion detector) that could measure and record the physical motion of a person walking. In this way, the teacher engaged the students in generating verbal descriptions of simulated motion that were explicitly connected to physical motions that the students had experienced earlier.

Using Representations to Clarify Situations

In this episode, we illustrate how the teacher pressed a student to insert a representation into an argument so as to support and clarify his reasoning about a specific situation. The teacher had posed the following question to the class for homework: "If two people take a walk and end together, have the same velocity throughout the walk, then both must have walked for the same amount of time. True or false?" This task was designed with some intentional ambiguity around what it means to "have the same velocity" that the students would need to resolve in answering the question. In the class discussion the next day, the teacher polled the students and made public the result of the poll: all of the students, except one, thought that the claim in the posed question was true. The teacher decided to hear about the false argument:

1 *Vic:* It says take a walk. It doesn't say that they started the same time, so one [person] can have already been going at… that for a while so… they could have…. at the same time so… Let's say [inaudible] one's going somewhat faster and the other one could be going somewhat slower, but the slower one started earlier… so they end together, at the same place at the same time… but… this does not seem, I mean… they had their own velocities, uh for the walk… that is to say that, they both had the exact same velocities.
2 *Tchr:* Is this bouncing off of Vic or new idea? [to Jorge who is holding up his hand]
3 *Jorge:* I have a new idea. Uh, it says that they "have the same velocity". If they didn't have the same velocity and one person was already ahead of the other then they would never end up at the same time.
4 *Tchr:* Uh huh
5 *Jorge:* Like if two people are walking at 4 meters per second – how are they gonna end up at the same place in the same amount of time if one already started walking.
6 *Tchr:* So what do you take "same" to mean?
7 *Jorge:* That… basically two people are walking at the same time, and one walks for a longer dist[ance], for a longer amount of time, then he'll walk more distance.
8 *Tchr:* Okay.
9 *Vic:* Um
10 *Tchr:* [To *Vic*] Do you have a rebuttle to that?
11 *Vic:* Uh huh
12 *Tchr:* You want to argue with that?
13 *Vic:* Yes, um, that's still not taking into account that someone could have already been ahead of the other [person]. But going into, the velocity, um, but it's still, making the velocity constant. It isn't saying that it has, that is, that it has to have the exact same velocity. It says "have the same velocity throughout the walk." That could mean anything. That could even just mean constant velocity.

In the first turn, Vic offers the argument that the "same" velocity means that the walkers each had their own "same" constant velocity throughout the walk. But, in turn 5, Jorge makes clear that he has interpreted "same" velocity to mean the same as each other: both are walking at a constant velocity of "4 meters per second." In turn 6, the teacher acknowledges the ambiguity of the meaning of the "same" and in turn 10 invites Vic to further his argument.

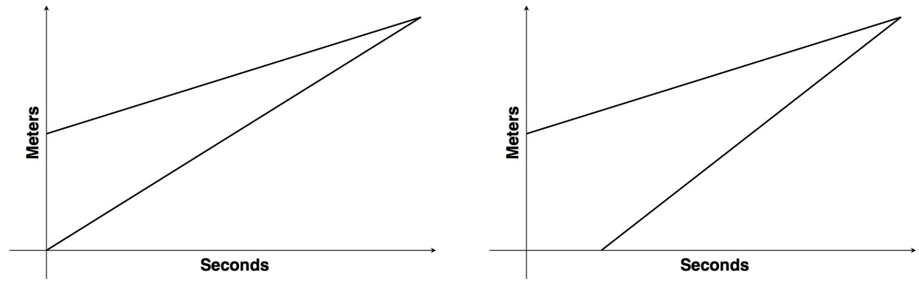

Figure 3: Representing the "same velocity" with different times

After checking with the students in the class for their understanding of Vic's argument the teacher asked Vic: "Do you think that you can demonstrate what you are talking about?," a suggestion Vic quickly takes up; he goes to the blackboard and draws the graph shown on the left in Figure 3. This graph shows "the slower one" (as Vic expressed in turn 1) starting behind the other walker in terms of position (as expressed in turn 13), but both walkers walk the same amount of time and hence this is not a counterargument to the original claim. As Vic elaborates his thinking, he correctly revises his graph to the one shown on the right in Figure 3, which shows the slow walker being ahead of the fast walker, but the walkers walk for different amounts of time, an argument that convinces many students that the original claim is false. The teacher had not (and could not) fully anticipate all of the students' arguments and pressing for representations was helpful to her in understanding the complexity of the students' arguments.

Discussion and Conclusions

Students' difficulties in learning to interpret rates of change, particularly in the context of one-dimensional motion, are well known in the research literature. Computing technology would seem to hold great potential for helping students to understand this rich and yet challenging concept. However, the relationship between pedagogy and student learning with technology is still an area in need of research (Hoyles & Noss, 1992; Ruthven et al., 2009). The computer technology provided a flexible way for students to represent their ideas and to manipulate them. As students engaged with the tasks in the environment, and the related non-computer tasks where they had to interpret the meaning of graphs and give verbal descriptions or arguments justifying their representation, more student ideas were generated and conflicts among interpretations arose that needed to be resolved by mathematical reasoning. The technology also provided a common frame of reference for small group conversations and whole class discussions. However, as Hoyles and Noss (1992) warned, one cannot assume that the students fully understand the representations in the computing environment. The generation of student ideas and the need for students to interpret and give meaning to the representations in the computer environment place new demands on the craft knowledge of the teacher. In this study, we found the emergence of a teaching practice that responded to these new demands, namely pressing for representations. Through this practice, the teacher pressed the students to articulate the connections among representations, to make interpretations of their representations while giving arguments to justify their interpretations, and to use representations to clarify situations and resolve questions.

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