TEACHERS' ATTRIBUTIONS FOR STUDENTS' MATHEMATICAL WORK

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This paper reports on a design experiment within a professional development context purposefully planned to teach teachers about students' mathematics thinking and learning. We examine the factors to which participating elementary teachers attributed student mathematics success or failure when engaging with the projects' professional learning tasks.

Statement of the Problem

In his attempt to explain how people think, Schoenfeld (2011) put forth the following claim: "People's decision making in well practiced, knowledge-intensive domains can be fully characterized as a function of their orientations, resources, and goals" (p. 182). Defining orientations as including a myriad of concepts such as dispositions, beliefs, values, tastes, and preferences, Schoenfeld explained that orientations shape what we perceive, the meaning we make of what we see as relevant, the goals we establish in a particular situation, and the resources we bring to bear to achieve those goals. Further, he claimed that in mathematics classrooms, teachers' actions were shaped by their orientations toward mathematics, students, learning and teaching.

Using the broad definition of orientation that Schoenfeld put forth, our study examines elementary teachers' orientations toward students' mathematics. More specifically, we attend to teachers' attribution as one aspect of orientation and examine the following question: *to what factors do elementary teachers' attribute students' mathematical work when working on professional learning tasks designed to teach them about students' mathematics thinking and learning?* The project consisted of a design experiment within a professional development setting purposefully planned to teach teachers' about students' mathematical thinking and learning. This design experiment allowed for the study of changes over time in teachers' attributions for students' mathematical successes and failures. The initial conjecture under investigation stated that learning about students' mathematical thinking would add a new attribution to teachers' repertoire, thus changing the array of attributions available for teachers to use as they examined student work within the professional learning tasks used in the professional development. As a first step in the investigation of this conjecture, the various factors teachers used in the professional development to attribute students' successes or failures were documented.

We begin this paper by briefly reviewing the literature that defines the theoretical framework of our study. Then, we present our research methodology, describing the professional development setting in which we work. Next, we define the attribution factors we observed in our professional development and share examples of how these attributions were present in our work with elementary teachers. We conclude with a set of next steps for our research.

Framework

Thompson, Phillip, Thompson, and Boyd (1994) first used the concept of orientation to describe what they called a calculational and a conceptual approach to teaching mathematics. The authors incorporated teachers' knowledge, beliefs and values within the concept of orientation and, much like Schoenfeld (2011), proposed that these orientations shape teachers' images, views, intentions, and goals for mathematics instruction. Magnusson, Krajcik, and Borko (1999) included orientation as a component of teachers' pedagogical content knowledge. They considered that teachers' orientation influenced instructional practice by shaping teachers' knowledge and beliefs about curriculum, students, teaching, and assessment. Phillip (2007) noted teachers' orientations were operationalized through attention to teachers' language and actions.

When analysing teachers' language in a professional development organized around student work, Kazemi and Franke (2004) reported a shift in the ways teachers attended to the details of student mathematical thinking. They explained that in the initial meetings of the professional development group, teachers focused their analysis of student work on students' mistakes, could not provide detailed explanations on how students completed the problem posed to them, and were surprised that the problem posed was difficult for the students. However overtime, teachers conversations became more detailed regarding the work their students were doing and teachers able to note various levels of sophistication in students' mathematics reasoning.

Sowder (2007) cited various examples of professional development projects in mathematics that used student thinking to promote teacher learning and noted that these projects often provided teachers with opportunities to examine student work. As indicated in Little (1999), the sustained and systematic study of student work provides one of the most powerful and least expensive opportunities for teacher learning.

When working with teachers in professional development that offered opportunities to examine student work, we observed that an important aspect of teachers' orientation toward students was the ways in which they talked about students' successes or failures in completing the mathematics tasks under examination. The attribution aspect of teachers' discourse turned our attention to attribution theories as one facet of teachers' language when examining student work.

Bar-Tal (1978) defined attributions as the inferences made about the causes of one's own or someone else's behaviours. Weiner (1985) noted that attributions were classified in relation to its locus of causality (internal or external) as well as stability (fixed or not) and controllability (who can change it). Classification of attribution along these dimensions usually leads to the examination of ability, effort, luck or the difficulty of the task as the causes for one's successes or failure.

Middleton (1999) noted that teachers' attributions of their students' successes and failures were reflected in the ways teachers interacted with their students during mathematics instruction. Examining pre-school settings, Dobbs and Arnold (2009) claimed that teacher's attributions of the students' behavior shaped the teacher's behavior toward the child, which in turn often elicited the expected behavior from the child, having a self-fulfilling prophecy effect.

Because our work is in professional development settings, we extend the discussion of the role of teachers' orientations and attributions in instruction to professional development settings. We consider that teachers' orientations toward students' mathematics play a fundamental role in teachers' engagement with professional learning tasks, with teachers' attributions of students' successes and failures shaping professional conversations around student work used in these learning tasks.

Methods

Professional Development

Our work with teachers is based on the concept of learning trajectories (LTs). When Simon (1995) coined the expression "hypothetical learning trajectory," he indicated that teachers create representations of the "paths by which learning might proceed" (p. 135) when students progress from their own starting points toward an intended learning goal. He named these trajectories hypothetical because each student individual learning path was not knowable in advance. However, he suggested that these learning paths represented expected tendencies and that commonalities across students allowed teachers to develop expectations about how learning might proceed.

Over time, the concept of LTs has developed to go beyond the notion that teachers have expectations about how learning might proceed to include an empirical search for the highly probable sets of levels through which students progress as their learning of specific mathematics topics evolve. Thus, current work on LTs uses research on student learning from clinical interviews and large-scale assessment trials to seek clarification of the intermediate steps students take as learning proceeds from informal conjectures into sophisticated mathematics.

Recently, research on LTs has progressed from an agenda for studying student learning to an agenda for research on teaching. Daro, Mosher, and Corcoran (2011) called for the translation of LTs into "usable tools for teachers" (p. 57). They indicated the need to make these trajectories available to teachers so that they can guide classroom instruction.

Research Design

The overarching purpose of our research is to understand the ways in which teachers come to learn about one particular LT as a representation of students' mathematics in the context of a professional development setting. Inasmuch, we examine both teacher learning and the set of professional learning tasks that support their learning experiences. As we teach teachers' about students' mathematics through the concept of LTs, teachers' orientations toward students shape the ways in which teachers engage with the professional learning tasks proposed to them, with teachers' attributions playing an important role in their discourse.

We use a design experiment methodology within a school-based professional development setting to accomplish our research goals. Design experiments are "iterative, situated, and theory-based attempts simultaneously to understand and improve education processes" (diSessa & Cobb, 2004, p. 80). They are used to develop "a class of theories about both the process of learning and the means that are designed to support that learning" and they "entail both 'engineering' particular forms of learning and systematically studying those forms of learning within the context defined by the means of supporting them" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003, p. 9).

In order to analyze the data, we engaged in a grounded theory approach to data analysis (Strauss & Corbin, 1989). In doing so, we coded our data (field notes and group discussion transcripts) using open coding, which enabled us to create concepts from raw data. In addition to creating data-driven codes, we also used theory and research goals to help create several of the codes. Once all of the codes were created, we then engaged in axial coding in order to make connections between the initial codes. This allowed us to create larger categories or themes (see results section for categories). In line with the grounded theory approach to data analysis, we used the constant comparison method in that we were comparing various project data sources including field notes and group discussion transcripts as well as the research literature (Glaser & Strauss, 1967). The constant comparison method allows for the creation of emerging categories in the data analysis and the refinement of these categories as they are contrasted with new project data. Various sources of data are used for the ongoing analysis and for triangulating information (Miles & Huberman, 1994) in search of both confirming and disconfirming evidences.

Context and Participants

The professional development comprised of both a summer institute and academic-year monthly meetings. These two components of the intervention were designed with different goals in mind. The summer institute offered teachers opportunities to learn about the LT and develop an appreciation for the role of the trajectory in understanding student mathematics. In contrast, the academic-year monthly meetings focused on establishing connections between the trajectory and instructional practices. The two components of the professional development totalled 60 hours of face-to-face, whole group interactions over one school year.

The professional development was offered in partnership with one elementary school in a mid-size urban area in the southeast of the United States. The school had approximately 600 students, 35% Caucasian, 29% Hispanic, 25% African American, 7% Asian, and 4% other; 54% of the children qualified for free or reduced lunch. Teachers at the school volunteered to participate and all professional development meetings were conducted at the school, in times selected based on convenience to the teachers. Of the 24 teachers who started the professional development in July 2010, 21 completed the program one year later in June 2011. The initial group of teachers included six Kindergarten teachers, three

Grade 1, five Grade 2, three Grade 3, two Grade 4, and one Grade 5 teacher. Four teachers taught multiple grade levels.

Results

Teachers' attributions of students' mathematics emerged very early on in the professional development, calling our attention to its importance for our work. Here, we offer one example from a professional learning task posed to teachers in the beginning of the professional development to demonstrate how teachers' attributions shaped discourse in the professional development setting.

The task we are using as an example engaged teachers in watching videos of clinical interviews with students from different grade levels solving similar mathematical problems. Teachers were asked to describe the ways in which each child solved the problem, conjecture about each student reasoning for that particular solution, consider the sophistication of the various strategies, and examine what surprised them about each student work. In the discussion that followed, despite the facilitator's effort to focus the discussion on what each child did and why, teachers' discourse focused mostly on whether what each child did aligned (or not) with what teachers' thought a student at that grade level was expected to do. That is, they attributed what the children did to grade level. The information about each child's grade level, offered to teachers as part of the context for the clinical interviews, became the center of the discussion as if grade level defined for the teachers what a child could or could not do mathematically. Thus, in the case of this particular task, teachers' attributions for students' work shaped the discussion around the professional learning task.

Through the examination of teachers' discourse when asked to engage with a collection of professional learning tasks, we documented the various attributions teachers' brought forth. In what follows, we present each attribution, a short working definition for it, and two or three quotes that exemplify how the attribution was represented in our data.

1. *Ability*: Considers personal traits of students and characteristics that define the student as fixed qualities related to students' aptitude in mathematics. Often times, teachers use achievement to consider students' abilities, attributing students' performance to an innate capacity.

   "We had evaluated this student and we were convinced there was a learning disability. The work was really low. But we were working on tangrams and this student put the 7 shapes into a square; he did immediately, first one to have done it and did it quickly."

   "I had a lot of math genius and they can figure things out when they are so young."

2. *Effort*: Refers to the level of student attention and engagement with a particular task at a particular moment. It indicates that performance does not always represent a fixed characteristic of the student, but depends on how carefully or how speedy that particular student progressed through the work at a particular moment.

   "Well, he just zipped through all this, so, no wonder…"

   "He worked on this so carefully."

   "In my mind, this kid just wasn't paying attention to me while I was teaching and he played connect the dots."

3. *Luck*: Includes the idea that what students do has no intentionality behind it. Also implies that students do things that have no real explanation for what or why they did something, or knew what they were doing.

   "I thought she was just guessing and she was just lucky."

   "When questioned how did you know, that is when I realized she really randomly chose to give each one two pieces. It was not that she had the number fact or she understood."

4. *Difficulty of task*: Expresses the notion that what students do is determined by the clarity or lack of clarity of the question posed to them. Has an embedded idea that there is a perfect way to ask a question so that students would not make a mistake.

   "The proctor asked her to put things together and then divide them, so, she shared differently because the proctor asked a different question."

   "When we teach a group of students and over half of them make the same mistake, then we have to go back and look at the way we presented it and ask ourselves…is it some fault in the way the question was presented?"

5. *Grade level*: Includes the notion of development and the expectations teachers have for students' performance given normalized definitions for what the generic student should be able to do at certain point in his or her development. Indicates that grade level groups students at similar developmental levels.

   "I taught Kindergarten and I would have guessed she would share using one for you, one for you, one for you; what she did was more advanced because she counted two plus two plus two."

   "I had expected the third grader to not share dealing it one by one."

6. *Cultural context*: Indicates that teachers take into account the experiences students bring with them from their own lives. Includes outside school understandings and explanations that students generalize to the academic context.

   "She just shared and she thought "it is fair because we each got some", and that is because of how we use the word share in the real world. She thought we both have some so we have shared."

   "I think that was a problem for a lot of these kids, dishing out the whole birthday cake (to fair share it). I just wonder if you called it something else besides a birthday cake if they would have seen the whole differently."

7. *Teaching*: Implies that what students do depends on what teachers have presented to them. Depending on whether or not a teacher has already taught a particular topic to the students, teachers expect students to know a topic taught. On the other hand, it indicates that teachers consider that students have no way of knowing a topic not yet taught.

   "Sometimes students can say something even when we had not taught it, like, this is ½ of 10 so that part has to be 5 as well. It seems simplistic, but I don't know how they would have known that already."

   "It used to be that students would do what teachers taught and we would follow it. But now students generate their own ideas and can do it in a way that is different from my own. They know how to come with the right answer by themselves."

Next Steps

In this paper, we documented seven different factors brought forth in the context of our professional development as teachers attributed students' mathematics successes or failures when examining student work. These attributions go beyond ability, effort, luck and difficulty of tasks to also include grade level, cultural context and teaching. They represent teachers' orientation toward students, and indicate the knowledge, dispositions, beliefs, and values teachers activated to examine student work in the context of our professional learning task.

In continuing our research, our conjecture is that the array of attributions available for teachers examining student work will change as teachers learn about student mathematics represented by LTs. Thus, we will examine whether our professional development on LTs added a new attribution to teachers' repertoire, one that includes recognition of students' mathematics successes and failures in relation to the level represented in LTs. This attribution recognizes that students' mathematics requires interactions between internal and external factors such as previous knowledge and opportunities to learn. Further, this attribution is not fixed and both students and teachers are responsible for changing it. We also conjecture that as the professional development unfolds and teachers come to better understand LTs, they will use the learning trajectory attribution more often. Examining these conjectures are the next step in the development of our work.

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