TEACHERS' NOTICING OF CHILDREN'S UNDERSTANDING OF LINEAR MEASUREMENT

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This article is a report on the findings of a case study that focuses on a first grade teacher's noticing of children's understanding of linear measurement along a learning trajectory, extending Jacobs and her colleagues' framework (Jacobs, Lamb, & Philipp, 2010). It documents what the teacher noticed in terms of attending to and interpreting student strategies in four different contexts during her participation in a lesson study. The findings indicate that the teacher was overall more successful in attending to student strategies than interpreting mathematical understanding reflected in the strategies when she used a learning trajectory as a tool to notice student understanding. More interestingly, we found that her level of noticing differed depending of the role that she took in the process of lesson study.

Keywords*:* Learning Trajectories (or Progressions); Teacher Education–Inservice/Professional Development; Teacher Knowledge)

Understanding children's mathematical thinking is one of the key factors for teachers to provide effective instruction. More specifically, teachers' knowledge about how children's thinking progresses over time and what conceptual mile stones indicate is critical to support children's mathematical learning. The National Research Council (NRC, 2001) asserted, "Familiarity with the trajectories along which fundamental mathematical ideas develop is crucial if a teacher is to promote students' movement along those trajectories" (p. 370). Many research studies (e.g., Cobb et al., 1991; Confrey, Mojica, & Wilson, 2009; Gearhart & Saxe, 2004; Schifter, 1998, 2001) investigated teachers' instruction that builds on children's mathematical thinking and its progression in the domain of numbers and operations, and some studies reported improvement in student learning by building on children's thinking (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema et al., 1996; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Villaseñor & Kepner, 1993).

Several groups of researchers (Barrett & Clements, 2003; Barrett et al., 2012; Sarama & Clements, 2009) have documented children's thinking and learning trajectories in the domain of measurement, more specifically linear measurement. A study by Barrett and his colleagues (Barrett, Jones, Thornton, & Dickson, 2003) discussed benefits of instruction when teachers design tasks and questions that recognize where students are in the learning trajectory and help move children to a more sophisticated level. However, more research studies are needed to document how teachers make sense of a learning trajectory and how it may impact their teaching. This initial study may help to fill this gap through a case study of a first grade teacher who participated in a professional development program that focused on knowledge of children's thinking about linear measurement through the use of a learning trajectory. The teacher was supported in her effort to put the knowledge into practice through a lesson study.

To capture how teachers use their knowledge of a learning trajectory of linear measurement in practice, we used *noticing* (Mason, 2002) as a main framework. Noticing allows us to highlight the nature of knowledge that teachers need to actively respond to complex and challenging environment in practice. Teachers may have knowledge on children's thinking, but if it is not active they may not notice it in practice, which in turn will result in difficulty in taking appropriate instructional actions to improve it. In mathematics education, Jacobs and her colleagues (Jacobs, Lamb, & Philipp, 2010) recently studied what teachers notice in terms of children's mathematical thinking in the domain of whole numbers and operations with a goal of unpacking teachers' in-the-moment decision making. They defined the

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professional noticing as a set of skills including how they attend to children's strategies, how they interpret mathematical understanding reflected in the strategies, and what decisions teachers make to respond to the understanding in the strategies. Jacobs et al. analyzed what teachers noticed in children's strategies presented in a video clip and also a collection of students' written work during their professional development activities. They then compared teacher noticing with different levels of teaching and professional development experiences. They concluded that teachers' noticing expertise grew with teaching and professional development experiences, which indicate that this expertise can be learned and supported through professional development. Although Jacobs' study provides a framework on how to analyze teachers' noticing of children's thinking, the question of how teachers' noticing in a professional development context is related to teaching context still remains unanswered. In this study, we aimed to extend Jacobs' study to an actual classroom, and to put a step closer to teachers' in-the-moment decision making.

This study examines teachers' noticing in the context of lesson study as a part of a professional development program. The context of lesson study allows teachers to develop *knowledge-in-practice*, which Cochran-Smith and Lytle (1999) described as the practical knowledge of teaching "embedded in practice and in teachers' reflections on practice" (p. 250). Sowder (2007) discussed lesson study as an example of learning-in-practice because "in lesson study teachers deliberate on the practices they observe with others." Fernandez and Yoshida (2004) described lesson study as a well-defined common practice in Japanese schools. This process involves three processes and three additional processes that some groups follow. The first three processes are for teachers and professionals in the community to collaboratively plan a study lesson, to observe the lesson study in action, and to discuss the lesson. Sowder (2007) pointed out that the first two steps are not new to U.S. teachers, although they rarely involve other teachers, but the last step is uncommon. Ball (2002) and Lewis (2000) discussed that these processes allow teachers to attend to and learn what each child understands, organize instructional tasks based on mathematics, and make adjustments as needed. Three additional processes that are optional are to revise the lesson, to teach the new version of the lesson, and to share reflections about the new version of the lesson. Our study situates teacher's noticing of children's understanding of linear measurement concepts along a learning trajectory in the context of all six processes of lesson study.

The purpose of this study is to contribute in making sense of teachers' learning of children's thinking in the domain of linear measurement. More specifically, we aim to examine one teacher's noticing through a case study in the context of a lesson study, which supported teachers' learning and use of a learning trajectory as tool to make sense of children's understanding. Our research question is:

- How do teachers use a learning trajectory as a tool to notice students' measurement understanding in the context of lesson study? In their noticing, how do teachers attend to and interpret students' strategies?

Methods

Participant

Here we report a case study that focuses on one teacher, Ms. Smith, from a larger study involving 24 teachers. At the time of the study, Ms. Smith was teaching first grade with 16 years of teaching experience. Ms. Smith taught at a K–4 elementary school. The school was classified as a Title 1 school, where 34% of the students were qualified for free or reduced lunch, and 59% were minority.

Professional Development

The aim of the larger study was to introduce teachers to a learning trajectory on length measurement and support their use of it in assessing students and designing instructional tasks. All of the participants were from an urban school district in the Midwest. The teachers participated in two summer professional development conferences for a total of ten days. During the first professional development, which lasted six days in June, the teachers were introduced to the Length Learning Trajectory developed by Sarama and Clements (2009). The teachers learned about each level of the trajectory and student understanding at each

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level. They designed assessment tasks using the trajectory and also tested their tasks with children from a local summer program. The second professional development, which lasted four days in August, introduced the concept of lesson study.

The teachers worked together in six groups of four to develop lesson plans. Ms. Smith's lesson study group designed a lesson to develop students' understanding of linear measure focusing on non-standard units. The teachers in Ms. Smith's lesson study group designed a lesson about a postman delivering mail. The students were asked to measure and compare routes on their classroom floor using cutouts of the postman's foot.

Prior to the first instruction of the lesson, each teacher was asked to interview six students of varying abilities from their classroom using length tasks he or she designed or tasks given during the summer professional development. Following the interviews, each group of teachers participated in the processes of teaching or observing the lesson, and discussing the lesson. Based on the discussion, the teachers revised the lesson and iterated the processes four times. In this process, each teacher was asked to re-interview his or her six students following the classroom lesson. The teachers were asked to write reflections for each iteration of the lesson study as well as reflections on pre- and post-student interviews. These reflections prompted the teachers to describe the tasks or lesson posed, discuss student responses and thinking in relation to the learning trajectory, and prescribe future instructional tasks for the students.

Data

In this study, we analyzed video and journal accounts of Ms. Smith's reflections for the second and third iterations of the lesson. Ms. Smith taught the second iteration of the lesson and observed the third iteration. We transcribed the videotapes of the second lesson that Ms. Smith taught and the post lesson discussions of the second and third lessons that she participated in. The journal accounts included her reflections of the lessons and discussions as well as the pre- and post-student interviews. This provided us with four main data sources: Ms. Smith's report of pre-lesson interviews with the six students, her reflection of her own teaching, post-lesson interview with the six students, and Ms. Smith's reflection of third iteration of the lesson taught by another teacher in her group.

Data Analyses

Each of the four main data sources was analyzed with attention to two of the professional-noticing skills from Jacobs, Lamb and Philipp (2010), including *attending* to student strategies and *interpreting* children's understanding. With regard to attending to student strategies, we used two codes of showing or not showing evidence when we analyzed her reflections on pre- and post-student interviews. If she was able to provide mathematically significant details on how a student measured or used tools to measure then it was coded as showing evidence of attending to student strategies. We used three codes, attending to individual student strategies, attending to group strategies, or not showing evidence when we analyzed her reflections on discussions or lessons during the lesson study. This two-tier coding scheme was used because in the interview context, Ms. Smith worked with the students one-on-one, and in the lesson study context she worked with a classroom of students. We decided to use the additional code for the data from the lesson study context to account for the difference in the nature of the contexts.

With regard to interpreting student strategies, we used three codes including robust evidence of interpretation, limited evidence of interpretation, or lack of evidence of interpretation. We used the same set of codes for both contexts. When Ms. Smith made specific comments about her interpretation of mathematics in students' strategies, it was coded as robust evidence. When Ms. Smith made general comments of mathematics in student strategies, it was coded as limited evidence. When Ms. Smith provided little to no comments of mathematics in student strategies, it was coded as lack of evidence. For instance, her comments focusing on other issues within her classroom such as her teaching style, improving teaching, or student behavior were coded as lack of evidence.

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Results

In this section, we share Ms. Smith's noticing of children's understanding of linear measurement from the four different contexts. We describe our observation of her noticing with sample statements from her reflections.

Ms. Smith's Noticing in the Context of Pre-Lesson Student Interviews

In the pre-student interviews, Ms. Smith interviewed six students one-on-one and described their responses to each of the three tasks. After describing the student response to the task, Ms. Smith provided her interpretation of their responses.

Attending to student strategies. Ms. Smith's reflections showed evidence of attention to student strategies for each of the six students. When describing student strategies, she noted how individual students responded to the task with very detailed descriptions of the strategy. She typed up about one-page descriptions of each student. Consider Ms. Smith's following statements that showed evidence of attention to student strategies:

> With the *Length Comparer* activities, she lined up the first two objects and identified them correctly as a big one and a small one. Then she took the five objects and lined them up in correct order but there were not all starting at the same zero point.

In the statements, Ms. Smith captured mathematically important details. Specifically, she included descriptions about how the student compared the length of multiple objects and made a mathematically significant note that the student did not line them up with the same starting point.

Interpreting students' understanding. Ms. Smith exhibited limited evidence of interpretation of students' understanding. Her statements showed her intention of interpretation but they were rather broad and general. The following is an example of showing limited evidence of interpretation:

> In the Indirect Length Tasks, he was able to identify the shorter and taller of two fixed objects…he took the thread and measured the first cabinet and saved his place on the string. When he held it up to the longer cabinet, he said it was longer because it was longer than his arms… I would place student 6 in the Indirect Length activities.

Ms. Smith provided detail descriptions of what the student did to compare the height of two cabinets, but she concluded that the student's strategy would be at level 6 without providing evidence or justifying why she came to the conclusion.

Ms. Smith's Noticing in the Context of Teaching

Attending to children's strategies. When her group met after she taught the postman "Bob" lesson as the second of the four iterations of the lesson, Ms. Smith shared what children's strategies she noticed during the lesson. Unlike her detailed descriptions of individual student's strategies in the context of pre-student interview, Ms. Smith provided description of strategies that she noticed a group of students used:

> Most of them just slid the foot [paper cutout] along counting as they went. Some of them slid it longer than other ones. … Students seem to be at the beginning of the end-to-end trajectory. They were moving their foot [paper cutout] along the street [marked on the floor] and counting as they went. Some were actually putting a finger down to mark their place but most were just moving it in jerky, supposedly iterated movements.

Although she thought that children's strategy of sliding the foot cutout to measure lengths of delivery routes was invalid, Ms. Smith provided a detailed description of the strategy including, the motion that children took, length of the motion, and jerkiness of it. However, she did not discuss which students used the strategy, but rather said "most of them," referring to a large of group of students. We found that in her journal account Ms. Smith also reported her observation of the whole class, instead of individual students. We coded her noticing of students' strategies as attending to group strategies.

Interpreting children's understanding. Ms. Smith demonstrated lack of evidence in interpreting student understanding during the discussion following the teaching and in her written reflection. In both contexts, she focused on student behaviors unrelated to mathematical understanding or aspects of the lesson related to her teaching. The following are examples of Ms. Smith's responses to children's understanding:

> I was pleased with how the lesson flowed. The students were enthusiastic…. In retrospect, I guess I needed to model that a little more thoroughly…. I expected some of the students to use this as a time to play more than focus on the learning part of what they were doing and this is precisely what happened.

Ms. Smith's Noticing in the Context of Post-Lesson Student Interview

Attending to children's strategies. After teaching the lesson, Ms. Smith was also asked to re-interview the six students she initially interviewed and reflect on what they said about how they attempted the task and learned from the lesson. During the second interview, Ms. Smith showed evidence of attending to student strategies for only one student. For the other five Ms. Smith did not comment on how the student attempted the task. Ms. Smith did not reference her findings from the initial interview. Again she wrote about each student individually but this time she only wrote a few sentences and rarely referenced students' mathematical strategies. In this reflection she shifted from making specific comments about students' understanding to commenting about general behavior and teaching and learning. These are several of her comments from her post-interview with students.

> JH said it was fun. She said she had worked as a team with her friend who helped her measure the lines…TD did not iterate. He said he had compared it to driving and counted up that way as he moved his foot.

Interpreting children's understanding. In the post-lesson interviews, Ms. Smith demonstrated lack of evidence of interpreting student understanding. Miss Smith mainly focused on non-mathematical student behavior and she did not try to link student's individual behaviors to the levels in the trajectory following the lesson.

Ms. Smith's Noticing in the Context of Classroom Observer

Attending to children's strategies. As an observer Ms. Smith demonstrated evidence of attention to individual student strategies. Ms. Smith commented during the reflection that she was able to watch several students closely as she followed them around the classroom as they attempted to measure the length of several paths. In this instance, Miss Smith considered individual students within the group and the mathematical strategies that they used to measure a line.

> The team that I followed used their fingers to mark where they needed to move the foot forward from and count. One girl was more accurate with this than others… One of the boys didn't iterate, instead he just moved his foot along and counted… At one point they realized that it did not matter if they started at one end or the other when counting"

Interpreting children's understanding. Following the lesson, Ms. Smith demonstrated robust evidence of interpreting student strategies. Ms. Smith discussed with the group that that she had considered why the students were measuring in different ways and had formed a hypothesis based on student reasoning. She was able to link interpretations to specific student behaviors. In the post-discussion, she reflects on one student's struggles with measuring the path and she attributes this to his understanding of the number line.

> *Ms Smith*: I got the feeling that they were confusing how they were measuring with the foot. The student (that demonstrated) that came up with the incorrect answer was thinking of that first foot placement not as something he would count but he was using that as a starting off point and that is why their answer is less than the other. Instead of saying its one, two, three (moving her hand along a line). He started here, and you know how we teach the first step is one, two, three so he ended up

saying its three…it is kind of how you teach the number line counting to the kids at the start of the year. So, that's one observation I made on that initial thing. Trying to look at what the kids were thinking in their minds.

Discussion

In this case study of Ms. Smith, several themes emerge. First, there is evidence that when Mrs. Smith was introduced to a learning trajectory, it provided her with a language to describe student thinking. The findings indicates that Ms. Smith was able to use the learning trajectory to focus on student strategies, share knowledge about students with other teachers, and reflect on student strategies and responses. Ms. Smith was able to use appropriate mathematical language from the trajectory to communicate her understanding of students. Although Ms. Smith was not always proficient in using the trajectory, in several instances, she was able to correctly link student strategies with the appropriate level in the learning trajectory.

A second finding that emerged was that the lesson study provided a context that allowed the researchers to see differences in Ms. Smith's ability to notice student strategies. During the lesson study process Ms. Smith was able to take on several roles apart from her normal role as classroom teacher. Throughout the lesson study, Ms. Smith's noticing varied depending on the role that she took in the processes. Ms. Smith was more successful in attending to student noticing when she assumed the role of interviewer or observer. It may have been easier for her to observe and record student behavior because her focus was solely on one student at each interview. In the observer role during the third iteration of the lesson study, Ms. Smith focused on a small group of several students, instead of a whole class. Her attention to students' thinking may have been better because she was not responsible for student learning or classroom management. It seemed that this role of the observer allowed her to direct her focus to a few students for the entire class period and pay closer attention to their strategies and responses. When Ms. Smith taught the lesson, she did not attend to student strategies as well as in other contexts. This could be because the complexity of a classroom environment made it difficult for Ms. Smith to notice details of students' strategies or recall them in reflection.

Third, there seems to be connection between attending to student strategies and interpreting student understanding. When Ms. Smith provided more clear evidence of individual student strategies, she was more successful at interpreting student strategies. When Ms. Smith was able to attend to individual student strategies in the assumed role of interviewer or observer, she was able to interpret mathematics reflected in the strategies. We wonder if her close attention to individual strategies allowed an access to more concrete examples, which in turn helped her interpretations of student thinking. When Ms. Smith taught, she had difficulty attending to individual student strategies. In that context, she provided limited interpretation of student thinking and instead the focus was on her teaching or children's non-mathematical responses.

Lastly, we note the challenge of prompting teachers to use a learning trajectory as a longitudinal tool to assess children's progression over time. In the initial interview, Ms. Smith was able to use the trajectory to evaluate what level of the trajectory she thought students exemplified. However, we observed no evidence of her making connections of the information she gained from the pre-lesson student interviews to reflecting on the same students' thinking in a classroom lesson, and then to the post-lesson student interviews, although we had called on teachers to do so. It makes us wonder if she thought of the trajectory as an assessment tool prior to the lesson and not a tool to help promote student growth before, during, and following the lesson.

This case study of Ms. Smith provided us with a preliminary but very complex picture of what and how teachers notice children's thinking and how they use a trajectory to assess and make sense of student thinking. The results signify that the act of teacher noticing using a learning trajectory may become increasingly more complex when teachers move from observing and analyzing one or two students to working with an entire classroom. Further studies need to be conducted to analyze how classroom teachers develop in their ability to notice using a learning trajectory and how teachers connect knowledge of individual student strategies to classroom instruction. The findings and themes that emerged in this initial

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study gave us a glimpse of the multiple factors involved in improving teachers' noticing using a learning trajectory and provide a direction for future research.

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