THE ROLE OF TECHNOLOGICAL TOOLS IN RELATION TO STUDENTS' MATHEMATICAL THINKING DURING CLASSROOM TASKS

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This study uses the Mathematical Tasks Framework (Stein & Smith, 1998) to assess the cognitive demand of mathematical tasks implemented in four mathematics classrooms, and to investigate the role of technology in both low- and high-level cognitive demand tasks. The metaphor of using technology as an amplifier or reorganizer (Pea, 1987) is used to characterize technology use. Results indicate that when technology is used as an amplifier, it has no influence on the thinking demands of a mathematical task, but when used as a reorganizer it is intimately related to the supporting students' high-level mathematical thinking. Furthermore, this distinction can be an important part of mathematics teachers' technological pedagogical content knowledge (TPACK) (Mishra & Koehler, 2006; Niess et al., 2009) by providing ways to distinguish uses of technology along dimensions that matter for students' mathematical thinking and learning.

Research on the use of instructional technology in secondary mathematics education has proliferated over the last 20 years (e.g., Heid & Blume, 2008; Zbiek, Heid, Blume, & Dick, 2007). There has also been an increased awareness of and interest in students' mathematical thinking and reasoning (Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2009), However, little research has focused on how the use of technology can support students' mathematical thinking and reasoning more generally.

Theoretical Framework

As the purpose of the this paper is to characterize the use of common classroom technologies in relation to students' mathematical thinking, an array of classroom technologies are considered. The interest in the present study is with digital technologies used specifically as cognitive technologies (Pea, 1987). Pea defines cognitive technologies as those that "help transcend the limitations of the mind (e.g., attention to goals, short-term memory span) in thinking, learning, and problem-solving activities" (Pea, 1987, p. 91). By mediating human thought, cognitive technologies both assist and influence thought and learning. The focus of the present study is on *digital* cognitive technologies that may support students' mathematical activity. A distinction that Pea makes within cognitive technologies is between its use as an *amplifier* or a *reorganizer* of mental activity (1987). That is the focus of the next section.

Amplifier and Reorganizer Metaphors

When technology is used as an amplifier, it performs more accurately or efficiently tedious or time consuming processes that might be done by hand, like arithmetic computations or the generation of standard mathematical representations. In this use of technology, what students do or think about is not changed, but can be done with significantly less time and effort, and more accurately. The use of a scientific calculator for computations while students set up and solve proportions can make their work more efficient and help to avoid basic arithmetic errors in their solutions. However, what students are doing is not changed by the use of the calculator; their cognitive focus is still on setting up and solving proportions whether the calculator is used or not.

As a reorganizer, technology has the potential to support a shift in the focus of students' mathematical thinking and behavior, by producing novel representations which make salient some aspect of a concept which is difficult to make explicit without it, or by providing feedback to students that they would otherwise not have access to. For example, students might use dynamic geometry software (DGS) to construct a triangle and manipulate it in order to look for and make conjectures about the relationship between the lengths of the sides, with the goal of discovering the Triangle Inequality Theorem. By using

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technological tools to generate dynamic and interactive representation, students are able to focus on looking for patterns and making and testing conjectures, rather than on drawing and measuring triangles. This use of technology supports a shift in the focus of students' mathematical activity and thinking.

An important aspect of the type of thinking afforded by the use of technology is the kind of problem or task that calls for its use. Whether technology is used or not, one way that teachers shape students' learning and view of the discipline of mathematics is by the choice of mathematical tasks for instruction (National Council of Teachers of Mathematics, 1991). However, with the introduction of technology comes the need to understand what kinds of tasks utilize the resources provided by the technology to support students' high-level thinking (Hollebrands, Laborde, & StraBer, 2008). A framework for understanding the influence of tasks on students' mathematical thinking is described in the next section.

The Mathematical Tasks Framework

The *Mathematical Tasks Framework* (Stein & Smith, 1998) has been used to describe and differentiate the type of thinking that is called for by a given mathematical task, defined as "a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea" (Stein, Grover, & Henningsen, 1996, p. 460). This framework distinguishes between low-level cognitive demand, including *memorization* and the use of *procedures without connections* to meaning or concepts, and high-level cognitive demand, including the use *of procedures with connections* to meaning or concepts, and *doing mathematics*, of which non-algorithmic thinking is characteristic. An important characteristic of this taxonomy is that it is not related to specific mathematical content, but rather characterizes different types of thinking that students may engage in while working on a mathematical task.

An important contribution of the *Mathematical Tasks Framework* is the recognition that the thinking requirements of a task may change during its enactment. The task as it appears in curricular materials does not directly influence students' learning by the type of thinking it requires, as those demands may be altered by the teacher when announcing the task to students during instruction, known as the *set up* phase, and again while students are working on the task, referred to as the *implementation* phase. This element of the *Mathematical Tasks Framework* makes it especially suitable for describing the impact of using technology on students' thinking in a classroom context. The research question investigated by this study is: what is the role of technology in relation to the cognitive demand of mathematical tasks?

Research Methods

This study uses a qualitative, observational research design with the goal of understanding the role of technology in supporting the mathematical thinking of students. Four teachers were recruited primarily based on their use of technology for mathematics instruction. One or two units of instruction, as designated by the teacher, were observed in each classroom. Each of the teachers had three years of teaching experience, and had taught the observed unit at least once previously. An overview of the data collection classrooms is given in Table 1.

Data collected at each site included lesson observation field notes, task artifacts and student work on the task, and audio recorded post-lesson interviews with the teacher. Hand-written jottings taken during the observation were developed into a detailed narrative of the lesson immediately following the observation (Emerson, Fretz, & Shaw, 1995), and all post-lesson interviews were transcribed. Using the *Task Analysis Guide* (Stein & Smith, 1998), each task was coded with respect to the cognitive demand of the task as stated in the curriculum, as introduced to the class (set-up), and implementation. In addition, for those tasks which utilized technology, the use of technology was coded as amplifier, reorganizer, both, or neither during the set up and implementation phases. Approximately one-fourth (24%) of the observed tasks were double coded for reliability, with 89% agreement on the cognitive demand, and 86% with regard to the use of technology. All discrepancies were resolved and the consensus code was assigned to the task.

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Table 1: Summary of Data Collection Classrooms

	Tasks Observed	Grade/Class Level	Topics	Technologies
Ms. Jones[1]	12	9th grade Integrated Math	• Angle relations • Triangle Inequality • Similarity	• DGS • Scientific Calculators
Ms. Young	17	11th grade Inclusion	• Angle relations • Triangle Inequality	• DGS • Interactive Whiteboard • Scientific Calculators
Mr. Mack	17	6th grade Regular	• Order of operations • Fractions	• Interactive Whiteboard • Scientific Calculators
Ms. Lowe	17	10th grade Advanced	• Points of concurrency in a triangle	• DGS • Interactive Whiteboard • Graphing Calculators

Following data collection and coding, the coding results were summarized in order to observe patterns in the data that could guide qualitative analysis. For example, it had been hypothesized that the use of technology as an amplifier would be associated with low-level cognitive demand tasks. However, the summary of the coding results revealed that technology was used as an amplifier in both low- and high-level cognitive demand tasks across sites. These tasks were analyzed qualitatively in order to understand the role that technology played in these tasks, and how it was related to the thinking demands of the task. The constant comparative method (Glaser, 1965) was used in analyzing different tasks in the same classroom, as well as across classrooms, in order to make generalizations about the relationship between the role of technology and students' mathematical thinking.

Results

A primary concern in this study is how the use of technology might be correlated with the cognitive demand of the mathematical tasks within which that use is situated, and the meaning of those correlations. The hypotheses for this study were that technology is used as an amplifier in low-level tasks, and substantial evidence for the hypothesis that technology is used as a reorganizer (or both) in high-level tasks. The reasoning behind these hypotheses was that teachers would use technology to support a change in students' focus to high-level thinking by offloading computations or the generation of representations to the technological tools, or by providing novel representations capable of supporting conceptual connections that would be difficult or impossible to produce by hand. According to this logic, a teacher that did not utilize technology as a reorganizer would not be attempting to support such a shift, and thus the task would remain at a low-level. Results of the study provide partial evidence for the hypothesis that technology is used as an amplifier in low-level tasks, and substantial evidence for the hypothesis that technology is used as a reorganizer (or both) in high-level tasks.

The results of the coding of tasks in terms of the cognitive demand and the use of technology are reported in Tables 2 and 3. Table 2 shows the distribution of tasks using technology as an amplifier during set up at a low- or high-level, and during the implementation at a low- or high-level, while Table 3 depicts the same for the use of technology as both an amplifier and reorganizer.[2] Technology use during the set up phase refers to the way in which technology was designed to be used in the task as set up by the teacher, but prior to students actually engaging with the task. For example, if the teacher introduced as task in which students were to use DGS to investigate the properties of medians of a triangle, the use of technology was coded as an amplifier and reorganizer during set up.

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Table 2: Amplifier Technology Use in Relation to Cognitive Demand[3]

Amplifier Use of Technology	Low-level	High-level
Set up	24	7
Implementation	46	2

Table 3. Reorganizer Technology Use in Relation to Cognitive Demand

Technology Use as Reorganizer	Low-level	High-level
Set up	0	16
Implementation	2	6

Amplifier Use of Technology

As Table 2 indicates, within the sample of tasks set up at a low-level, the use of technology was always intended as an amplifier. Although there exists an association of low-level tasks with amplifier use during set up and implementation, qualitative analysis of these tasks revealed that the way the technology was used was not directly related to the low-level demands of the task. Indeed, technology was also used as an amplifier in high-level tasks, and likewise qualitative analysis revealed no relationship with the cognitive demand of these tasks; it was merely used for displaying the statement or description of a task that would have been high-level without it.

A primary way in which technology was used as an amplifier was in tasks in which the interactive whiteboard (IWB) was used to display lecture notes or practice problems, to project a worksheet while discussing problems or solutions, and in a few cases, it was used in conjunction with DGS in order to provide a dynamic demonstration or example. Another common amplifier use of technology included the use of a calculator for computations while practicing a procedure. For example, students used scientific calculators for arithmetic computations while solving for missing angles in a diagram of parallel lines cut by a transversal.

What all of these tasks had in common is that the cognitive demand of the tasks in which they appeared would not have changed if technology had been used in the way that it was, i.e., as an amplifier. Given the way that the amplifier use of technology is defined, i.e., making some process more accurate or efficient that could be accomplished without it, it makes sense that such a use of technology is not directly related to the cognitive demand. Rather, the association revealed in these data seems to be mediated through the teachers, and the affordances they perceive of the technology available to them in relation to low-level tasks. Thus, the selection of the task may be the primary factor in the cognitive demand when technology is used as an amplifier.

Reorganizer Use of Technology

The use of technology as a reorganizer was strongly associated with the set up and implementation of high-level tasks. As hypothesized, its use as a reorganizer was in all cases related to its use as an amplifier, in the sense that by offloading the construction, labeling, and measuring of mathematical objects to the technological tools there existed the potential for students to shift the focus of their mental activity to such behaviors such as dragging, observing, generalizing, and making and testing conjectures. In general, teachers used a dynamic geometry software package such as GeoGebra or Geometer's Sketchpad to have students investigate and explore the properties of geometric objects such as triangles.

Three of the four teachers this study used technology as both an amplifier and reorganizer to set up tasks at a high-level using DGS within a student-centered exploration. In general, the purpose of using technology in these tasks was to support students in constructing meaning for a mathematical concept or procedure, or to engage in mathematical behavior, such as observing, reasoning, generalizing, and conjecturing.

An example of a task that was set up and implemented at a high level using technology as a reorganizer is taken from Ms. Lowe's classroom. Ms. Lowe created a worksheet to guide students in using

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GeoGebra individually at their own computer for most of the period in order to investigate the properties of the centroid of a triangle, i.e., the intersection of the medians[4] of a triangle. She guided students to construct a triangle and the medians of the triangle, to construct the centroid, to measure the segments from the vertex to the centroid, and from the centroid to the midpoint of the opposite side, and then to record these measurements in a table in order to look for a relationship[5] and make and test conjectures. In this case, the opportunity to drag and explore the properties of the medians individually was directly connected to the cognitive demand of the task.

As an example of the type of thinking that students engaged in while working on the task, the following conversation between two students was observed while working on the task:

Nick and Brian are dragging their figures and discussing what it is that they're supposed to be noticing.
Nick: I'm going to make it a right triangle. What would that do? It would stay at the center of the triangle, right?
Brian: look at this.
Brian shows Nick his table, pointing out the 6.17 and the 3.08.
Brian: this one is almost exactly double that one.
Nick: you can't make assumptions from one triangle
Both start dragging their triangles.
Nick: I see something like that, but if you stretch it far enough…
They continue dragging their triangles and looking at the measurements.
Nick: one is always half of the other
Brian: the distance from the vertex is always double the distance to the midpoint.
Ms. Lowe: change it, see if you can disprove it.
Starting over with a new triangle, Brian begins to measure the distances from the centroid to the vertex and from the centroid to the midpoint for each median.
Brian: (as he measures each segment) that is double that, and that is double that, and that is double that.
Nick drags his figure.
Nick: yes, it does stand true. (Field note, 2/7/11)

This excerpt demonstrates how technology can be used as both an amplifier and reorganizer. As an amplifier, students constructed a triangle, the medians of the triangle, and the centroid quickly and precisely, and measured and labeled the angles, the lengths of the medians, and the lengths of the segments. Most students had completed this part of the task within 10 minutes. While students might be able to construct the centroid of a triangle and use a protractor and ruler to make the same measurements, this could be difficult for most students to do accurately in 10 minutes. Furthermore, by dragging the triangle, students are essentially creating many triangles, medians, and centroids. As a reorganizer, dragging does more than just create multiple examples quickly and accurately. One can observe, for example, how the centroid moves in response to a vertex being dragged, or how the location of the centroid is changed as the triangle is changed from an acute triangle, to a right triangle, to an obtuse triangle, and back again. This sort of "real-time" motion of one object in relation to another is simply not possible in a pencil-and-paper environment.

Further evidence of the reorganizer use of technology is that students are not focused on making the measurements, but on using them to discern regularities in the behavior of the segments and on understanding what they mean. Nick's statement, "I'm going to make it a right triangle. What would that do? It would stay at the center of the triangle, right?" indicates the open-ended nature of having students directly manipulating the object created within a DGS, that there are many possibilities to choose from in terms of how to drag the object. It also reveals the making and testing of conjectures that is inherent in the development of a more strategic investigation of an object using dragging. Students must consider the purpose of dragging in terms of an overarching goal, what information would be helpful in achieving that goal, and what sort of dragging might provide that information. Once that move is made, students must

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assess if the object behaved in the anticipated manner, and if not, why, and what the next move should be in light of this information. The technology acts as a reorganizer by supporting these students' focus on looking for relationships, and making and testing conjectures, which constitute the high-level thinking demands of the task.

The Role of Technology in the Decline of Cognitive Demand

One explanation for the correlation of amplifier use of technology with low-level tasks during implementation is that many tasks that intended to use technology as both an amplifier and reorganizer in a high-level task during set up were implemented at a low-level when students used technology as an amplifier only. In these tasks it was the use of technology as a reorganizer that was intimately connected with the high cognitive demand of these tasks as set up. Thus, when technology failed to act as a reorganizer of students' thinking, the cognitive demand declined during implementation.

In these tasks, students constructed, measured, and manipulated figures, but did not engage in making mathematically meaningful observations, generalizations, or conjectures. For example, in Ms. Jones class, students created triangles and measured side lengths in order to explore the Triangle Inequality Theorem.[6] However, when asked if it were possible to create a triangle in which the sum of two side lengths could be less than the third, some students replied "yes," and very few students wrote a conjecture about the relationship between the lengths of the sides.

In general, these teachers seemed to underestimate the support that students would need in connecting their work with DGS to the mathematical thinking and behavior required by the task. While the affordances of DGS can support high-level thinking, there is nothing about the use of a DGS for an exploratory task that *causes* students to engage in high-level thinking. For example, if students have never been asked to make a conjecture before, providing them with technological tools will not necessarily result in their ability to do so. DGS can support students' ability to make conjectures by providing the opportunity to examine numerous examples to analyze as the basis for a conjecture, and strategically manipulate objects in order to test a conjecture. However, it does not inherently support students' understanding of the importance of examining a variety of examples, what is mathematically meaningful to look for across those examples, how to make a mathematically precise statement as a conjecture, the importance of testing a conjecture or looking for counterexamples, or the difference between a conjecture and a proof. Ultimately, when technology is used as both an amplifier and a reorganizer, teachers must support the shift entailed by its use as a reorganizer. What that support may consist of has been discussed elsewhere (Sherman, in press).

Discussion

The present analysis builds on previous work that makes use of the amplifier and reorganizer distinction (Ben-Zvi, 2000; Laborde, 2002), but extends the distinction by considering how technology might act as an amplifier or reorganizer during the implementation of classroom tasks. The use of technology as an amplifier was generally associated with the interactive whiteboard and calculator, while its use as a reorganizer was almost always in the context of using DGS. It is tempting to explain the difference in technology to the differences in the affordances of these classroom technologies. Research points to the potential of calculators to be used in ways that can support and influence students' thinking (Burrill et al., 2002). Thus, the real issue may be how the affordances of these technologies are perceived by teachers.

A way in which the results of the present study may contribute to research in mathematics education is by characterizing the use of technology in relation to students' thinking in a way that can differentiate superficial from meaningful use of technology for mathematical instruction and learning. These results provide empirical evidence that the mere inclusion of technology does not have any inherent implications for students' opportunity for high-level thinking, but how it is used does.

An understanding of this distinction may be an important element of mathematics teachers' TPACK (Mishra & Koehler, 2006; Niess et al., 2009), by providing a way to critically examine the role of technology in the tasks they enact with their students. Anecdotal evidence indicates that preservice

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teachers in a secondary methods course were able to learn and use this distinction in evaluating and selecting tasks. Research is needed to examine this claim more carefully, and to determine how it may influence in-service mathematics teachers' selection and design of classroom tasks.

Endnotes

[1] Pseudonyms

[2] For the sake of simplicity, "both amplifier and reorganizer" is used interchangeably with "reorganizer" for the remainder, since no cases of using technology were coded as reorganizer only.

[3] The discrepancy in the number of tasks set up and implemented using technology as an amplifier has two sources. Some tasks were set up by the teacher without any explicit mention of technology as part of the set up of the task, but students initiated its use during while working on the task. In other cases, the task was set up to use technology as both an amplifier and a reorganizer, but utilized technology as only an amplifier during implementation. This also explains why more tasks were set up than implemented using technology as a reorganizer, as shown in Table 3.

[4] A segment connecting the midpoint of a side of a triangle to the opposite vertex.

[5] The relationship that students were intended to discover is that the segment from the midpoint to the centroid is 1/3 the length of the median, and the segment from the centroid to the opposite vertex is 2/3 the length of the median.

[6] The sum of the lengths of any two sides of a triangle is always greater than the length of the third side.

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