AT THE CROSSROADS OF MATHEMATICAL VOCABULARY, WRITING, AND THE SECONDARY MATHEMATICS TEACHER

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Although much research has been done citing the benefits of using writing in mathematics lessons, little has been done that examines teachers' responses to writing about mathematics and how those responses may shape teacher attitudes about using writing in the classroom. In this study, I examined the experiences of six teachers as they explored mathematics in a graduate class in secondary mathematics education and prepared written reports for Internet publication. After analyzing their work and their responses on questionnaires and in interviews, I found that several of the participants struggled with using the technical vocabulary of mathematics in their writings and that some of them had differing beliefs about the use of the vocabulary. Based on these findings, I recommend that mathematics educators use activities that challenge teachers to view vocabulary as mathematical content and that hone their skills in using it in their writings.

Keywords: Teacher Education–Preservice; Teacher Education–Inservice/Professional Development; Teacher Knowledge; Teacher Beliefs

Objective

During their careers, teachers will frequently transition between being a student and being a teacher. It is a necessary move that offers teachers the chance to experience for themselves the lessons they want their students to learn. As mathematics educators, we bear the burden of insuring that those moments the teachers spend as students are filled with rich, thought-provoking experiences that provide them with the knowledge they will need to help their students learn. This need for teachers to have rich experiences as students is particularly important in the area of mathematical language and writing in which teachers are asked to navigate yet another transition—when they are asked to transition from using informal language in writing about mathematics to using the formal language of mathematics.

For many years now, mathematics educators and researchers have promoted the use of writing in the mathematics classroom. In 1977, Geeslin reported on the benefits of using writing in mathematics "as a learning device for the student" (p. 113). In 1989, the National Council of the Teachers of Mathematics (NCTM) suggested in its *Curriculum & Evaluation Standards* that "all students need extensive experience…writing about…mathematical ideas" (p. 140). Since those early years, much research has been done noting the benefits of using writing in the mathematics classroom (Porter & Masingila, 2001), but there is evidence that suggests these recommendations have not been embraced by a majority of secondary mathematics teachers. In a national survey of secondary mathematics teachers conducted in the United States in 2000, 55% of those teachers surveyed indicated that they never use reflective writing in their classrooms (Weiss, Banilower, McMahon, & Smith, 2001).

As intuitively expected, Flores and Britain (2003) suggested that mathematics teachers are likely not to use writing in their lessons "unless they have had the experience themselves of writing in relation to mathematics" (p. 112). However, this suggestion seems to overlook the nature of the experience and creates a question about how teachers respond to writing about mathematics. Before we as mathematics educators can help preservice and inservice mathematics teachers transition to an effective use of writing in their classrooms, we must first understand how teachers themselves respond to the writing in terms of what they can do and what they believe. Principally, we need to know how they respond to writing about mathematics when they are acting from the perspective of a student. In this study, I endeavored to explore and examine those responses. In this paper, I report on one area of the study in which I focused on how the participants responded to writing in terms of the language they used and what they believed about the type of language they should use.

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Theoretical Framework

I conducted this exploratory study from the view that to effectively write about mathematics, teachers and students must give attention not only to accurate descriptions of concepts and procedures but also to the proper use of mathematical language. Arguably, students do not have a competent understanding of mathematics unless they are fluent in its language which Ball and Sleep (2007) characterized as "both mathematical content to be learned and [a] medium for learning mathematical content" (p. 19). Essentially, to write about mathematics in a manner which showcases understanding, students must first have a working knowledge of mathematical language.

The language of mathematics is often defined as the mathematics register. Foley (2008) characterized the mathematics register as "the formal academic approach to mathematical speaking and writing" (p. 1). Schleppegrell (2007) separated the mathematics register into two categories: multiple semiotic representations and grammatical patterns. Multiple semiotic representations address symbolic notation, oral and written language, graphs, and other visual displays. Grammatical patterns cover technical vocabulary, dense noun phrases, and "implicit logical relationships" (p. 141). In this paper, I focus on elements drawn from both categories. I specifically focus on the use of technical vocabulary within the written language of mathematics.

In terms of using the technical language of mathematics in the classroom, teachers can often feel two opposing forces at work within themselves: the urge to use students' informal language in order to be relevant and the need to foster the development of the technical vocabulary of mathematics. Jill Adler (1997) characterized this delicate balancing act as one of the "dilemmas of mediation" (p. 235) in which mathematics teachers have the burden of "shaping informal, expressive and sometimes incomplete and confusing language, while aiming towards the abstract and formal language of mathematics" (p. 236). How teachers balance this tension, however, is often influenced by what they believe about the use of mathematical language in the classroom.

In this study, the word *belief* is being used in a broad sense to encompass the idea of *attitude* which Philip (2007) defined as "manners of acting, feeling, or thinking that show one's disposition or opinion" (p. 259). Although there are distinctions between the two concepts, it can be argued that belief and attitude are deeply connected and that what people believe does influence how they act and what they say. In teacher education, beliefs play an important role in how preservice and inservice teachers approach their training and what they glean from it. Cooney (1998) stated that mathematics educators must consider such beliefs in order to "create activities that encourage teachers to wonder, to doubt, to consider what might be, to reflect, and most important, to be adaptive" (p. 332). In this paper, I focus on those beliefs about the use of the technical language of mathematics that seemed to influence how the participants in the study wrote about the mathematics.

Methodology

In this qualitative study, I examined the responses of five preservice teachers and one inservice teacher in a graduate course in secondary mathematics education as they completed 11 explorations of various mathematical topics using technology. After completing the explorations, they posted their findings on the Internet in written reports called "write-ups." In addition to preparing these formal reports, I asked the participants to take notes while they explored the mathematics and to complete a written reflection after they finished each activity. My objective was to have three forms of writing to which the participants could respond: formal, informal, and reflective. In this paper, I focus on their responses to the formal writing or to the write-ups they prepared for Internet publication.

Maxwell (2005) noted in his book *Qualitative Research Design: An Interactive Approach* that "the typical way of selecting setting and individuals" (p. 88) is "purposeful selection" (p. 88). He described this method as "a strategy in which particular settings, person, or activities are selected deliberately in order to provide information that can't be gotten as well from other choices" (p. 88). In an effort to collect unbiased data, I solicited participation from a class in secondary mathematics education in which writing about mathematics was frequently used but was not a focus of instruction. In so doing, I diminished the risk that

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the biases of the instructor about writing in mathematics were frequently passed on to the participants. Throughout the study, I also endeavored to refrain from offering my opinion about writing in mathematics, about what the participants had to say, or about the quality of their work. My objective was to study the responses of the participants in an atmosphere as free as possible from instructor or researcher bias.

The semester-long class met weekly for three hours, and after a brief introduction of the relevant topic by the instructor, most of the class time was devoted to individual explorations of mathematical topics at computers. Students prepared their Internet reports based on 11 activities covering topics in algebra, geometry, data analysis, precalculus, and calculus. These activities presented a wide range of tasks that students could explore using software such as Geometer's Sketchpad (Version 4.07) and Graphing Calculator (Version 3.5). In each activity, students were given several tasks from which they could choose one to explore and about which they could write a report. For example, in one activity they could choose to describe what happens to the graph of a quadratic equation in standard form when the value of a, b, or c is varied as the other two values are held constant in the equation. Students were free to work through the activities at their own pace and post their reports to the Internet at any time throughout the semester.

At the first class meeting, I requested that all master's level students in a class of 31 complete the initial questionnaire. Twenty-three students signed a consent form and 18 students returned their responses via email or at the next class meeting. From these 18 students, I asked 10 if they would agree to participate based upon their responses to the questionnaire. My goal was to ask participants to volunteer who offered differing opinions on the use of writing in the mathematics classroom. Throughout the semester-long class, I tracked the participants' progress with the write-ups by checking their Internet postings, informally speaking with each participant during class, and by conducting formal interviews of each participant at the beginning, midpoint and end of the semester. At the end of the semester, I also asked the participants to complete a post-questionnaire about their experiences with the various writings in the class.

Near the end of the study, I determined that four of the participants had finished less than half of the write-ups. This lack of progress meant that they would complete the bulk of the course in two weeks which ran counter to an initial request I had made at the first meeting that they work at a steady pace throughout the semester to insure they had an adequate amount of time for reflection. In good faith, I could not compare their work with those who had steadily worked their way through the course and were primarily done with the course at the end of the study; therefore, I eliminated these four participants from the study. After data collection, I began the analysis of the data collected from the six remaining participants: Gwen, Amy, Claire, Grace, Lisa, and Kim.

During the analysis phase of my study, I performed two different types of examinations. During the first examination, I studied all notes, interview transcriptions, questionnaire responses, and written reflections to categorize participant responses according to various topics such as background, experiences with the course and the writings, and their beliefs about writing in mathematics. This categorization allowed me to situate the participants according to their various experiences. I prepared a report for each participant in outline form which addressed these topics. After I completed a report for each participant, I carefully examined each report noting emerging themes across the documents about participant responses to writing in mathematics. Once I identified these themes, I reexamined all the data, making note of any new evidence to support or contradict these major ideas. During the second type of examination, I studied each write-up posted on the Internet to determine the soundness of the mathematics used and the quality of the writing in terms of style, grammar, and language usage. In this paper, I specifically focus on two themes which emerged from these two examinations: the quality of the participants' use of technical vocabulary in the write-ups and the participants' differing beliefs about the use of technical vocabulary when writing about mathematics.

Results

Several participants struggled with the use of technical vocabulary in their write-ups. In one write-up, Grace described ellipses as "tall up and down" or "long left to right" rather than as *vertical* or *horizontal*. In another write-up, she characterized the areas of triangles as congruent. Kim characterized the graph of an inverted parabola as a "negative" graph. Gwen described the number of "humps" in the graph of a

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parametric equation. Amy described graphs as merging "after the domain of –3 [and] 3" which she seemed to want to mean that the graphs merged after the points with the *x-coordinates* of –3 and 3 (see Figure 1). However, her phrasing technically means that the domain consisted only of –3 and 3 which is not a true statement. Although a knowledgeable reader could reasonably infer what these participants intended when they used these words and phrases, the use of mathematical vocabulary in the write-ups ranged from informal at best, imprecise on average, and incorrect at worst. For example, the word *congruent* is customarily used in reference to two geometric figures that have the same size and shape. The concept of area, as Grace used it, is typically not included in that description.

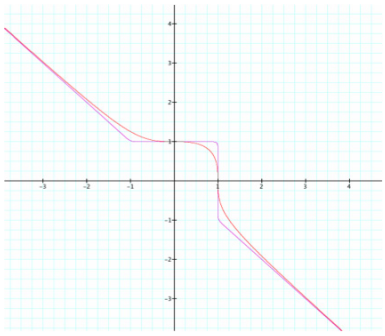

Figure 1: Amy's merging graphs

The use of technical vocabulary also brought out differing beliefs and attitudes in three of the participants. Claire stated in the second interview that she had recently learned about the mathematics register in one of her graduate classes and implied that the lesson had helped her to become more aware of the language she was using in her write-ups. She implied it was important that the readers understood the technical language she was using in the write-ups so they could make sense of her explanations. Kim, however, expressly stated that she wanted to avoid the use of technical language. During the second interview, she offered the opinion that she thought her write-ups were "mathematically written" but were not like a textbook which seemed to be what she wanted. She stated during the interview that she believed textbooks contain "just too much mathematical language" and implied that she wanted to "use just normal conversational language." In the final interview at the end of the study, she noted again that she "wanted to make sure that [her] words were universal."

In contrast to Kim, Amy showed a desire to use technical vocabulary in her writing, but she noted on several occasions that she struggled with the language. During the second interview, she commented that "a lot of why I can't communicate mathematically [is] sometimes I don't know the language." She clarified the comment by stating that she had a problem with "using the right math terminology" but conceded that doing the write-ups up to that point had helped her to build her mathematical vocabulary. She commented that completing the write-ups was helping her "think about the math language and how should I write this or how should I explain what's going on with this in words…." It was not entirely clear, however, that Amy believed that the mastery of the vocabulary was part of the mathematical content she needed to know. She stated in the final interview that "my…my problem isn't math, it's writing about math or writing, I think, about anything period…." When asked if she would have preferred to have done oral rather than written presentations, she stated that she would have preferred the oral presentations because it would have been easier for her to "show you why versus trying to explain in words why."

Discussion and Conclusions

Although researchers have claimed for many years that writing is a beneficial tool to help students learn mathematics, a gap exists in the research which informs us about how teachers respond as students to writing about mathematics. In this study, I sought to examine the responses of preservice and inservice teachers as they engaged in an intensive exploration of various mathematical topics and published their written findings on the Internet. In this paper, I focus on participant response in terms of technical

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vocabulary. The results of this study tend to suggest that preservice and inservice teachers struggle with the use of mathematical language in terms of vocabulary and that they also have varying beliefs about the role technical vocabulary should play in writing about mathematics.

Ball and Sleep (2007) described mathematical language as "mathematical content to be learned" (p. 19). Viewed from this perspective, this study shows that several of the participants were deficient in this area of their mathematical content knowledge in varying degrees. For example, Grace knew how to describe the orientation of the ellipse as "tall and long" but she did not reach for the content word *vertical*. In effect, she did not take her informal language and translate it into mathematical content. At the other end of the spectrum, Amy's struggle with the use of mathematical language showed a clear deficiency in mathematical understanding. In other words, she could not use the content word *domain* properly in her writing because she did not appear to fully understand the concept.

The study also tends to show that not all of the participants agreed with the notion that mathematical language is mathematical content as demonstrated by their attitudes or their "manners of acting, feeling, or thinking…" (p. 259). After learning about the mathematics register in one of her graduate classes, Claire embraced technical vocabulary as part of mathematical content and endeavored to make her writing precise and technically correct. Kim, on the other hand, expressly fought against it. Kim desired that her writings contain what she called "universal" language. For her, there seemed to be no "dilemmas of mediation" (Adler, 1997) between formal and informal language but instead a belief that technical language is an unnecessary obstacle to understanding mathematics. Amy seemed to share a similar opinion. Although Amy frequently acknowledged her struggles with knowing the vocabulary of mathematics, she nevertheless declared at the end of the study that she knew the mathematics but had problems expressing it in writing. This stance seems to imply that she believed someone could know the mathematics without being able to use its technical language in writing.

When we as mathematics educators ask secondary teachers to use writing in their classrooms, we sometimes assume that they believe in the use of technical vocabulary in writing about mathematics and that they have mastered the skill themselves. However, this study indicates we must first consider the importance of teachers' beliefs about the use of technical language in writing about mathematics. We must do as Cooney advised and "create activities that encourage teachers…to consider what might be…and most important, to be adaptive" (p. 332). Specifically, we need to develop lessons that directly challenge both preservice and inservice teachers to confront their beliefs about the role of technical language in mathematics and that encourage a consideration of mathematical language as an integral part of mathematical content knowledge. The simple inclusion of lessons about the mathematics register may raise awareness for some teachers such as Claire experienced in one of her classes. Next, we need to provide guidance and practice through activities and assessments designed to help teachers transition from the use of informal language in their writing to the effective use of the formal language of mathematics. We also need to raise their awareness that a misuse of technical vocabulary in writing may indicate that teachers do not fully understand the mathematical concept behind the terms. In providing these types of activities, we better equip teachers to navigate the "dilemmas of mediation" (Adler, 1997, p. 235) which they will face in their own classrooms.

The results of this study also suggest an area in need of further research. We need studies devoted to how teacher beliefs about the use of technical vocabulary in writing may influence their teaching. Specifically, we need to probe the depths of how those beliefs may influence how teachers structure their lessons, what they expect from their students in terms of language use, and how those decisions and expectations influence student learning. In pursuing such research, we provide a connection to practice that informs both the researcher and the practitioner in the ways language use in writing may shape what students learn and do not learn in their mathematics classes.

The most important transition educators make is between the roles of student and teacher. It is a transition that occurs frequently during one's career from the preservice phase to the inservice stage, to advanced schooling, and then on to years of professional development. It is during those times when teachers are in the role of students that we as mathematics educators must make the most of their experiences. Essentially, we cannot ask teachers to teach what they have not experienced themselves as

students. This is particularly true in the area of mathematical language and writing. Teachers at all stages in their development need a rich, thorough experience of using the technical vocabulary of mathematics in their writings. By participating in activities and lessons focused on the use of technical vocabulary in writing about mathematics, they have a chance to confront their beliefs and work on their skills. In essence, we as mathematics educators help them navigate the transition from the use of informal language to the use of the formal language of mathematics. By providing these rich experiences, we also increase the odds that these students will become master teachers who feel more comfortable with writing about mathematics and, in turn, are more likely to use writing in their classrooms. Ultimately, we create teachers who can effectively guide their students in becoming fluent in the use of mathematical language in writing about mathematics.

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