MATHEMATICS, THE COMMON CORE STANDARDS, AND LANGUAGE: MATHEMATICS INSTRUCTION FOR ELS ALIGNED WITH THE COMMON CORE

Judit N. Moschkovich
University of California, Santa Cruz
jmoschko@ucsc.edu

This paper outlines research-based recommendations for mathematics instruction for English Learners (ELs) aligned with the Common Core Standards. The recommendations focus on improving mathematics learning and teaching through language for all students, and especially for ELs. These recommendations are intended to guide teachers and teacher educators in developing approaches to support mathematical reasoning and sense making for ELs.

Keywords: Classroom Discourse; Equity and Diversity; Standards

Introduction

This paper outlines recommendations for meeting the challenges in developing mathematics instruction for English Learners (ELs) that is aligned with the Common Core Standards. These recommendations for teaching practices are based on research that often runs counter to commonsense notions of language. The first issue is the term language. There are multiple uses of the term language: to refer to the language used in classrooms, in the home and community, by mathematicians, in textbooks, or in test items (Moschkovich, 2010). It is crucial to clarify how we use the term, what phenomena we are referring to, and which aspects of these phenomena we are focusing on. Many recommendations for teaching academic language in mathematics classrooms reduce the meaning of "language" to single words and the proper use of grammar (for an example, see Cavanagh, 2005). In contrast, work on the language of specific disciplines provides a more complex view of mathematical language (e.g., Pimm, 1987) as not only specialized vocabulary (new words or new meanings for familiar words) but also as extended discourse that includes syntax and organization (Crowhurst, 1994), the mathematics register (Halliday, 1978), and discourse practices (Moschkovich, 2007b). I use a socio-cultural and situated framework to frame these recommendations (Moschkovich, 2002). From this perspective, language is a socio-cultural-historical activity. I use the phrase "the language of mathematics" not to mean a list of vocabulary words with precise meanings but the communicative competence necessary and sufficient for competent participation in mathematical discourse practices.

It is difficult to make generalizations about the instructional needs of all students who are learning English. Information about students' previous instructional experiences in mathematics is crucial for understanding how ELs communicate in mathematics classrooms. Classroom instruction should be informed by knowledge of students' experiences with mathematics instruction, language history, and educational background (Moschkovich, 2010). In addition to knowing the details of students' experiences, research suggests that high-quality instruction for ELs that supports student achievement has two general characteristics: a view of language as a resource, rather than a deficiency, and an emphasis on academic achievement, not only on learning English (Gándara & Contreras 2009). Research provides general guidelines for instruction for this student population. Overall, students who are labeled as ELs are from non-dominant communities and they need access to curricula, instruction, and teachers proven to be effective in supporting the academic success of these students. The general characteristics of such environments are that curricula provide "abundant and diverse opportunities for speaking, listening, reading, and writing" and that instruction "encourage students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts" (Garcia & Gonzalez, 1995, p. 424).

Research on language and mathematics education provides several guidelines for instructional practices for teaching ELs mathematics (Moschkovich, 2010). Mathematics instruction for ELs should (1) address much more than vocabulary; (2) support EL's participation in mathematical discussions as they learn English; and (3) draw on multiple resources available in classrooms (objects, drawings, graphs, and gestures) as well as home languages and experiences outside of school. Research shows that ELs, even as they are learning English, can participate in discussions where they grapple with important mathematical content. Instruction for this population should not emphasize low-level language skills over opportunities to actively communicate about mathematical ideas. One of the goals of mathematics instruction for ELs should be to support all students, regardless of their proficiency in English, in participating in discussions that focus on important mathematical concepts and reasoning, rather than on pronunciation, vocabulary, or low-level linguistic skills. By learning to recognize how ELs express their mathematical ideas as they are learning English, teachers can maintain a focus on mathematical reasoning as well as on language development.

Alignment with Common Core State Standards

The recommendations provided here describe teaching practices that simultaneously align with the Common Core State Standards (CCSS) for mathematics, support students in learning English, and support students in learning important mathematical content. Mathematics instruction for ELs should align with the CCSS, particularly in these four ways: (1) *Balance conceptual understanding and procedural fluency*. Instruction should balance student activities that address important conceptual and procedural knowledge and connect the two types of knowledge; (2) *Maintain high cognitive demand*. Instruction should use high cognitive demand math tasks and maintain the rigor of tasks throughout lessons and units; (3) *Develop beliefs*. Instruction should support students in developing beliefs that mathematics is sensible, worthwhile, and doable; (4) *Engage students in mathematical practices*. Instruction should provide opportunities for students to engage in mathematical practices such as solving problems, making connections, understanding multiple representations of mathematical concepts, communicating their thinking, justifying their reasoning, and critiquing arguments.

According to a review of the research (Hiebert & Grouws, 2007), mathematics teaching that makes a difference in student achievement and promotes conceptual development in mathematics has two central features: one is that teachers and students attend explicitly to concepts and the other is that teachers give students the time to wrestle with important mathematics. Mathematics instruction for ELs should follow these general recommendations for high quality mathematics instruction to focus on mathematical concepts and the connections among those concepts and to use and maintain high cognitive demand mathematical tasks, for example, by encouraging students to explain their problem-solving and reasoning (AERA, 2006; Stein, Grover, & Henningsen, 1996).

The CCSS and the NCTM Standards provide examples of how instruction can focus on important mathematical concepts (i.e. the meaning of equivalent fractions or the meaning of fraction multiplication, etc.) and how students can show their understanding of concepts (conceptual understanding) not by giving a definition or describing a procedure, but by using multiple representations, reasoning, and justification. For example, students can show conceptual understanding by using a picture of a rectangle as an area model to *show* that two fractions are equivalent or how multiplication by a fraction smaller than one makes the result smaller, and pictures can be accompanied by oral or written explanations.

The preceding examples point to several challenges in connecting *language* to content that students face in mathematics classrooms focused on conceptual understanding. Since conceptual understanding and mathematical practices are often made visible by showing a solution, describing reasoning, or explaining "why," instead of simply providing an answer, the CCSS implies an expectation that students will communicate their reasoning. Students are expected to (a) communicate their reasoning through multiple representations (including objects, pictures, words, symbols, tables, graphs, etc.); (b) engage in productive pictorial, symbolic, oral, and written group work with peers; (c) engage in effective pictorial, symbolic, oral, and written interactions with teachers; (d) explain and demonstrate their knowledge using emerging language; and (e) extract meaning from written mathematical texts. The main challenges teachers of ELs face are, first, to teach for understanding and then to support students in using multiple representations and emerging language to communicate about mathematical concepts. Since the CCSS documents already provide descriptions of how to teach mathematics for understanding, below I will focus on how to connect mathematical content to language, in particular through "Engaging students in mathematical practices" (Focus #4 above).

A Classroom Transcript

This transcript is intended to illustrate the recommendations and show how they play out in classroom interactions. The excerpt (Moschkovich, 1999) comes from a third-grade bilingual classroom in an urban California school with 33 students identified as Limited English Proficiency. In general, this teacher introduced students to topics in Spanish and then later conducted lessons in English. For several weeks the students had been working on a unit on two-dimensional geometric figures. Instruction had included using vocabulary such as "radius," "diameter," "congruent," "hypotenuse" and the names of quadrilaterals in both Spanish and English. Students had been talking about shapes and the teacher had asked them to point, touch, and identify different shapes. The teacher identified this lesson as an English as a Second Language mathematics lesson, where students would be using English in the context of describing and talking about geometric shapes.

1. *Teacher*: Today we are going to have a very special lesson in which you really gonna have to listen. You're going to put on your best, best listening ears because I'm only going to speak in English. Nothing else. Only English. Let's see how much we remembered from Monday. Hold up your rectangles . . . high as you can. (Students hold up rectangles) Good, now. Who can describe a rectangle? Eric, can you describe it [a rectangle]? Can you tell me about it?
2. *Eric*: A rectangle has . . . two . . . short sides, and two . . . long sides.
3. *Teacher*: Two short sides and two long sides. Can somebody tell me something else about this rectangle, if somebody didn't know what it looked like, what, what . . . how would you say it.
4. *Julian*: Paralela [holding up a rectangle, voice trails off].
5. *Teacher*: It's parallel. Very interesting word. Parallel. Wow! Pretty interesting word, isn't it? Parallel. Can you describe what that is?
6. *Julian*: Never get together. They never get together [runs his finger over the top side of the rectangle].
7. *Teacher*: What never gets together?
8. *Julian*: The parallela . . . they . . . when they go, they go higher [runs two fingers parallel to each other first along the top and base of the rectangle and then continues along those lines], they never get together.
9. *Antonio*: Yeah!
10. *Teacher*: Very interesting. The rectangle then has sides that will never meet. Those sides will be parallel. Good work. Excellent work.

The transcript shows that English language learners can participate in discussions where they grapple with important mathematical content. Students were grappling not only with definitions for quadrilaterals but also with the concept of parallelism. Student were also engaged in mathematical practices as they were making claims, generalizing, imagining, hypothesizing, and predicting what will happen to two lines segments if they are extended indefinitely. To communicate about these mathematical concepts students used words, objects, gestures, and other student's utterances as resources. This transcript illustrates several instructional strategies that can be useful in supporting student participation in mathematical discussions: asking for clarification, re-phrasing student statements, accepting and building on what students say, and probing what students mean. It is important to notice that this teacher did *not* focus directly on vocabulary development but instead on mathematical ideas and arguments as he interpreted, clarified, and rephrased what students were saying. This teacher provided opportunities for discussion by moving past student grammatical or vocabulary errors, listening to students, and trying to understand the mathematics in what students said. He kept the discussion mathematical by focusing on the mathematical content of what students said and did.

Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). (2012). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Kalamazoo, MI: Western Michigan University.

Recommendations for Connecting Mathematical Content to Language

Recommendation #1: Focus on students' mathematical reasoning, not accuracy in using language

Instruction should focus on uncovering and supporting students' mathematical reasoning, not on accuracy in using language (Moschkovich, 2010). Understanding the mathematical ideas in student's talk can be difficult. However, it is possible to take time after a discussion to reflect on the mathematical content of student contributions and design subsequent lessons to address these mathematical concepts. But, it is only possible to uncover the mathematical ideas in what students say if students have the opportunity to participate in a discussion and if this discussion is focused on mathematics. For teachers, understanding (and re-phrasing) student contributions can also be a challenge, perhaps especially when working with students who are learning English. It may not be easy (or even possible) to sort out which aspects of a student utterance are due to the student's conceptual understanding or the student's English language proficiency. However, if the goal is to support student participation in a mathematical discussion, determining the origin of an error is less important than listening to students to uncover the mathematics in what they are saying.

As we can see in the transcript, uncovering the mathematical content in Julian's contributions was certainly a complex endeavor. Julian's utterances in turns 4, 6, and 8 are difficult both to hear and interpret. He uttered the word "parallela" in a halting manner, sounding unsure of the choice of word or of its pronunciation. His voice trailed off, so it is difficult to tell whether he said "parallelo" or "parallela." His pronunciation could be interpreted as a mixture of English and Spanish; the "ll" sound being pronounced in English and the addition of the "o" or "a" being pronounced in Spanish. The grammatical structure of the utterance in line 8 is intriguing. The apparently singular "parallela" is preceded by the word "the" which can be either plural or singular and then followed with a plural "when they go higher." In any case, it is clear that Julian made several attempts to communicate a mathematical idea in his emerging second language. If we only focus on accuracy, we would miss his mathematical reasoning. Julian is, in fact, participating in mathematical practices and attempting to describe a property of parallel lines. This teacher moved past Julian's unclear utterance, he focused on uncovering the mathematical content in what Julian had said. He did not correct Julian's English, but instead asked questions to probe what the student meant.

Recommendation #2: Shift to a focus on mathematical discourse practices, move away from simplified views of language

In keeping with the CC focus on *mathematical practices* (Focus #4) and research in mathematics education, the focus of classroom activity should be on student participation in mathematical discourse practices (explaining, conjecturing, justifying, etc.). Instruction should move away from simplified views of language as lists of words, phrases, vocabulary, or definitions (Moschkovich, 2010). In particular, teaching practices need to move away from oversimplified views of *language as vocabulary*. An overemphasis on correct vocabulary and formal language limits the linguistic resources teachers and students can use in the classroom to learn mathematics with understanding. Work on the language of disciplines provides a complex view of mathematical language as not only specialized vocabulary (new words and new meanings for familiar words) but also as extended discourse that includes syntax, organization, the mathematics register, and discourse practices. Instruction needs to move beyond interpretations of the mathematics register as merely a set of words or phrases that are particular to mathematics. The mathematics register includes styles of meaning, modes of argument, and mathematical practices. Looking at the transcript, we can ask: What mathematical practices did Julian display? Julian was participating in three central mathematical practices, abstracting, generalizing, and imagining. He was describing an abstract property of parallel lines and making a generalization saying that parallel lines will never meet. He was also imagining what happens when the parallel sides of a rectangle are extended. If we only focused on vocabulary, we would miss Julian's participation in these important mathematical practices.

While vocabulary is necessary, it is not sufficient. Learning to communicate mathematically is not merely or primarily a matter of learning vocabulary. The question is not whether students who are ELs should learn vocabulary but, instead, how instruction can best support students as they learn both vocabulary and mathematics. Vocabulary drill and practice is not the most effective instructional practice for learning vocabulary. Instead, vocabulary and second-language acquisition experts describe vocabulary acquisition as occurring most successfully in instructional contexts that are language-rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz & Fisher, 2000). To develop written and oral communication skills students need to participate in negotiating meaning and in tasks that require output from students (Swain, 2001). In sum, instruction should provide opportunities for students to participate in mathematical practices, actively using mathematical language to communicate about and negotiate meaning for mathematical situations.

Recommendation #3: Recognize and support students to engage with the complexity of language in math classrooms

Language in mathematics classrooms is complex and involves multiple modes (oral, written, receptive, expressive, etc.), multiple representations (objects, pictures, words, symbols, tables, graphs, etc.), different types of written texts (textbooks, word problems, student explanations, teacher explanations, etc., different types of talk (exploratory and expository), and different audiences (presentations to the teacher, to peers, by the teacher, by peers, etc.). "Language" needs to expand beyond talk to consider the interaction of the three semiotic systems involved in mathematical discourse—natural language, mathematics symbol systems, and visual displays. Instruction should recognize and strategically support EL students' opportunity to engage with this linguistic complexity. Looking at the transcript, we can ask: What modes of expression did Julian and the teacher use? Julian used gestures and objects in his description, running his fingers along the parallel sides of a paper rectangle. The teacher also used gestures and visual displays of geometric figures on the blackboard. This example shows some of the complexity of language in the mathematics classroom.

Instruction needs to distinguish among multiple modalities (written and oral) as well as between receptive and productive skills. Other important distinctions are between listening and oral comprehension, comprehending and producing oral contributions, and comprehending and producing written text. Different mathematical domains, genres of mathematical texts, for example word problems and textbooks. Materials need to support and consider how artifacts serve as mediators. Instruction should support movement between and among different types of texts, spoken and written, among texts such as homework, blackboard diagrams, textbooks, interactions between teacher and students, and interactions among students.[1] Instruction should recognize the multimodal and multi-semiotic nature of mathematical communication, move from viewing language as autonomous and instead recognize language as a complex meaning-making system, and embrace the nature of mathematical activity as multimodal and multi-semiotic (Gutierrez et al., 2010; O'Halloran, 2005; Schleppegrell, 2010).

Recommendation #4: Treat everyday language and experiences as resources, not as obstacles

Everyday language and experiences are not necessarily obstacles to developing academic ways of communicating in mathematics (Moschkovich, 2002, 2007c). It is not useful to dichotomize everyday and academic language (Gutierrez et al., 2010; Moschkovich, 2010). Instead, instruction needs to consider how to support students in connecting the two ways of communicating, building on everyday communication, and contrasting the two when necessary. In looking for mathematical practices, we need to consider the spectrum of mathematical activity as a continuum rather than reifying the separation between practices in out-of-school settings and the practices in school (Gutierrez et al., 2010). Rather than debating whether an utterance, lesson, or discussion is or is not mathematical discourse, teachers should instead explore what practices, inscriptions, and talk mean to students and how they use these to accomplish their goals. Instruction needs to shift from monolithic views of mathematical discourse and dichotomized views of discourse practices and consider everyday and scientific discourses as interdependent, dialectical, and related rather than assume they are mutually exclusive. Looking at the transcript, we can ask: What language resources did Julian use to communicate his mathematical ideas? He used colloquial expressions such as "go higher" and "get together" rather than the formal terms "extended" or "meet." These everyday expressions were not obstacles but resources.[2]

Recommendation #5: Uncover the mathematics in what students say and do

Looking at the transcript, we can ask several questions that illustrate this recommendation: How did the teacher respond to Julian's contributions? The teacher moved past Julian's confusing uses of the word "parallela" to focus on the mathematical content of Julian's contribution. He did not correct Julian's English, but instead asked questions to probe what the student meant. This is significant in that it represents a stance towards student contributions during mathematical discussion: listen to students and try to figure out what they are saying. When teaching English learners, this means moving beyond vocabulary, pronunciation, or grammatical errors to listen for the mathematical content in student contributions. (For a discussion of the tensions between these two, see Adler, 2001.) What instructional strategies did the teacher use? The teacher used gestures and objects, such as the cardboard geometric shapes, to clarify what he meant. For example, he pointed to vertices and sides when speaking about these parts of a figure. Although using objects to clarify meanings is an important ESL instructional strategy, it is crucial to understand that these objects do not have meaning that is *separate* from language. Objects acquire meaning as students talk about them and these meanings are negotiated through talk. Although the teacher and the students had the geometric figures in front of them, and it seemed helpful to use the objects and gestures for clarification, students still needed to sort out what "parallelogram" and "parallel" meant by using language and negotiating common meanings for these words.

Overall, the teacher did not focus on vocabulary instruction but instead supported students' participation in mathematical arguments by using three instructional strategies that focus more on mathematical discourse: (1) Building on student responses: The teacher accepted and built on student responses. For example in turns 4–5, the teacher accepted Julian's response and probed what he meant by "parallel." (2) Asking for clarification: The teacher prompted the students for clarification. For example, in turn 7 the teacher asked Julian to clarify what he meant by "they." (3) Re-phrasing: The teacher re-phrased (or re-voiced) student statements, by interpreting and rephrasing what students said. For example, in turn 10 the teacher rephrased what Julian had said in turn 8. Julian's "the parallela, they" became the teacher's "sides" and Julian's "they never get together" became "will never meet." The teacher thus built on Julian's everyday language as he re-voiced Julian's contributions using more academic language.

Researchers and practitioners alike need to recognize the emerging mathematical reasoning that English learners construct in, through, and with emerging language. To focus on the mathematical meanings English learners construct—rather than the mistakes they make or the obstacles they face—curriculum materials, professional development, and training for researchers needs to focus on recognizing emerging mathematical reasoning that expressed through emerging language. Professional development should support teachers in uncovering the mathematics in student contributions, when to move from everyday to more mathematical ways of communicating, and when and how to develop mathematical precision (Schleppegrell, 2010).

Acknowledgments

This work was supported by Grants #REC-9896129 and #ROLE-0096065 from NSF. The Math Discourse Project at Arizona State University videotaped this lesson with support by an NSF grant.

Endnotes

[1] Topics for further research include defining linguistic complexity for mathematical texts and providing examples of linguistic complexity that go beyond readability (such as the syntactic structure of sentences, underlying semantic structures, or frequency of technical vocabulary, verb phrases, conditional clauses, relative clauses, and so on).

[2] The question of whether mathematical ideas are as clear when expressed in colloquial terms as when expressed in more formal language is highly contested and not yet, by any means, settled.

References

Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Netherlands: Kluwer Academic Press.

AERA. (2006). Do the math: Cognitive demand makes a difference. *Research Points, 4*(2).

Blachowicz, C., & Fisher, P. (2000). Vocabulary instruction. In M. Kamil, P. Mosenthal, P. D. .Pearson, & R. Barr (Eds.), *Handbook of reading research* (Vol. 3, pp. 503–523). Mahwah, NJ: Lawrence Erlbaum Associates.

Cavanagh, S. (2005, July). Math: The not-so-universal language. *Education Week*. Retrieved August 2, 2009, from http://www.barrow.k12.ga.us/esol/Math_The_Not_So_Universal_Language.pdf

Crowhurst, M. (1994). *Language and learning across the curriculum*. Scarborough, Ontario: Allyn and Bacon.

Gándara, P., & Contreras, F. (2009). *The Latino education crisis: The consequences of failed social policies*. Cambridge, MA: Harvard University Press.

Garcia, E., & Gonzalez, R. (1995). Issues in systemic reform for culturally and linguistically diverse students. *Teachers College Record, 96*(3), 418–431.

Gutierrez, K., Sengupta-Irving, T., & Dieckmann, J. (2010). Developing a mathematical vision: Mathematics as a discursive and embodied practice. In J. N. Moschkovich (Ed.). *Language and mathematics education: Multiple perspectives and directions for research*. Charlotte, NC: Information Age.

Halliday, M. A. K. (1978). Sociolinguistics aspects of mathematical education. In M. Halliday (Ed.), *The social interpretation of language and meaning* (pp. 194–204). London: University Park Press.

Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Reston, VA: NCTM.

Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.

Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*(2–3), 189–212.

Moschkovich, J. N. (2007a). Beyond words to mathematical content: Assessing English learners in the mathematics classroom. In A. Schoenfeld (Ed.), *Assessing mathematical proficiency* (pp. 345–352). New York: Cambridge University Press.

Moschkovich, J. N. (2007b). Bilingual mathematics learners: How views of language, bilingual learners, and mathematical communication impact instruction. In N. S. Nasir & P. Cobb (Eds.), *Diversity, equity, and access to mathematical ideas* (pp. 89–104). New York: Teachers College Press.

Moschkovich, J. N. (2007c). Examining mathematical discourse practices. *For The Learning of Mathematics*, *27*(1), 24–30.

Moschkovich, J. N. (2010). *Language and mathematics education: Multiple perspectives and directions for research*. Charlotte, NC: Information Age.

O'Halloran, K. L. (2005). *Mathematical discourse: Language, symbolism and visual images*. London: Continuum.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London: Routledge.

Schleppegrell, M. (2010). Language in mathematics teaching and learning: A research review. In J. N. Moschkovich (Ed.), *Language and mathematics education: Multiple perspectives and directions for research*. Charlotte, NC: Information Age.

Stein, M. K. Grover, B., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal,* 33(2), 455–488.

Swain, M. (2001). Integrating language and content teaching through collaborative tasks. *Canadian Modern Language Review, 58*(1), 44–63.