# CONTRASTING CHARACTERIZATIONS OF CHANGE AMONG PROMINENT THEORETICAL PERSPECTIVES IN MATHEMATICS EDUCATION

**Michael A. Tallman**                                     **Eric Weber**
Arizona State University                              Oregon State University
michael.tallman@asu.edu                          eric.weber@oregonstate.edu

*In this paper, we problematize an ontological characterization of change within a complex system by illustrating how epistemological premises of interactionist, individualist, and collectivist theoretical perspectives reveal only specific aspects of a changing system. Methodological considerations resulting from our recognition that change is characterized subjectively within various theoretical perspectives are made.*

Keywords: Research Methods; Measurement

## Introduction

A central focus of work in mathematics education research is the characterization of change—which we view as the entailment of three processes: (1) identifying whether or not change has occurred, (2) identifying the amount of change that has occurred, and (3) identifying the potential causes of change. One may be interested in characterizing change in students' ways of understanding as they progress through a particular instructional sequence. Another may focus on explaining how the norms of a classroom environment evolve over the course of a semester and how individual students' perceptions of, and activity within, these norms change in tandem. Still another may focus on describing how institutions change in response to educational policies or reform initiatives. Change is everywhere, is occurring all the time, and whether one is studying students, teachers, or institutions, the characterization of change is an integral part of contemporary mathematics education research.

If one seeks to characterize change in a complex system, a key consideration must be to recognize the system within which that change occurs and to make assumptions about how that system can undergo change. The identification and explanation of change within a complex system depends largely on the epistemological assumptions one makes about knowledge and about learning as well as the ways in which agents of change are characterized within the system. While there have been articles that focus on describing the assumptions and practices of various theoretical perspectives (e.g., Cobb, 2007), none have explicitly focused on how specific theories of learning characterize change. The focus of the present article is to explain the basic assumptions regarding learning and knowledge, and its growth held by prominent theoretical perspectives in mathematics education research. In addition, we show how the assumptions of various learning theories serve as a lens through which change is perceived in order to illustrate how change within a single system can be characterized in different ways depending on the premises of the theoretical perspective that a researcher assumes.

In our view, any characterization of change will necessarily entail limiting the scope of analysis to specific aspects of the changing system. As a result, the epistemological assumptions made by the researcher as well as the unit of analysis chosen, constrain the type of change that can be characterized within a system, such as a classroom. Hence, we use the word *comprehensive change* to convey all aspects of the changing system, including all *anticipated* representations and causal conditions of change, that are hypothesized to collectively comprise some measurable change in a system that occurs over some interval of time.

A major focus in this article will be to problematize the notion that empirical mathematics education research characterizes change as if it is an ontological and agreed upon construct. Our review of the literature suggests that taken as a whole, current work has not rigorously addressed or defined comprehensive change. As a result, there have been few efforts that examine change in a learning system by conscientiously defining the learning system and its boundaries in order to characterize the mechanisms by which different variables within the system interact. We believe that any attempt to study change

requires one to characterize a learning system and the interactions that take place within the learning system based on the assumptions of particular theoretical orientations. We propose that the theoretical perspective that one assumes serves as a lens through which one attempts to "control" specific aspects of the changing system to construct a viable characterization of change with respect to specific features of the changing system. We argue that the specific features of the changing system that one attends to, and their hypothesized effects on other aspects of the system, are largely determined by the epistemology of one's theoretical orientation.

## Survey of Theoretical Perspectives

Mathematics education researchers currently use a plethora of theoretical perspectives that originate from fields such as cognitive science, sociology, anthropology, and psychology. Such perspectives include but are not limited to, radical constructivism, behaviorism, sociocultural theory, situated cognition, cognitive information processing theory, cognitive psychology, experimental psychology, social cognitive theory, and social constructivism. To efficiently contrast the characterization of change among these perspectives, we have organized them into three general categories that account for their historical treatment of learning, and the unit of analysis by which these traditions assume learning can be understood: *interactionism*, *individualism*, and *collectivism*. Our rationale for this partition resulted from recognizing that the epistemological tenets in the theoretical perspectives prevalent in mathematics education research are not distinct, but instead are championed, shared, and modified by researchers in a learning system. This partition is helpful to understand the ways in which the field is conflicted in its message about change, the subsequent claims we can make about how change occurs, and the recommendations we can make regarding potential levers for enhancing growth in mathematical teaching and learning.

We begin by outlining the characteristics of interactionism, individualism, and collectivism; providing brief summaries of select theoretical perspectives that are encompassed within each class of perspectives relative to their characterization of individual agents in a learning system as well as their epistemological stance. This discussion highlights the potential conflicts arising in our field from people speaking the same words, but not meaning the same things regarding the study of change in teaching, learning, and policy. We conclude with methodological considerations resulting from our recognition that change is characterized differently within different theoretical perspectives.

### Interactionism

Interactionism encompasses theoretical perspectives that consider cognizing agents as subjective interpreters situated in social and societal contexts. In interactionism, individual behavior is dictated by subjective interpretations of social experiences that cannot resemble an objective existence. To the interactionist, individual agents create their experiential world and act within their own experiential world. However, interactionism does not disregard an outside world, but at the same time makes no claims about the existence of a single ontological reality. This is because perspectives in interactionism assume that we cannot step outside of ourselves to observe a "real world", as the world is a subjective reality. Thinking about the external world as a subjective reality allows interactionists to describe students' learning as taking place within an experiential world that they are simultaneously organizing as they learn and create new knowledge. Perspectives that comprise interactionism include situated cognition, radical constructivism, and social constructivism.

**Situated cognition.** Situated cognition (Brown, Collins, & Duguid, 1989) frames the individual as a component of a reasoning *system* that is comprised of the individual's *immediate* social, physical, and psychological environment. The external influence on one's cognition is immediate in the sense that there is a consistent interaction between the individual and the reasoning system. Moreover, the external influence of one's cognition is also dynamic in the sense that the reasoning system within which one participates is amenable to rapid change (i.e., is responsive to feedback from the environment). Learning, then, is characterized by the extent to which an individual is able to effectively coordinate elements of their

immediate social, physical, or psychological environment as a reasoning aid. In other words, learning in the situated cognition perspective is characterized by an individual's ability to become a productive component of the reasoning system by using their immediate external resources productively.

Further, situated cognition assumes that knowing or understanding is inseparable from doing. Knowledge is characterized as competence with respect to norms of the setting in which one operates and accrues as one gains experience working within the constraints of a particular context. It is the individual's interaction with social norms, however, that takes precedence over group dynamics. Learning is characterized, then, as increasingly effective performance and higher levels of competency across situations (Wenger, 1998). As a result of these assumptions, a situated cognitivist might describe change as the level of familiarity one has working in a particular context or job, or the degree to which one is able to skillfully manipulate tools and representations in a discussion in the classroom. This is because change is not characterized by an accumulation of associations but instead is the attunement of actions between the agent and its environment, and that dynamic is necessary for the characteristics of learning and knowledge to be made manifest.

In the interest of full disclosure, we recognize that aspects of the situated cognition perspective lend themselves to the collectivist paradigm. However, as mentioned above, the analytical unit within the situated cognition perspective is the cognitive behavior of individuals situated within a reasoning system. Hence, we find it more appropriate for situated cognition to be considered principally a subset of interactionism rather than collectivism.

**Constructivism.** Radical and social constructivism are variants of a more general learning theory of constructivism. Constructivism is an epistemology asserting that humans construct knowledge and meaning from their perceptions of their own interactions with the experiential world. Formalization of constructivism is typically attributed to Piaget, who focused on the mechanisms by which learners internalize knowledge. Piaget's genetic epistemology emphasized how a cognitive organism, such as a human, becomes a cognizing agent. Piaget described adaptation and organization as the key principles to biological development. These principles of adaptation and organization are the key components of constructivism, but are interpreted in different ways by radical and social constructivists. For the sake of brevity, we detail only radical constructivism in this article.

**Radical constructivism.** Radical constructivism is a philosophical perspective on learning based on Piaget's more general notion of constructivism, which is concerned with the paradox of how one comes to "know" an ontological reality when one cannot step outside of his or her own ways of thinking and ways of perceiving reality. Radical constructivism posits that one perceives a subjective reality through adaptation and organization of ways of thinking, which von Glasersfeld (1995) operationalized as assimilation and accommodation.

von Glasersfeld (1995) and others (e.g., Thompson, 2000), often based on Piaget's genetic epistemology, have focused on the constructs of assimilation, accommodation, and equilibration to explain how one comes to create, refine, and evaluate a viable mental model of the world around them by focusing on conceptual analysis. Conceptual analysis is the construction of a scheme of meanings and ways of understanding that make one's actions sensible and coherent. In short, conceptual analysis allows an observer, who cannot observe another's subjective reality, to nonetheless create a viable model that makes ones actions coherent. The developmental of this mental model allows one not only to describe and explain, but also predict one's actions based on the model of one's ways of understanding a particular idea. Thus, a model can be a viable representation of the assimilation, accommodation, and equilibrium states of the student. At the same time, radical constructivists are constrained in explaining the thinking of another because they are dependent on making inferences about one's mental model from the language and actions. Thus, a characterization of change within the radical constructivist tradition depends on tracking the changes in the mental model of one's ways of understanding and ways of thinking.

## Individualism

Individualist theoretical perspectives assert that individual's comprise the primary unit of reality and that societies emerge as a consequence of individual behavior. However, individualists contend that societies do not determine the identity, or govern the behavior of individuals within societies. Theoretical perspectives that comprise interactionism include cognitive information processing theory and experimental psychology.

**Cognitive information processing theory.** The essence of cognitive information processing is that human thought and cognition are treated as computational in nature. This theory assumes that existing mental structures process stimuli, and that knowledge is structured *in memory* as an association between concepts that have numerous branches to other concepts.

Cognitive information processing theory holds that attention is the primary mechanism by which knowledge is developed. Since individuals maintain the inherent propensity to organize information obtained from sensory input, the stimuli that individuals attend to among many potential inputs necessarily determines what information has the potential to be stored in working memory.

Information processing theory treats the processing of stimuli much like a computer program. In particular, our nervous system registers a sensory input, which is perceived and filtered through attention and interpretative structures into working memory. As a consequence, learning can be thought of as the process where new information is "fitted" into existing cognitive structures, often characterized as long term memory. Thus, the development of existing networks of understandings stored in long-term memory characterizes change within the cognitive information processing paradigm (Gagne, 1985).

**Experimental psychology.** Research within experimental psychology aims to develop a *collective abstract individual*. A collective abstract individual is collective in the sense that it is devised from a statistical aggregate of quantifiable attributes, and abstract in the sense that the individual need not correspond to the attributes of any particular individual in the group that comprised the statistical aggregate (Cobb, 2007). In the experimental psychology perspective, measurable characteristics of individual students are perceived to consist of discrete, isolatable attributes that can be measured with some fidelity and aggregated using quantitative methods. Thus, the amount that one has learned is measured by the extent to which one deviates from the statistical aggregate that comprises the collective abstract individual. More specifically, an aim of experimental psychology is to determine one's discrete, isolatable attributes at two or more moments in time and compare these attributes to those of the collective abstract individual. As a result of these assumptions, experimental psychology allows one to make probability estimates in the population regarding student thinking, motivation, or reactions. A decrease in deviation over time, which can be quantified, serves as evidence of learning within this perspective. However, experimental psychology does not explicitly define a lens through which causal factors for change of an individual within a learning system are identified. Rather, educational research within the experimental psychology paradigm has traditionally assumed a process-product orientation in which desired learning outcomes are attributed to observable teaching behaviors with an inattention to the cognitive or affective causal factors of learning. Hence, experimental psychologists limit the potential causal factors of change by considering only the independent variables that are hypothesized at the outset of an experiment. Therefore, causal factors of change do not have the opportunity of manifesting themselves throughout the conduct of research as a consequence of experimental psychology methodology.

## Collectivism

Collectivist perspectives consider individual behavior and cognition to be fundamentally influenced by their situation within social and societal contexts. Accordingly, the analytical unit within collectivist theoretical perspectives is the activity of the culture or *collective*. Individuals serve as contributing agents in the collective as they participate in established cultural practices. As a complex system, the collective activity is an emergent property of the individual actions of its members and their interaction (Cobb &

Yackel, 1996).  Norms and other social behaviors form the basis for understanding learning.  Sociocultural theory is the predominant collectivist theoretical perspective.

**Sociocultural theory.** Sociocultural theory situates the individual within a general social environment and considers the individual's cognition inseparable from their more general social circumstances. Accordingly, many sociocultural theorists consider the individual-as-situated-in-a-cultural-practice as the appropriate analytical unit. Hence, learning in the sociocultural perspective is evidenced by "changes that occur in people's activity as they move from relatively peripheral participation to increasingly substantial participation in the practices of established communities" (Cobb, 2007, p. 24). That is, sociocultural theorists hold that cognitive behavior and participation in cultural practices co-participate in each other's evolution. This perspective differs from that of situated cognition in that situated cognitive theorists consider the relationship between cognizing subject and external environment to remain fixed. It is the recognition that intellectual development and cultural participation co-evolve that characterizes sociocultural theory as a collectivist perspective.

Sociocultural theorists identify change of an individual within a learning system by whether or not a social participant's activity is modified as they increase their participation in established cultural practices. The interaction between a participant and their social and cultural environment always serves as the causal factor for change within sociocultural theory.

## Methodological Implications of Studying Change

We have thus far described major theoretical perspectives through the lenses of individualism, interactionism, and collectivism, and in doing so have shown that if one seeks to describe change within a complex system, the boundaries and assumptions about interaction of variables within the system constrain the type and amount of change that one can characterize.

We believe individualist, interactionist, and collectivist paradigms are uniquely powerful for characterizing various aspects of change within a complex learning environment, and claim that problematizing comprehensive change has important methodological implications. It is critical to understand the type of change at play, and we believe the individualist, interactionist, and collectivist perspectives are helpful in making this distinction. In this section, we consider methodological implications that one must consider in order to rigorously study change in a learning environment.

### Research Question

Since a variety of aspects of a complex system are changing in tandem, and as we have argued, they cannot all be characterized simultaneously, researchers must assume the responsibility to explicate the ways in which the theoretical perspective they assume imposes a limit on the nature of change they are able to characterize. Demonstrating the recognition that one's theoretical orientation imposes conceptual blinders on specific aspects of the changing system in the statement of one's research questions is an essential aspect of communicating one's research in a way that promotes intersubjectivity among author and reader.

Because it is impossible to simultaneously characterize every type of change, a research question must address three issues. First, it must be specific enough so that the unit of analysis is unambiguous. Second, it must characterize the system within which the unit of analysis is to be studied. Third, it must specify a particular aspect of the complex system to be studied, including relevant variables and their interactions. These three considerations permit the researcher to specify what is to be studied, to determine at what grain level it is to be studied, and to demarcate boundaries and constraints within which the unit of analysis operates. These considerations not only confirm epistemological and theoretical coherence, but also allow the researcher to classify their characterization of change as individualist, interactionist or collectivist. This classification accordingly results in the recognition of changing aspects of a complex system that are not recognized by the researcher's method.

A research question that clearly identifies the unit of analysis and demarcates the boundaries within which the unit of analysis operates constrains the type of change that one can claim to characterize.

Constraining the type of change under consideration allows the researcher to identify a theoretical framework composed of descriptive and explanatory components that can characterize change in the unit of analysis. We do not claim that any of these frameworks are more appropriate than another. Instead, the usefulness of the framework in a study focused on characterizing arises from its ability to describe, explain, and even predict aspects of the complex system under study while fitting within the constraints of the boundaries of the system.

We recognize that in many cases the theoretical framing may constrain the development of the research question instead of the research question constraining the theoretical framework. In this case, one might start with the desire to characterize change using a collective, interactionist, or individualist perspective. Whatever research question develops from these constraints must still meet our three proposed specifications. This promotes the theoretical coherence of the framework and research question.

**Design of Experiment: Data Collection**

We believe the focus of experimental design must address the type of data that should be collected to adequately address the proposed research questions. Addressing this concern is critical to generating a data corpus that allows the researcher to characterize change within a particular component of a complex system. Accordingly, we describe the types of data collection crucial to characterizing change within the individualist, interactionist and collectivist paradigms.

Individualism recognizes change as a modification of an individual's behavior independent of their social practices and attributes the change to an individual's orientation to focus on behavior without regard to social influence. The amount of change can be measured by the displacement in alignment between an individual's behavior and idealized behavior between two or more moments in time. Thus, any data collected within the individualist paradigm must allow the researcher to make inferences about student's behavior patterns to generate a working model of those behavior patterns. Development of this working model is crucial to identifying any robust changes in behavior. Behavior patterns can be documented by tracking verbal cues, gestures, and written work as the student reasons through a particular problem, in a group of students, or with a computer program. A shift in verbal cues or gestures can suggest a change in behavior, which can then be studied in more detail. Whatever the setting in which the data is collected, when the focus of the data is on the student's individual actions, the data corpus can support characterizing change in an individualist paradigm.

Interactionism considers change as a modification of one's interpretation of experiential reality and attributes this change to a reorganization of cognitive structures initiated by an interaction with external stimuli. The amount of change is given by a displacement between one's interpretation and an intended interpretation between two or more moments in time. Any data collected within the interactionist paradigm must allow the researcher to make inferences about a student's model of the experiential world because change cannot be identified and explained without an initial working model. As with individualism, verbal cues, written text and gestures are most useful. In order to create a model of the student's experiential world, the researcher must create situations in which the student experiences constraints on their perception or thinking. It is not until the researcher experiences the constraints of the student that he or she can make a claim about the boundaries of a student's experiential world. Change then, can be characterized when the boundaries of the student's mental model of the world or a particular mathematical idea begins to shift. By focusing on the boundaries of a student's thinking, the researcher can continually generate and test hypotheses in order to create an increasingly viable and explanatory model of a student's mental model of the world.

Collectivism considers change as a modification of a social participant's activity as they participate in established cultural practices and attributes this change to the interaction between a participant and his or her culture. Accordingly, the amount of change is measured by the displacement in alignment between a social participant's activity and the established cultural practices between two or more moments in time. Data collected within the constraints of the collectivist paradigm must allow the researcher to characterize the social participant's activity as well as the cultural practices and the social participant's perception of those cultural practices. The collectivist paradigm requires the researcher to think about the social

participant's perception and interaction as part of a collective, which might be the classroom in which they participate. The researcher must document the actions, including verbal cues, gestures, discussions, and written work of not only the individual student, but also the classroom as a whole. In collectivism, the classroom, not the individual, defines the boundaries of the system. The individual works within the boundaries of this larger system, but is not the focus within the collectivist paradigm. Thus, the researcher must be systematic about creating situations in which he or she can experience the boundaries of the classroom as a collective. The researcher can, at best, create a model that describes and explains the boundaries of the classroom as a collective, and this model can only come from the actions of the classroom as a whole. As the model of the classroom as a collective becomes more viable, just as in interactionism, the researcher is able to identify more subtle shifts (change) in the system.

**Design of Experiment: Microgenetics and Density of Observations**

Assuming that one has specified a type of data that adequately attends to the research question, how do we know if the amount of data is sufficient for creating a viable model of the individual or collective? The density and duration of time over which the observations are taken is critical. Siegler and Crowley (1991) addressed this issue with microgenetics, which has three properties. First, observations span the period from the initiation of a change to the end of a change, marked by the stability of a system under study. Second, the density of observations is high relative to the rate of change of the phenomenon. In short, the rate of change of number of observations with respect to time increases if one anticipates the system to be at a point of a critical change. Third, observed behavior undergoes trial-by-trial analysis with the goal of attributing causal agents to particular aspects of change in a system. (Siegler & Crowley, 1991, p. 606).

For example, suppose that a researcher is attempting to create a mental model of a student's thinking as he or she participates in a two-week long instructional sequence. The researcher believes that the major shifts in student's thinking will occur on days 1, 4 and 9 based on analysis of the instructional sequence. Thus, the researcher may increase the density of observations (i.e. number of documented actions, verbal cues, or gestures) on days 1, 4 and 9 relative to the other days in the instructional sequence. These observations take place at the moment the researcher anticipates a major shift to begin occurring and ends when the researcher's model of the student's thinking becomes relatively stable.

## Discussion

In this paper, we have problematized an ontological characterization of change within a complex system by illustrating how epistemological premises of interactionist, individualist, and collectivist theoretical perspectives reveal only specific aspects of a changing system. Moreover, methodological considerations resulting from our recognition that change is characterized subjectively within various theoretical perspectives were made. The methodological recommendations advanced in this paper intend to support the intersubjective interpretation of research findings by promoting researchers' clarification of the ways in which their theoretical orientation constrains their recognition of various aspects of the changing system under study.

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