HOW STUDENTS REASON DIFFERENTLY IN EVERYDAY AND MATHEMATICAL CONTEXTS: TYPICALITY AND EXAMPLE CHOICE IN JUSTIFICATION

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Middle school students bring with them to the classroom powerful, informal resources for reasoning about mathematical ideas. However, little research has examined how these resources can interact with or support skills of mathematical justification. Here, we explore how middle school students consider inductive strategies—the use of examples in proof—when confronted with conjectures. We discuss ways in which these students might reason about mathematical objects like numbers and shapes strategically as they test examples. We argue that critical to such strategic reasoning is flexible application of mathematical and everyday ways of knowing.

Keywords: Reasoning and Proof; Middle School Education

Students bring with them to the mathematics classroom powerful intuitive ways of reasoning based on their everyday experiences interacting with the world. An important goal in mathematics education has been to find ways to leverage these resources or "funds of knowledge" (Moll & Gonzalez, 1997) to support mathematics learning. However, this search has proven to be problematic. Students not only have trouble applying their "school math" knowledge to complex, situated real world problems (e.g., Masingila, Davidenko, & Prus-Wisniowska, 1996; Walkington, Sherman, & Petrosino, 2012), but they struggle to productively use knowledge from their everyday experiences in school-based tasks (e.g., Reusser & Stebler, 1997; Walkington, Nathan, Wolfgram, Alibali, & Srisurichan, in press). A situated perspective on learning acknowledges that the interplay between the practices valued in school and everyday activity is complex, and that these two sets of practices will not always overlap. However, recent work has uncovered ways for students' concrete, situated experiences to support the learning of mathematical formalisms (e.g., Walkington & Sherman, 2012).

One area in which the interaction between everyday experience and formal mathematical knowledge has not been well-examined is mathematical justification. Here, we define justification or proving as "the process employed by an individual to remove or create doubts about the truth of an observation," and emphasize that this process is often based on intuition, internal conviction, and necessity (Harel & Sowder, 1998, p. 243). The importance of the construction and evaluation of mathematical arguments is accentuated in both the Common Core and NCTM standards (CCSSI, 2010; NCTM, 2000). But how are children's intuitive ways of reasoning important when considering mathematical justification, which has been traditionally characterized as a formal and disembodied chain of axiomatic, deductive statements?

Recent research has revealed the *inductive* or *example-based* reasoning strategies that children use when considering mathematical conjectures (Knuth et al., 2011). For instance, when presented with the conjecture "the sum of any two even numbers is even," a student might test different even numbers, like 2 and 20. Some studies have suggested that this kind of reasoning might allow students develop more general arguments (Knuth et al., 2011). Here we will examine how students' everyday and mathematical knowledge interacts with their evaluation of example-based justifications. We argue that students must navigate along a learning continuum as they gain expertise with mathematical arguments, which ultimately leads to flexible and appropriate application of everyday and mathematical knowledge. Gaining an understanding of this continuum, of the ways in which students think about the nature of evidence in inductive justification, may help mathematics educators in better supporting students' learning to prove.

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Theoretical Framework

Use of Example Objects in Justification

Many of the problems people face in life resist formal solution. There is no deductive proof for beliefs about friends, nor a valid algorithm for picking a spouse. Instead people must employ inductive reasoning strategies. Some of the most well studied inductive strategies in the cognitive science literature are example-based (see Feeney & Heit, 2007). One way to decide if a person will be a good friend is to compare them to others. But which others? Children and adults employ a number of strategies for selecting good examples in their everyday lives, strategies that often are in line with formal principles of inductive inference.

In mathematics, students also tend to use inductive reasoning when confronted with conjectures (Chazan, 1993; Knuth et al., 2011; Harel & Sowder, 1998). Such reasoning has sometimes been identified as problematic because students may use *only* examples, without moving towards more powerful general arguments. However, examples may still play a critical role in understanding conjectures and constructing more general justifications. For instance, mathematicians use examples as tools when confronted with conjectures (Alcock, 2004). Expert mathematicians ($N = 133$) indicated they use examples to verify and understand conjectures, generalize from examples to a proof, and seek counter-examples or try to "break" the conjecture (Lockwood et al., 2012). Examples play an important role in the development of proofs.

Typicality and Example Choice in Non-Mathematical and Mathematical Domains

In scientific domains, three principles of example selection (see Osherson et al., 1990) have been identified as useful when drawing conclusions about a class or type: *quantity*—more examples are better than fewer, *diversity*—a wide variety of examples are better than a set of very similar examples, and *typicality*—generic or "average" examples are better than special or "weird" examples. Thus in trying to decide whether all birds have hollow bones, one would want to check many birds, a diverse set of birds, and relatively typical birds. Here we focus on typicality—a typical example shares properties with many members of its class and has few distinctive properties. One challenge in developing accounts of example-based inference is identifying which features are used to compute typicality. In science, people have robust intuitions about features that are "biologically relevant." That cats and goldfish are both kept as pets does not seem relevant in determining their biological relatedness. However, untangling everyday notions of typicality from typicality based on properties of *mathematical* objects may be more difficult.

In previous work, we found it useful to distinguish two types of mathematical typicality (Williams et al., 2011). The *everyday typicality* of an object is how common it is in everyday life—i.e., how many experiences a person has with objects of that kind in their day-to-day activity. The *mathematical typicality* of an object is how typical it is when its mathematical properties are considered in relation to the properties of *all* objects of that type. The number "0" would be a mathematically *atypical* number because it has properties that no other integers share (e.g., additive identity). The number 322 might be a *typical* number in a mathematical context because it does not have many properties that make it distinct from the set of whole numbers. Middle school mathematics is an interesting site for exploring these two types of typicality, as many of the objects that are highly *atypical* mathematically (e.g., numbers like 0 or 1) are highly *typical* in everyday life. Students may struggle to reconcile these two different conceptions of typicality. But do typicality judgments really matter when considering mathematical justifications?

When the expert mathematicians ($N = 133$) were asked how they choose examples when exploring conjectures, many responses referenced the typicality of their examples. They reported choosing common examples with no special properties or generic or general examples, unusual, obscure, or "tricky" examples, examples with special properties, and examples that are boundary cases (Lockwood et al., 2012). These mathematicians seemed to have found ways to use typicality *strategically*—to allow typicality judgments to support and inform their exploration of mathematical conjectures. But what about middle school students? Do they consider typicality when exploring conjectures with examples, and if so, what type of typicality?

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We presented middle school students (*N* = 20) with conjectures about numbers, and students reported purposefully varying the typicality of the examples they chose when testing conjectures. Students reported trying to test both typical and atypical numbers, or trying to test unusual numbers to see if the conjecture would hold (Cooper et al., 2011). Students' reports of what made a number typical varied—some were attuned to whether the number was prime or the relative size of the number, while others identified typical numbers based on their everyday experiences. Overall, it seemed that students were reasoning strategically about the typicality of their examples. In the present study, we implement a large-scale survey to assess how students use mathematical and everyday typicality when considering examples in justification.

Research Questions

Our research questions are: (1) How do middle school students use typicality *strategically* when considering examples? and (2) How are students' conceptions of mathematical typicality consistent or at odds with their everyday notions of typicality?

There are two dimensions along which middle school students might demonstrate using mathematical typicality strategically. First, students might realize that conjectures that hold for mathematically atypical objects (i.e., objects with mathematically special properties) may not hold for all objects. For instance, a conjecture holding for the number "1" may not be strong evidence that the conjecture would hold for all numbers, since 1 has special properties (e.g., multiplicative identity). However, this conception of mathematical typicality might be directly at odds with students' everyday notions of typicality, because although 1 is highly atypical in a mathematical context, it is highly typical in students' everyday life. Thus if typicality is used strategically, we might see a reversal. Students may recognize that a number like 1 is highly *atypical* in a mathematical context, despite being highly *typical* in an everyday context.

Second, students might use mathematical typicality strategically if they realize that superficial features of a mathematical object are not particularly important when considering whether conjectures that hold for that object will hold for most objects. Students might realize that when a parallelogram is in a non-standard orientation, this is unlikely to impact most mathematical conjectures in middle school mathematics. Similarly, a student might realize that the relative size of a number (e.g., 3 or 103) or the cultural significance of a number (e.g., 13) might not be particularly important. This strategic use of mathematical typicality may be at odds with everyday notions of typicality—in daily life, students are accustomed to seeing shapes in standard orientation and working with relatively small numbers, so objects that do not conform to these experiences might be considered atypical. Thus we argue that strategic use of typicality requires students to flexibly switch between their "everyday" and "mathematical" lenses.

Methods

A total of 475 middle school students (46% female) from a suburban middle school in a Midwestern state were included in the study. Students were distributed across grades 6 (144 students), 7 (160 students), and 8 (163 students), and mathematics classes used reform texts. The school demographics were 48% Caucasian, 21% African American, 14% Asian, 11% Hispanic, and 1% Native American, with 37% low income, and 10% English Language Leaners (ELL).

A survey was administered to all participants during their normal math classes. Each survey contained questions relating to two of four different domains: numbers, parallelograms, triangles, and birds (birds are omitted here). For each domain, students were presented with mathematical objects or items in that domain (e.g., a small equilateral triangle or the number "6") and asked to rate each item's typicality on a 1–7 scale in a mathematical context and in an everyday context. Figure 1 gives an example of the instructions students received on the survey (left) and actual survey items (right). Mathematical objects were selected by the researchers to either cover the space of possible mathematical properties in the domain (e.g., the parallelogram in Figure 1 is a rectangle; we also included squares, rhombi, etc.), or to be completely devoid of any property that would distinguish the object mathematically (e.g., a long, skinny rhomboid with no 90 degree angles). The order of the 9 items within each context and the 2 domains was randomized.

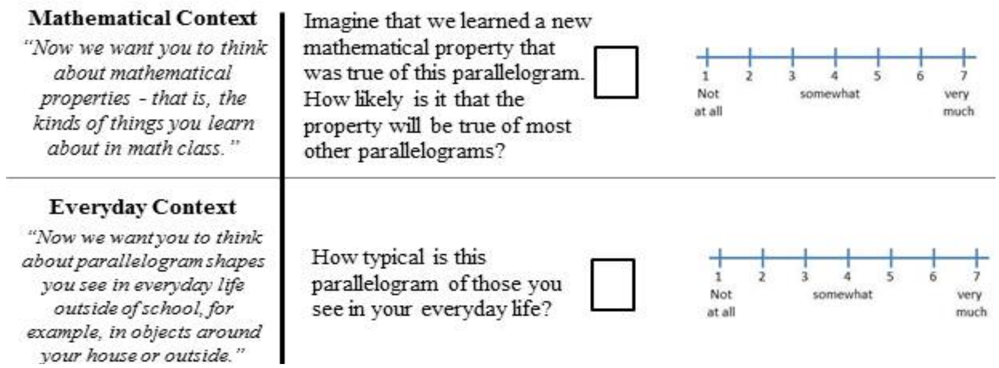

Figure 1: Example of questions on middle school survey

Table 1: Mathematical (italics) and Everyday (underline) Properties Entered into Model

Numbers	Parallelograms	Triangles
Prime or perfect square *Power of 2 or 10* *Multiple of 5 or 10* *Identity properties (i.e., 0 or 1)* Relative magnitude (small or large)	*Square, rectangle, or rhombus* Size (small, large, "skinny") Orientation (standard, non-standard, left-leaning)	*Isosceles, equilateral, scalene* *Obtuse, acute, right* Size (small, large, "skinny") Orientation (standard, non-standard)

The data were analyzed using hierarchical linear regression models (Snijders & Bosker, 1999) where repeated observations were nested within students nested within teachers. Three different models, one per domain, were fit to the data. Random effects included student, teacher, and which mathematical object (i.e., which specific number or shape) the question referenced. Fixed effects included context (Figure 1), the mathematical and everyday properties of the object (Table 1), and the interaction of these two terms. Properties that did not have significant main effects or interactions with context were removed. Fixed effects for gender and grade were not significant in any of the models. Mathematical and everyday properties of numbers, triangles, and parallelograms entered into the model are in Table 1. These properties were chosen based on the mathematical knowledge of a team of mathematicians, psychologists, and mathematics educators and former K–12 teachers, as well as based on previous results from our studies of inductive reasoning (Knuth et al., 2011; Cooper et al., 2011).

Results and Discussion

Number

As can be seen from Table 2, across mathematical and everyday contexts, students rated small numbers ($p < .001$), numbers ending in 5 ($p = .020$), and powers of 10 as being more typical ($p < .001$). This suggests two ways in which students might *not* be considering mathematical typicality strategically. First, students seemed to believe that conjectures that hold for mathematically-special numbers, like powers of 10 or multiples of 5, would be *more* likely to hold for other numbers. From a mathematical standpoint, properties that hold for these numbers may be *less* likely to hold for other numbers. Second, students rated that conjectures that held for small numbers were *more* likely to hold for other numbers. Here, students may have been considering a superficial or mathematically irrelevant feature when considering mathematical conjectures. In both cases, students' everyday notions of typicality, their familiarity encountering small numbers, multiples of 5, and powers of 10 in their lives, may have influenced their mathematical notions of typicality—whether it makes sense for properties that hold for these numbers to hold for most other numbers. We also see no evidence of the desired reversal for mathematical typicality that might evidence strategic thinking. Students did not indicate that numbers with special properties—like prime numbers—were atypical in a mathematical context.

Table 2: HLM Analysis of Students' Typicality Ratings for Number

	Estimate	S.E.	*t*	*p*	Sig.
(Intercept)	3.61	0.46	7.70	< .001	***
Mathematical Context	(ref.)				
Everyday Context	0.19	0.17	1.10	0.274	
Large	(ref.)				
Small	0.77	0.17	4.42	< .001	***
Ends with 5	0.68	0.27	2.55	0.020	*
Power of 10	1.32	0.25	5.39	< .001	***
Prime	-0.17	0.22	-0.76	0.445	
Identity (0 or 1)	-0.37	0.35	-1.05	0.298	***
Everyday Context: Small	0.59	0.09	6.67	< .001	***
Everyday Context: Prime	0.33	0.11	3.07	0.001	**
Everyday Context: Identity	0.55	0.16	3.44	< .001	***

* $p < .05$. ** $p < .01$. *** $p < .001$.

However, looking at the interaction terms in Table 2, we do see evidence that students are at times using mathematical typicality strategically. First, although students rated small numbers as typical regardless of the context, being small had a larger impact on typicality in an everyday context ($p < .001$). This suggests that students may realize that superficial characteristics, like relative size, are less important when considering a number mathematically. Second, students found both prime and the identity numbers more typical in an everyday context ($p = .001$ and $p < .001$). Thus although students expressed their familiarity with these numbers by giving them high everyday typicality ratings, this familiarity did not inflate mathematical typicality ratings.

Parallelograms

Across mathematical and everyday contexts, students rated squares as being more typical (Table 3; $p = .015$). This again suggests that students might not be considering mathematical typicality strategically—these ratings suggest that properties that hold for squares are more likely to hold for other parallelograms. Students' everyday familiarity with squares might be interfering with viewing a square as a mathematical object that has special properties (e.g., 90° angles). We again do not see evidence of the desired reversal—students do not rate mathematically special parallelograms (like squares) as being less typical in a mathematical context.

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Table 3: HLM Analysis of Students' Typicality Ratings for Parallelograms

	Estimate	**S.E.**	***t***	***p***	Sig.
(Intercept)	2.88	0.61	4.7	< .001	***
Mathematical Context	(ref.)				
Everyday Context	0.39	0.27	1.41	0.165	
Standard Orientation	0.43	0.24	1.78	0.080	
Large	(ref.)				
Small	0.19	0.55	0.34	0.710	
Leans Left	-0.51	0.34	-1.52	0.134	
Square	0.88	0.34	2.58	0.015	*
Rectangle	0.63	0.32	1.99	0.055	
Rhombus	0.57	0.40	1.44	0.154	
Everyday Context: Standard Orientation	0.49	0.11	4.49	< .001	***
Everyday Context: Small	-1.04	0.25	-4.20	< .001	***
Everyday Context: Leans Left	0.52	0.15	3.48	< .001	***
Everyday Context: Square	0.55	0.15	3.64	< .001	***
Everyday Context: Rectangle	1.31	0.14	9.30	< .001	***
Everyday Context: Rhombus	0.56	0.18	3.14	0.001	**

* $p < .05$. ** $p < .01$. *** $p < .001$.

However, looking at the interaction terms, we see considerable evidence that students can use mathematical typicality strategically. Although students rated squares as being typical regardless of the context, squares were considered even more typical in an everyday context ($p < .001$). Similarly, students rated rectangles and rhombi as more typical in an everyday context ($p < .001$ and $p = .001$). Students seemed to recognize that although these shapes were common in their everyday lives, this consideration should not inflate their ratings when determining whether properties that hold for these shapes will hold for other shapes. Students also allowed superficial properties of parallelograms—like size and orientation—to influence their everyday typicality ratings ($p < .001$), but not their mathematical typicality ratings.

Triangles

Across mathematical and everyday contexts, students found equilateral, isosceles, and standard orientation triangles more typical (Table 4; $p = .037$, $p = .004$, $p < .001$, respectively) and skinny triangles less typical ($p = .002$). This suggests that students may not be using mathematical typicality strategically. Students expressed that conjectures that hold for special triangles like isosceles and equilateral triangles are more likely to hold in general, and that conjectures that hold for skinny or non-standard orientation triangles, superficial considerations, are less likely to hold in general. Here, again, students do not seem to be differentiating between everyday typicality (the commonness of equilateral and isosceles triangles in their everyday life, and the rarity of skinny and non-standard orientation triangles) and mathematical typicality (whether conjectures that hold for certain triangles are likely to hold for other triangles). We also see no evidence of the desired reversal—students did not rate mathematically special triangles as atypical in a mathematical context. However, looking at the interaction terms, students seem to sometimes reason strategically about mathematical typicality. Although students rated equilateral triangles as typical regardless of context, they were even more typical in an everyday context ($p = .002$). Further, right triangles were typical in an everyday context ($p = .004$), but students did not let everyday familiarity inflate ratings in a mathematical context. Students may realize that although these triangles are highly salient in their everyday experiences, this familiarity should not affect whether conjectures that hold for these triangles will hold for other triangles.

Table 4: HLM Analysis of Students' Typicality Ratings for Triangles

	Estimate	S.E.	*t*	*p*	Sig.
(Intercept)	4.98	0.41	12.16	< .001	***
Mathematical Context	(ref.)				
Everyday Context	0.15	0.20	0.75	0.457	
Skinny	-0.74	0.17	-4.28	0.002	**
Isosceles	0.63	0.17	3.64	0.004	**
Equilateral	0.75	0.33	2.28	0.037	*
Acute	(ref.)				
Obtuse	-0.049	0.19	-0.25	0.794	
Right	0.20	0.27	0.75	0.453	
Standard Orientation	0.59	0.18	3.19	0.009	**
Everyday Context: Equilateral	0.68	0.22	3.19	0.002	**
Everyday Context: Obtuse	-0.16	0.10	-1.56	0.124	
Everyday Context: Right	0.46	0.16	2.91	0.004	**

* $p < .05$. ** $p < .01$. *** $p < .001$.

Summary and Conclusions

We examined whether middle school students use typicality *strategically* when considering conjectures, and found mixed results. When numbers or shapes had special mathematical properties, students considered them *more* typical in a mathematical context. However, properties that hold for these special objects should be *less* likely to hold for other objects. In other cases, superficial characteristics impacted whether students thought that conjectures that held for an object would hold for other objects. Both behaviors suggest that students might be conflating everyday typicality with mathematical typicality. Despite these results, students did sometimes distinguish mathematical and everyday contexts; they appropriately recognized the relevance of mathematically special and surface-level properties in each domain. This suggests that students have important resources for using typicality strategically, and for differentiating how objects should be considered in the math classroom and everyday life. But are these behaviors really characteristic of mathematical expertise? We recently presented the survey to 339 mathematicians. Initial analyses suggest that mathematicians do use typicality strategically in the ways we predicted, and they recognize everyday and mathematical typicality as two distinct entities that are often in opposition. This stands in contrast to how middle school students considered typicality, as they had difficulty reconciling mathematical and everyday contexts.

Our results suggest that students must negotiate an important learning continuum regarding mathematical conjectures. Initially, students appear to have difficulty reconciling their mathematical experiences with numbers and shapes with their concrete, salient everyday experiences. However, expertise in mathematics is characterized by flexible application of formal mathematical knowledge and everyday experience, based on the features of the problem and the social context. Thus students should be encouraged to critically reflect on how mathematical objects like numbers and shapes are considered differently in the mathematics classroom when exploring conjectures, compared to interacting with these objects in day-to-day life. Our work suggests that mathematicians are able to move flexibly between each of these two viewpoints, and use both examples and typicality judgments as resources in their work. Strategic use of examples and considerations of typicality may thus be important in helping students think more critically about the nature of mathematical evidence and in moving students towards making important generalizations about why mathematical conjectures hold, both of which ultimately could support deductive reasoning and formal mathematical proof.

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References

Alcock, L. J. (2004). Uses of example objects in proving. In M. J. Hines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Annual Conference of the International Group for the Psychology of Mathematics Education (Vol. 2)*. Bergen, Norway: Bergen University College.

Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics*, *24*(4), 359–387.

Common Core State Standards Initiative. (2010). *Common Core State Standards (Mathematics Standards)*. Retrieved from corestandards.org

Cooper, J., Walkington, C., Williams, C., Akinsiku, O., Kalish, C., Ellis, A., & Knuth, E. (2011). Adolescent reasoning in mathematics: Exploring middle school students' strategic approaches to empirical-based justifications. In *Proceedings of the 33rd Annual Conference of the Cognitive Science Society*. Boston, MA.

Feeny, A., & Heit, E. (2007). *Inductive reasoning: Experimental, developmental, and computational approaches*. Cambridge: Cambridge University Press.

Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. *Issues in Mathematics Education*, *7*, 234–283.

Knuth, E., Kalish, C., Ellis, A., Williams, C., & Felton, M. (2011). Adolescent reasoning in mathematical and non-mathematical domains: Exploring the paradox. In V. Reyna, S. Chapman, M. Dougherty, & J. Confrey (Eds.), *The adolescent brain: Learning, reasoning, and decision making*. Washington, DC: APA.

Lockwood, E., Ellis, A. B., Dogan, M. F., Williams, C. C., & Knuth, E. (2012). A framework for mathematicians' example-related activity when exploring and proving mathematical conjectures. In *Proceedings of the 34th Annual Conference of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Kalamazoo, MI: Western Michigan University.

Masingila, J., Davidenko, S., & Prus-Wisniowska, E. (1996). Mathematics learning and practice in and out of school: A framework for connecting these experiences. *Educational Studies in Mathematics*, *31*, 175–200.

Moll, L., & Gonzalez, N. (1997). Teachers as social scientists: Learning about culture from household research. In P. M. Hall (Ed.), *Race, ethnicity, and multiculturalism: Policy and practice* (pp. 89–114). New York: Garland.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Osherson, D. N., Smith, E. E., Wilkie, O., Lopez, A., & Shafir, E. (1990). Category-based induction. *Psychological Review*, *97*, 185–200.

Reusser, K., & Stebler, R. (1997). Every word problem has a solution: The social rationality of mathematical modeling in schools. *Learning and Instruction*, *7*(4), 309–327.

Snijders, T., & Bosker, R. (1999). *Multilevel analysis: An introduction to basic and advanced multilevel modeling*. Thousand Oaks, CA: Sage.

Walkington, C., Nathan, M., Wolfgram, M., Alibali, M., & Srisurichan, R. (in press). Bridges and barriers to constructing conceptual cohesion across modalities and temporalities: Challenges of STEM integration in the precollege engineering classroom. In J. Strobel, S. Purzer, & M. Cardella (Eds.), *Engineering in precollege settings: Research into practice*. Sense Publishers.

Walkington, C., & Sherman, M. (2012). Using adaptive learning technologies to personalize instruction: The impact of interest-based scenarios on performance in algebra. To appear in *Proceedings of the 10th International Conference of the Learning Sciences*. Sydney, Australia.

Walkington, C., Sherman, M., & Petrosino, A. (2012). 'Playing the game' of story problems: Coordinating situation-based reasoning with algebraic representation. *Journal of Mathematical Behavior 31*(2), 174–195.

Williams, C., Akinsiku, O., Walkington, C., Cooper, J., Ellis, A., Kalish, C., & Knuth, E. (2011). Understanding students' similarity and typicality judgments in and out of mathematics. In *Proceedings of the 32nd Annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Reno, NV.