LONGITUDINALLY INVESTIGATING THE IMPACT OF CURRICULA AND CLASSROOM EMPHASES ON EQUITY IN ALGEBRA LEARNING[1]

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This paper explores how curriculum and classroom conceptual and procedural emphases affect the learning of algebra for students of color. Using data from a longitudinal study of the Connected Mathematics Program (CMP), we apply cross-sectional HLM to lend explanatory power to the longitudinal analysis afforded by Growth Curve Modeling that we have reported elsewhere. Overall, we find that the achievement gaps tend to decrease over the course of the middle grades. However, differences in open-ended problem solving and equation solving performance persist for African American students. Classroom conceptual and procedural emphases also appear to differentially influence the performance of Hispanic and African American students, depending on the aspect of algebra learning that is being measured.

Keywords: Curriculum Analysis; Equity and Diversity; Algebra and Algebraic Thinking

Classrooms in the United States are becoming increasingly ethnically diverse. However, disparities in the mathematics achievement of different ethnic groups remain a persistent challenge (Lubienski & Crockett, 2007). Since teaching and learning are cultural activities, students with different ethnic and cultural backgrounds may respond differently to the same curriculum. Given the development and implementation of curricula based on the *Standards* documents developed by the National Council of Teachers of Mathematics (NCTM, 1989, 2000), a key question about curriculum reform is: How does the use of a *Standards*-based curriculum like CMP impact the learning of students of color as compared to White students?

In our project, Longitudinal Investigation of the Effect of Curriculum on Algebra Learning (LieCal), we used a longitudinal design to examine the similarities and differences between a *Standards*-based curriculum called the Connected Mathematics Program (CMP), and more traditional curricula (non-CMP). We investigated not only the ways and circumstances under which the CMP and non-CMP curricula affected student achievement gains, but also the characteristics of these reform and traditional curricula that hindered or contributed to the gains. One aspect of the LieCal analysis was an examination of potentially differential effects of curriculum and procedural and conceptual emphases in the classroom on the achievement of students of color. In this paper, we present results from a cross-sectional analysis of student growth within each grade level. This analysis allows us to add depth to our previous analysis using Growth Curve Modeling by probing effects that are significant at individual grades but which were not uncovered in our longitudinal analysis.

Background

Algebra readiness has been characterized as the most important "gatekeeper" in school mathematics (Pelavin & Kane, 1990). In particular, success in algebra and geometry has been shown to help narrow the disparity between minority and non-minority participation in post-secondary opportunities (Loveless, 2008). Research shows that completion of an Algebra II course correlates significantly with success in college and with earnings from employment. The National Mathematics Advisory Panel (2008) found that students who complete Algebra II are more than twice as likely to graduate from college as students with less mathematical preparation. Furthermore, the African-American and Hispanic students who complete

Algebra II cut the gap between their college graduation rate and that of the general student population in half. However, success in high school algebra is dependent upon mathematics experiences in the middle grades. In fact, middle school is a critical turning point for students' development of algebraic thinking (College Board, 2000).

In a *Standards*-based curriculum like CMP, the focus is on conceptual understanding and problem solving rather than on procedural knowledge. Students are expected to learn algorithms and master basic skills as they engage in explorations of worthwhile problems. However, a persistent concern about Standards-based curricula is that the development of students' higher-order thinking skills comes at the expense of fluency in computational procedures and symbolic manipulation. In addition, it is not clear whether this potential trade-off might play out differently for students from different ethnic backgrounds. Some reports have suggested that Hispanic and African American students using the CMP curriculum may in fact show greater achievement gains than students from other backgrounds (Rivette, Grant, Ludema, & Rickard, 2003). Still, research is needed to assess whether and how the use of a Standards-based curriculum such as CMP can improve the mathematics achievement of all students while helping to close achievement gaps (Lubienski & Gutiérrez, 2008; Schoenfeld, 2002).

Since the effectiveness of a curriculum depends critically on how it is implemented by teachers in real classrooms, studies of the effectiveness of Standards-based curricula must examine how teachers use the curricula (Kilpatrick, 2003; NRC, 2004; Wilson & Floden, 2001). The data gathered must be analyzed in appropriate ways to control for variations in classroom instruction and the learning environment. In order to determine the effects of curriculum on learning, it is essential to examine the classroom experiences of the teachers and students who are using the different curricula. In this paper, we take features of classroom instruction into consideration when we examine the impact of curricula on students' learning of algebra. In particular, we examine the extent to which teachers emphasize concepts and procedures in the classroom. As was reported by Moyer, Cai, Nie, and Wang (2011), CMP teachers placed more emphasis on conceptual understanding whereas non-CMP teachers placed more emphasis on procedural knowledge.

Our previous longitudinal analyses of the LieCal data using Growth Curve Modeling showed that over the three middle school years, African American students experienced greater gain in symbol manipulation when they used a traditional curriculum. The use of either the CMP or a non-CMP curriculum improved the mathematics achievement of all students, including students of color. The use of CMP contributed to significantly higher problem-solving growth for all ethnic groups (Cai, Wang, Moyer, Wang, & Nie, 2011). In this paper, we take a cross-sectional approach and examine the achievement of students of color in each grade level while controlling for the conceptual and procedural emphases in classroom instruction.

Method

Sample

The LieCal project was conducted in 14 middle schools of an urban school district serving a diverse student population. When the project began, 27 of the 51 middle schools in the district had adopted the CMP curriculum, and the remaining 24 had adopted more traditional curricula. Seven schools were randomly selected from the 27 schools that had adopted the CMP curriculum. After the seven CMP schools were selected, seven non-CMP schools were chosen based on comparable demographics. In sixth grade, 695 CMP students in 25 classes and 589 non-CMP students in 22 classes participated in the study. We followed these 1,284 students as they progressed from grades 6 to 8. Approximately 85% of the participants were minority students: 64% African American, 16% Hispanic, 4% Asian, and 1% Native American. Male and female students were almost evenly distributed.

Assessing Students' Learning

Learning algebra involves honing procedural skills with computation and equation-solving, fostering a deep understanding of fundamental algebraic concepts and the connections between them, and developing the ability to use algebra to solve problems. Thus, to assess students' learning of algebra, it is important to consider their conceptual understanding, their symbol manipulation skills, and their ability to solve problems. We used state test scores in mathematics and reading as measures of prior achievement. We used LieCal-developed multiple-choice and open-ended assessment tests as dependent measures of procedural knowledge and conceptual understanding in algebra, respectively. The two LieCal-developed tests were administered four times, once as a baseline in the fall of 2005, and again each spring (2006, 2007, and 2008).

We used multiple-choice items to assess whether students had learned the basic knowledge required to perform competently in introductory algebra. Each version of the multiple-choice test was comprised of 32 questions that assessed five mathematics components (Mayer, 1987): translation, integration, planning, computation (or execution), and equation solving. For this paper, we report on the results from the translation, computation, and equation-solving components of the multiple-choice tasks. In addition, the open-ended tasks were designed to assess students' conceptual understanding and problem-solving skills. These tasks were adopted from various projects including Balanced Assessment, the QUASAR Project (Lane et al., 1995), and a cross-national study (Cai, 2000). Since only a small number of open-ended tasks can be administered in a testing period, and since grading students' responses to such items is labor-intensive, we distributed the non-baseline tasks over three forms (five items in each form) and used a matrix sampling design to administer them. Examples of the items and tasks used in the LieCal assessments can be found in Cai et al. (2011).

The multiple-choice items then were scored electronically, either right or wrong. The open-ended tasks were scored by middle school mathematics teachers, who were trained using previously developed holistic scoring rubrics. Two teachers scored each response. On average, perfect agreement between each pair of raters was nearly 80%, and agreement within one point difference out of 6 points (on average) was over 95% across tasks. Differences in scoring were arbitrated through discussion. The two-parameter Item Response Theory (IRT) model was used to scale student assessment data on both multiple-choice tasks and open-ended tasks (Hambleton, Swaminathan, & Rogers, 1991; Lord, 1980).

Conceptual and Procedural Emphases as Classroom-level Variables

Mathematical proficiency includes both conceptual and procedural aspects (NRC, 2001), and teachers can shape instruction in ways that emphasize either or both aspects. We used conceptual and procedural emphases as classroom variables when examining the impact of curriculum on students' learning. To do so, we estimated the levels of conceptual and procedural emphases in the CMP and non-CMP classrooms using data from 620 lesson observations of the LieCal teachers, which we conducted while the students were in grades 6, 7, and 8. Each class was observed four times, during two consecutive lessons in the fall and two in the spring. Further details about the observations are documented in Moyer et al. (2011). One component of the observation was a set of 21 items using a 5-point Likert scale to rate the nature of instruction for each lesson. Of the 21 items, four were designed to assess the extent to which a teacher's lesson had a conceptual emphasis. For example, observers rated a lesson's conceptual emphasis using the following item: "The teacher's questioning strategies were likely to enhance the development of student conceptual understanding/problem solving." Another four items were designed to determine the extent to which a teacher's lesson had a procedural emphasis. For example, observers rated a lesson's procedural emphasis using this item: "Students had opportunities to learn procedures (by teacher demonstration, class discussion, or some other means) before they practiced them." Factor analysis of the LieCal observation data confirmed that the four procedural-emphasis items loaded on a single factor, as did the four conceptual-emphasis items. Since students changed their classrooms and teachers as they moved from grade 6 to grade 7 and from grade 7 to grade 8, each student could have a different value each year for three years, but all students in the same classroom at each grade had the same value.

Quantitative Data Analysis

To examine student growth within each school year while controlling for multiple factors such as gender, ethnicity, and classroom conceptual and procedural emphases, we used hierarchical linear modeling (HLM). After unconditional models were fitted, two sets of conditional cross-sectional HLM analyses were conducted. The first set of models was composed of cross-sectional hierarchical linear models that included student-level variables and a curriculum variable. These models used four different student achievement measures: open-ended, translation, computation, and equation solving. Each HLM model used data from one of the four dependent achievement measures in one of three middle grades, together with an independent prior achievement measure, namely the results of the state mathematics testing in the fall of the corresponding year. So, each model examined a single type of learning within a specific grade level. Since we had four achievement measures at each of three grade levels, there were 12 cross-sectional models in this first group.

The next set of models built on the first group of models by adding two classroom-level variables: the conceptual emphasis of the classroom and the procedural emphasis of the classroom. These cross-sectional HLM models were of the following form:

Level-1 Model

$$\begin{aligned} Y_{ij} = p_{0j} &+ p_{1j}(\text{Prior Achievement}_{ijk} - \overline{X}_1) + p_{2j}(\text{Gender}_{ijk} - \overline{X}_2) \\ &+ p_{3j}(\text{African American}_{ijk} - \overline{X}_3) + p_{4j}(\text{Hispanic}_{ijk} - \overline{X}_4) \\ &+ p_{5j}(\text{Other Ethnicity}_{ijk} - \overline{X}_5) + r_{ijk} \end{aligned}$$

Level-2 Model

$$p_{0j} = b_{00} + b_{01}\text{CMP} + b_{02}\text{Conceptual Emphasis}_j + b_{03}\text{Procedural Emphasis}_j + r_{0j}$$

Interactions between conceptual emphasis, procedural emphasis, and curricula were tested, but found to be not significant.

Results

We present the results of our analysis in two parts. First, we report on the cross-sectional HLM models that included student-level and curriculum variables. Then, we examine the impact of including the classroom-level conceptual and procedural emphasis variables in the models.

Student-Level and Curriculum Cross-sectional HLM Models

Table 1 shows the standardized results from an examination of the performance of African-American and Hispanic students relative to White students, when controlling for prior achievement, gender, and curriculum (but not conceptual and procedural classroom emphases).

Table 1: Effect of Ethnicity on Mathematics Achievement

	Grade 6		Grade 7		Grade 8	
	African American	Hispanic	African American	Hispanic	African American	Hispanic
Open-ended	-0.50***	-0.21*	-0.26**	-0.22*	-0.28***	-0.13*
Translation	-0.24**	--	--	--	--	--
Computation	-0.37***	-0.22*	--	--	--	--
Equation solving	-0.35**	-0.23*	-0.24**	-0.22*	--	--

* $p < .05$. ** $p < .01$. *** $p < .001$.

Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). (2012). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Kalamazoo, MI: Western Michigan University.

In the sixth grade, an achievement gap was seen between African American students and White students on all four student achievement measures, and between Hispanic students and White students on the open-ended, computation, and equation solving measures. The gaps on the open-ended and equation solving measures remained in the seventh grade for both groups. However, performance on the computation and translation measures had equalized across the groups. In the eighth grade, the only gap that remained was on the open-ended items. The overall trend was a gradual decline or elimination of the achievement gap among the ethnic groups.

To better understand if using the CMP curriculum would reduce achievement gaps, we conducted separate parallel analyses for CMP and non-CMP students. The results are shown in Table 2. In the analysis of the combined CMP and non-CMP student sample, achievement gaps for the translation and computation measures occurred only in the 6th grade: White students outperformed African American students on both measures, and White students outperformed Hispanic students on computation. However, in the analyses of the separate student samples, we found that although all three of these gaps appeared for the non-CMP students, the only achievement gap for the 6th grade CMP students was in computation. In grades 7 and 8, the performance parity on computation and translation items observed in the combined sample of students was mostly preserved in the separate analyses, except for the appearance of a gap between CMP 8th grade African American students and White students on computation items.

Mirroring the results from the combined sample, White students outperformed African American students on open-ended items across all three grades regardless of curriculum. For students using CMP, White students also outperformed Hispanic students on these items in Grades 7 and 8. For non-CMP Hispanic students, however, there were no parallel achievement gaps. For the equation solving items in the combined analysis, White students outperformed African-American and Hispanic students in grades 6 and 7, with no achievement gap in grade 8. These gaps were attributable to the CMP students; there were no achievement gaps found for equation solving items among the non-CMP students. For CMP students, White students outperformed African American students in all three grades, and White students outperformed Hispanic students in grades 7 and 8.

Table 2: Effect of Ethnicity on Mathematics Achievement for CMP / Non-CMP Students

	Grade 6		Grade 7		Grade 8	
	African American	Hispanic	African American	Hispanic	African American	Hispanic
Open-ended	-0.40** / -0.91***	-- / --	-0.27** / -0.23*	-0.31**/ --	-0.36** / -0.26**	-0.28**/ --
Translation	-- / -0.37*	-- / --	-- / --	-- / --	-- / --	-- / --
Computation	-0.43** / -0.27**	-- / -0.35**	-- / --	-- / --	-0.22* / --	-- / --
Equation solving	-0.35* / --	--/ --	-0.44** / --	-0.45**/ --	-0.23**/ --	-0.22*/ --

* *p* < .05. ** *p* < .01. *** *p* < .001.

Student-Level, Classroom-Level and Curriculum HLM Models

We built on the results of Table 1 with the addition of the conceptual emphasis and procedural emphasis classroom-level variables. Our goal in adding these variables to the analysis was to begin to probe the complexity that underlies conclusions we might otherwise draw from one-dimensional comparisons of students in different ethnic groups. With respect to the analysis of the combined CMP and non-CMP students, however, the addition of the classroom-level variables did not greatly perturb the results save for the disappearance of the gap in Hispanic students' performance on open-ended tasks in the 8th grade.

We again conducted parallel analyses for the CMP and non-CMP students, this time including the conceptual and procedural emphasis classroom-level variables. The results are presented in Table 3. For the CMP students, two achievement gaps were no longer statistically significant with the addition of the classroom variables: 8th grade African American students on computation items, and 8th grade Hispanic students on equation solving items. For non-CMP students, the performance gap of 6th grade African American students on translation and computation items ceased to be significant.

Table 3: Effect of Ethnicity on Mathematics Achievement for CMP/Non-CMP Students Controlling for Conceptual and Procedural Emphases

	Grade 6		Grade 7		Grade 8	
	African American	Hispanic	African American	Hispanic	African American	Hispanic
Open-ended	-0.40** / -0.90***	--/ -0.33*	-0.28** / -0.27*	-0.30**/ --	-0.36** / -0.23**	-0.29**/ --
Translation	-- / --	-- / --	-- / --	-- / --	-- / --	-- / --
Computation	-0.37** / --	-- / -0.33**	-- / --	-- / --	-- / --	-- / --
Equation solving	-0.35*/ --	-- / --	-0.45** / --	-0.44**/ --	-0.21** / --	-- / --

* $p < .05$. ** $p < .01$. *** $p < .001$.

For the combined student groups, the performance of Hispanic students in the 8th grade was not significantly different from 8th grade White students for all four achievement measures. Similarly, the performances of 8th grade African American and White students were not significantly different except on the open-ended items; there was no achievement gap between African American and White students in the 8th grade on translation, computation, and equation solving items. When analyzed as separate groups, the CMP and non-CMP students of color generally showed achievement gaps on open-ended items compared to White students using the same curriculum. Within the CMP student group, there were also achievement gaps for African American students on equation solving items.

Discussion

In examining how Standards-based curricula such as CMP affect the mathematics learning of students of color, it is important to use nuanced analyses to look beyond one-dimensional comparisons (Lubienski, 2008). The longitudinal growth curve analysis of the LieCal data provided mixed conclusions regarding the use of the CMP curriculum with students of color (Cai et al., 2011). Though, over the course of the middle grades, African American and Hispanic students had growth rates similar to students not in their ethnic groups on the open-ended, translation, and equation solving measures, African American students had a smaller growth rate on the computation measure. The cross-sectional HLM analysis in this paper provides detail not captured in the longitudinal analysis.

Overall, the results of the cross-sectional analysis show a trend of decreasing gaps in achievement. Whereas Hispanic and African American students score significantly lower than White students on most or all of the measures at the end of 6th grade, by the end of 8th grade, only the open-ended measure still reflects a gap. Moreover, when classroom conceptual and procedural emphasis is taken into account, the only difference that remains at the end of 8th grade is in African-American students' performance on the open-ended tasks. Despite the slower growth rate in African American students' performance on computation tasks that was identified in the longitudinal analysis, the effect seems to be largely limited to the 6th grade.

When the cross-sectional analysis is limited to the CMP students, the open-ended measure reflects a persistent gap between White students and students of color. Similarly, for African American students in the CMP group, equation solving remains an area of challenge throughout the middle grades. Even when classroom conceptual and procedural emphasis variables are included, these gaps remain. Indeed, these performance gaps in the CMP analysis do not decrease with grade level, as many of the other performance gaps do. Thus, despite the fact that the longitudinal analysis showed comparable growth curves for White students and students of color on the open-ended measure, the cross-sectional analysis suggests that there may be opportunities within the CMP curriculum for developing open-ended problem-solving skills that are being differentially accessed by students of different ethnic backgrounds.

It is interesting to note how the influence of classroom emphasis variables played out differently for different student groups. For example, the profile of Hispanic CMP students' equation solving performance was somewhat different from the African American students'. For Hispanics, the negative CMP effect was limited to the 7th grade. Classroom conceptual and procedural emphases, not curriculum, appear to account for Hispanic student performance differences in the 8th grade. Moreover, the reverse appears to be the case with respect to the translation and computation measures in the 6th grade. When controlling for classroom emphasis, there was no longer an achievement gap for African American students. This difference in the effects of classroom emphasis on Hispanic and African American students merits exploration.

In conclusion, the longitudinal and cross-sectional analyses continue to paint a mixed picture of the effects of the CMP curriculum for students of color. By grade 8, most performance differences on the measures in this study were no longer significant. Though African-American students' computation skills appeared to grow more slowly across grades 6 through 8, the effect of this difference seems to have been primarily limited to grade 6. However, the persistent gaps between African American students and White students on the open-ended and equation solving measures, even when classroom emphases are taken into account, invite further investigation.

Endnote

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