WHY IS THE METAPHOR OF ENGLISH LEARNERS *BRINGING* RESOURCES TO MATHEMATICS CLASSROOMS NOT HELPING TEACHERS?

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While the metaphor of students bringing linguistic, cultural, and community resources to classrooms abounds in mathematics education research with non dominant students, teachers seem not to benefit from such metaphoric language as most of them struggle to figure out how such resources can be used during mathematics instruction. This paper urges a revision of this metaphor by proposing a new perspective on resources. The revised perspective is illustrated with an interaction with a third grade English learner working on a volume problem. During this interaction, the student and the interviewer recognized resources that did not pre-date the interaction but instead shared and sustained the life of the interaction. Revising this metaphor is important for helping teachers build common resources with English learners.

Keywords: Teacher Education-Inservice/Professional Development, Equity and Diversity, Elementary School Education, Measurement

Purpose

Research with English learners is replete with arguments that counter deficit views of these students. One common argument is that English learners bring resources to classrooms. Included in this argument is the metaphor of *bringing resources*. The purpose of this paper is not to question the intent of these arguments, as negative views on English learners persist even after years of counterevidence. Instead, the purpose is to revise this particular metaphor in order to construct stronger arguments in which everyone—particularly teachers—can participate. The revision—illustrated with an interaction with an English learner—is important because the metaphor has left many teachers of English learners wondering how they can begin to use such resources in classroom instruction.

Theoretical Approach

To revise the perspective that English learners bring resources to classrooms I propose a theoretical approach that shifts the emphasis on the *bringing* of resources *by* students to *recognizing* resources *with* English learners. Three interrelated ideas in this approach include: heterogeneity, togethering, and care. Each idea is elaborated next.

I begin this metaphor revision by making clear that I do not question the idea that students bring resources from their language, culture, and community. What I question is that it leaves unspecified the role of teachers in using these resources. Teachers can recognize these resources with the students, specifically in the diversity of ways in which children talk—in the *heterogeneity* of their talk (Rosebery, et al., 2010; Warren, et al., 2001)—as well as in what they do—the embodiment of knowledge that reflects the child as a linguistic, cultural, and communal being. We must start, in other words, where the child is (Mercer, 1995) and with what makes sense to them as young learners. For example, in Turner et al.'s (in review) study of English learners participating in mathematical discussion, one student said: "Cada 8 valía por 2" (Each [bag of] 8 is worth 2 [bags of 4]). This unusual way of talking about proportional reasoning—

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grounded in a common saying in Spanish—was recognized by the teacher in the moment of this interaction, becoming a resource that enhanced the class understanding of proportionality.

The second idea in this framework that supports recognizing resources with students is explained by Radford and Roth (2011) in his analytic category of *togethering***:** "[*T*]ogethering… accounts for the *ethical* manner in which individuals engage, respond, and tune to each other, despite their cognitive, emotional, and other differences" (p. 236). Togethering also addresses the issue that teachers and English learners too often do not share common resources (Author, in review), by urging teachers to decenter cognitively and join the students in learning processes that transcend cognitive and emotional differences. Radford and Roth illustrate how togethering works in a teacher-student interaction in which a student, Albert, is looking at a savings pattern problem by using a doubling strategy, whereas the teacher is thinking about the related algebraic formula $2n+1$. The researchers discuss the culminating moment of the teacher-student togethering process in which the teacher is about to bring the student to join her in seeing the formula: "She is tense. There is much at stake. If the interaction fails, the activity fails. It would not just be Albert's failure. It would be her failure as well." (p. 239).

Finally, if left unconnected, heterogeneity and togethering may not help teachers recognize resources with students. After all, heterogeneity emphasizes divergence of ideas whereas togethering focuses on participants transcending such differences. The category of *mathematical caring relationships* (Hackenberg, 2010) helps to conceptually connect these two categories. Defined as "a quality of interactions between a student and a teacher that conjoins affective and cognitive realms in the process of aiming for mathematical learning" (p. 237), mathematical caring relationships are evolving relationships in which the teacher continuously considers the students' heterogeneous perspectives, instantiating moments of togethering with the student aimed at understanding each other's ideas. The formation of caring relations between teachers and students in mathematics has not received enough research attention (Vithal, 2003; Hackeberg, 2010), much less in classrooms with English learners. An example is a case of a teacher who moved from unreciprocated to a responsive form of care about the students' difficulties in an estimation unit (Author, et al., in press).

Methods (Participants, Context, Data Collection, and Analysis)

Data for this paper come from a two-year professional development project aimed at supporting elementary teachers develop common resources with English learners in mathematics. The teachers learned to recognize resources with students by watching on video their own interactions with students but also by inviting me to teach some lessons that they observed. The school was located in a working class, predominantly Latino neighborhood. I videotaped an average of 2-3 days per week the interactions that these teachers and I had with Latino/a students (ELs, bilinguals, recent immigrants). Additional sources of data included analytic memos, e-mails, co-planning sessions with teachers, and interviews with teachers and students. I used these multiple data sources to triangulate findings and determine whether what we were recognizing were actual common resources between teachers and students. To illustrate the process of recognizing resource with English learners, I used a purposive sampling (Cohen, Manion, & Morrison, 2000) by selecting the case of Marifer, an English learner who participated in an interaction filled with moments in which resources were recognized.

Results

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Marifer's teacher, overwhelmed by the multiple demands of her first year of teaching, referred Marifer to me because "she needs help understanding the concept of volume" (field notes). The teacher's request was based on Marifer's poor performance on volume items on a benchmark test. I approached my interaction with Marifer with two questions in mind: (1) What does she know already about volume that I can recognize as possible resources to sustain our interaction? (2) How can I join her in this knowledge of volume so she and I can construct common resources for advancing her understanding of volume? The transcript of our conversation is presented in three parts. Each part is followed by analysis that illustrates the analytical categories of heterogeneity, togethering, and mathematical caring relationships. The English translation is bracketed in italics.

Interviewer: Muéstrame una pregunta con la que batallaste mucho. [*Show me one question with which you struggled a lot*.]
Marifer: (Opens test booklet and carefully looks for one question. When she finds one, she presses on it with index finger strongly and emphatically.)
Interviewer: Pues léemela tú primero. [*Well, you read it to me first.*
Marifer: El siguiente modelo está hecho con cubos de un centímetro. ¿Cuál es el volume de este modelo? [*The following model is made with one-centimeter cubes. What is the volume of this model?*]
Interviewer: Está hecho con cubos de un centímetro. ¿Me puedes encontrar un cubo de un centímetro? [*It's made with one centimeter cubes. Can you find me a one-centimeter cube?*]
Marifer: (Puts index finder on various cubes in the model)
Interviewer: OK. Éste es uno, y aquí hay otro. ¿Cuántos cubos de un centímetro crees que hay? [*OK, that's one, and here's another one. How many one-centimeter cubes do you think there are?*]
Marifer: Eighty-two.
Interviewer: ¿Cómo sabes? [*How do you know?*]
Marifer: Because…yo los conté ayer. [*...I counted them yesterday*]
Interviewer: ¿Y cómo los contaste? [*And how did you count them?*]
Marifer: De uno por uno. [*One by one*]
Interviewer: Pero éstos están atrás, ¿cómo sabes contar si no se ven, cómo le hiciste? [*But these are in the back, how do you know how to count them if they cannot be seen, how did you go about that?*]
Marifer: Yo nomás conté todos éstos, y éstos, y éstos [*I only counted all of these, and these, and these*] (Points to each of the 3 visible faces of the model)
Interviewer: Ah, contaste…[*Ah, you counted…*] (Marifer interjects)
Marifer: Como, conté esta parte (circula cara frontal), y esta parte (circles cara superior), y éstos de al lado (cara lateral), 82 en total. [*Like, I counted this part (circles front face), and then this part (circles top face), and the ones from the side (circles side face), 82 in total*]

By asking Marifer to pick one problem that was challenging for her, I wanted her to take me back to a moment when she experienced struggle and share that moment so we could begin to engage, respond, and tune to each other's ideas as suggested by togethering (Radford & Roth, 2010). Asking Marifer to find a 1 cm cube served to recognize a common object between us, given our cultural familiarity with the metric system. This common object I predicted was going to be pivotal for finding additional resources with Marifer. These resources, in turn, would move us beyond any cognitive and emotional differences possibly related to her failure on the test. First, the 1 cm cube served to reveal her misconception of

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volume as being only the three visible faces of the model. Noticing that Marifer was not seeing the intended (but hidden) three-dimensionality of the model drawn on the test, I embarked with Marifer in a new act of togethering by suggesting to co-construct a three-dimensional model using connecting cubes. In the next part of the transcript, I retake our conversation immediately after she and I had helped each other to reproduce the model on the test using connecting cubes.

Interviewer: ¿En qué se parece este modelo a éste? [*How does this model look like this on*

Marifer: Porque, está, está, uh, en cuadros, los dos están divididos en cuadritos. [*Because, it's, it's, uh, in squares, both are divided in squares*]

Interviewer: Uh, huh, en cubos de un centímetro. [*Uh, huh, in one centimeter cubes*]

Marifer: Sí. [*Yes*]

Interviewer: ¿Y en qué más se parece este modelo de los cubos al modelo del examen? [*And how else does the cubes model look like the model in the exam?*]

Marifer: Es la misma, uh (mueve ambas manos hacia arriba y abajo, con un espacio en medio) altura. [*It's the same,* (moves both hands up and down, with a space in between) *height*]

Interviewer: La misma altura, OK. ¿Cuál es la altura? [*The same height, OK. What's the height?*]

Marifer: Esto [*This*] (points to the top of the model on test).

Interviewer: Es como el techo. OK, ¿cuál es el largo? [*It's like the roof, OK. What's the length?*]

Marifer: That one. (Points to bottom of the model on test).

Interviewer: OK, aquí es el largo, y acá en los cubos, ¿cuál sería el largo? [*OK, here's the length, and over here with the cubes, which one is the length?*]

Marifer: Aquí el de abajo [*Here on the bottom*] (runs finger along the base of the block of cubes).

Interviewer: Uh-huh. Y el ancho, ¿cuál sería el ancho? [*Uh-huh. And the width, which would be the width?*]

Marifer: Aquí, como el ancho [*Here, like the width*] (points to base of block at one end).

Interviewer: OK, aquí es el ancho. Entonces tú me dices que aquí contaste 82, o sea ¿nadamás contando los que se ven? [*OK, here's the width. So you told me that here you counted 82, I mean, only counting the ones that are visible?*]

Marifer: Uh-huh.

Interviewer: Qué te parece aquí, en este modelo, ¿cuántos cubos de 1 centímetro hay aquí? [*What about here, on this model, how many one centimeter cubes are there?*]

Marifer: (Takes a careful look at front side for a while, then tilts head for an easier and closer look at one end side of the block, then announces): Cien treinta [*One hundred thirty*] (a common way among Mexican Americans to say numbers larger than one hundred; the standard way requires adding the suffix –to after the word cien, as in ciento treinta).

Interviewer: Ciento treinta. ¿Por qué ciento treinta? [*130. Why 130?*]

Marifer: Porque uh, conté uh, primero conté éstos, de esta línea, y había 10 cuadritos, entonces pensé que en cada línea había de éstos 10, y nomás conté todos. [*Because uh, I counted uh, I counted these first, in this line, and there were 10 little squares, and so I thought that in every line there were 10 of those, and so I just counted all of them*] (points to several of the sticks in descending order)

Interviewer: (Echoing Marifer as she speaks): A ver, ¿y por qué son ciento treinta? [*Let's*

see, and why there are 130?]
Marifer: Porque [*Because*], uh, I don't know if I'm right…(Points to each stick of ten as she skip counts by 10 in English): 10, 20, 30, 40, 50, 60, 70, 80, 90 (aspirates 90 as she runs out of breath, switches to Spanish after 100), 100, cien diez, cien veinte. [*One hundred ten, one hundred twenty*] Oh no, never mind, cien veinte. [*one hundred twenty*]
Interviewer: Estabas cerca, ¿verdad? (Marifer: Uh-huh). Ciento veinte. Pero cuando tú me dijiste aquí (le muestro el examen), no me dijiste que eran ciento veinte, ¿qué pasó ahi? [*You were close, right?* (Marifer: Uh-huh). *120. But when you told me right here* (show the test), *you didn't tell me it was 120, what happened there?*]
Marifer: I think, um, I counted, I count wrong.
Interviewer: You think you counted wrong (she nods) OK. ¿Qué más piensas? ¿Por qué son dos respuestas diferentes? [*OK, what else do you think? Why are these two different answers?*]
Marifer: Oh! Oh, porque, uh, en éstas (apunta a la pregunta del examen), hay como de cuatro (apunta a los 4 grupos en la parte superior del modelo con cubos) aquí 4 (apunta a los extremos de los 4 grupos en un lado del modelo con cubos) y aquí hay de diez (apunta al largo de un grupo). Como, en cada, en cada línea, como, el techo, como usted dijo, hay, están, tiene 4, y acá en el largo (ahora apunta al modelo con cubos) tiene 10, y allá (apunta al examen) tiene cuatro, como en cada línea está, como en cada lado tiene diferentes, uh, números, como así (coloca el filo de la mano en el ancho del modelo con cubos). [*Oh! Oh, because, uh, in these* (points to test item), *there are like (lines) of four* (points to 4 sticks on top of cubes model) *right here 4* (points to the ends of the 4 sticks on one side of the cubes model) *and here there are (sticks) of ten* (points to the length of one stick). *Like, in each, in each line there is, like on each side it has different, uh, numbers, like this* (puts edge of hand along the width of the cubes model)]
Interviewer: Uh-huh. Entonces ¿cuál crees que está bien, como los contaste aquí o como los contaste acá? [*Uh-huh. So, which one do you think is correct, the way you counted them here or the way you counted them there?*]
Marifer: Como los conté aquí [*The way I counted them here*] (points to cubes model).
Interviewer: ¿Por qué? [*Why?*]
Marifer: Porque…alli, aquí, uh, tenemos, la foto, y está como, aquí (redirige la atención del examen al modelo con cubos) tenemos las líneas que t-, como en cada uno está el mismo número de líneas. [*Because…there, here, uh, we have, the picture, and it's like, here* (shifts attention from test to cubes model) *we have the lines that, like in each one there's the same number of lines*]

The three-dimensional model that Marifer and I constructed served as a new resource that anchored the heterogeneity of our talk. The first evidence of Marifer's heterogeneous talk was when she began to recognize height—an important attribute of volume—first by embodying that knowledge as gesturing, then by pointing to the top of the model as the height, and finally by using the correct word *altura*. Her use of similar gestures to identify the length and the width of the model suggests that Marifer used everyday ways of engaging life experiences (Gutiérrez & Rogoff 2003) as resources to make sense of the school concept of volume. This resourceful way of making sense contrasted with her earlier isolated counting of visible faces that she reported using on the test. My initial question was, what does she know already about the concept of volume? Marifer certainly knew a lot and, perhaps more importantly, her knowledge was expressed in a variety of ways. For example, she "lifted" the attribute of width by gesturing with

her hands, leaving a space to indicate the width. This gesturing was enacted between the two models. She recognized that she had counted wrong in the two-dimensional model and was able to explain why she had counted wrong. In her explanation there was reference to "la foto" as something that prevented her from seeing all the dimensions in the test item that she was able to see in the constructed model.

Marifer's attention shifted back and forth between the two-dimensional and the three-dimensional model. I followed her as she moved back and forth between the two models, because I wanted her to decide which one was going to be our common resource. We also followed each other linguistically, sometimes talking in English, sometimes in Spanish, and sometimes bilingually, demonstrating to each other that our heterogeneous talk was a common resource. An important moment when Marifer "saw" the concept of volume occurred when she was mentally counting the connected cubes. She never touched the constructed model; instead, she was surveying the model, tilting her head to gain a different perspectival side view. Adler (2000) calls this the transparency of resources. Just like the paper version, the cubes model was not showing all the cubes that it was made of, but Marifer was seeing through it this time. For example, in her explanation of how she counted the invisible cubes, she declared: "*…and so I thought that in every line there were 10 of those*." The models clearly became transparent, and her reasoning through them became visible both for me and for her. In the final part of the transcript and as a result of noticing her miscount of 130, I asked Marifer to invent a different way of counting the cubes, one less prone to counting errors. Hearing my request, Marifer took a look at the model, pursed her lips slightly, then looked at me again and, while pointing to herself to emphasize her reaction to my request, exclaimed:

Marifer: Me?!
Interviewer: Uh-huh!
Marifer: (Continues looking at model for a while, then smiling exclaims): ¡No!
Interviewer: ¿No? Está bien, está muy bien lo que hiciste, de 10 en 10, pero… ¿Quieres pensar un poquito más, a ver si se puede más rápido? (Asiente) OK. [*No? That's alright, what you did is great, by tens, but… Do you want to keep thinking to see if you can do this faster? (She nods) OK*]
Marifer: (Looks at the corner of the block model where she can get a perspective of the three dimensions): Oh! uh, hay, en cada, en cada de éstos hay 4 uh de estas líneas, entonces, [*Oh! Uh, there's, in each, in every one of these there are 4 uh of these lines*] I think, y en cada linea hay 10 cuadritos [*and in each line there are 10 little squares*], so we can do, I think uh, ten times four, diez por cuatro.
Interviewer: Oh! Sí, está muy bien, y ¿cuánto es ten times four? Diez por cuatro, cuánto es diez por cuatro? [*Oh! Yes, that's great, and how much is 10 times 4?*]
Marifer: Forty.
Interviewer: Forty. OK.
Marifer: But I think it's not it.
Interviewer: Hu?
Marifer: I think it's not the answer.
Interviewer: Well, not the final answer, but I can see the forty right here. Can you see the 40 right here? (I lift the top layer of 4 sticks of ten, Marifer nods). OK, so you got like part of the answer, and then what? I like what you're doing! Y luego, ¿qué más harías, Marifer? [*So then what else would you do, Marifer?*] So you got 4x10. ¿Qué más? [*What else?*] *Marifer:* (Looks at model for a long time. I do not say a word. I am holding up the top layer: If she

fails, I fail, because we are doing this together.): We get another 4, (she lifts the layer of 4 sticks of ten that was in the middle) then these ones (points to the base layer).

By the time I asked Marifer to invent a different way of counting, we were already in a mathematical caring relationship (Hackenberg, 2010). We cared so much about each other's ideas and our common resources that now I wanted to move Marifer with me to a common understanding of a multiplicative way of thinking about volume, to prepare her to have a even more powerful resource that she could use in future tasks. Her surprise with my invitation indicated that she was not expecting this challenge. I asked the teacher for her interpretation of Marifer's reaction, and she explained: "She probably has not been asked that question before" (field notes). However, when students see themselves in mathematical caring relationships, they do not give up so easily when they face challenges. Marifer approached the challenge with care. She began the multiplicative counting first with one layer of cubes (4x10), but she knew that this was not the final answer. There was a final moment in our interaction when Marifer created yet another common resource: In that moment, Marifer lifted the layer that was in the middle, an action that I had initiated and that she appropriated. By doing this unprompted action, Marifer finished constructing our common resource that was helping us recognize the dimensions of the model and using these dimensions to count the one-centimeter cubes multiplicatively.

Discussion

For many teachers, certainly for Marifer's teacher, the metaphor of students bringing resources to classrooms is an esoteric idea that does not include the teacher in an active role. In this paper I have presented evidence from my instructional interaction with an English learner, to call for a revision of the metaphor in the sense that it is possible to recognize resources with students in interactions, instead of expecting that students will bring and spontaneously use such resources. The transcript of my interaction with Marifer includes multiple moments in which she and I recognized linguistic, cultural, and experiential resources that we used for advancing her understanding of volume. These resources did not pre-date our interaction. Instead, they shared and sustained the "life" of our interaction. The generative power of students' everyday experiences (Warren, et al., 2001) can be the teachers' best ally for recognizing resources with students. Finally in this paper I have suggested and used heterogeneity, togethering, and mathematical caring relationships as analytical categories that can help us recognize resources *with* our students. In revising the metaphor of students bringing resources to classrooms, I have rearranged the multiple pieces of this metaphor. First, the source of students' resources is not to be found outside the instructional interaction; instead, these resources are an important and recognizable part of any interaction. Second, teachers are not expected to know students' resources prior to interacting with students; rather, they are expected to recognize these resources with students by engaging, responding, and tuning to each other's ideas while forming caring relationships with students. Finally, this recognition of resources is urgent given that in many cases, the teachers of English learners share very little in common with the students. The revised metaphor is intended to guide teachers to recognize and construct these common resources with the students.

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