

TECHNOLOGY IN MATHEMATICS EDUCATION: A DISCUSSION OF AFFORDANCES

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This paper details a framework for evaluating the potential of particular forms of technology for supporting student engagement with mathematics. I propose that, focusing on the strength of affordances to engage in particular ways allows us to begin to build an analytic frame that serves two purposes: (1) to begin to document learning as an interaction with larger bodies of data; and (2) to make and evaluate conjectures about the potential of different technologies for supporting high quality engagement with mathematics. I describe design iterations from previous work that leveraged this framework as an example of how this framework could be used and why it has proven to be valuable.

Keywords: Technology, Data Analysis and Statistics, Learning Theory, Design Experiments

Introduction

We seem to be in an upswing of technology adoption, with the general belief that more technology is better. We can see this in one-laptop-per child programs, charter schools that advocate the use of technology such as “New Tech High,” the enthusiasm for “blended learning,” and even the popular BYOT programs at schools, which encourage children to bring in any form of technology that they have at their disposal. I find this enthusiasm puzzling, to say the least. Technology, as a general term, refers to such a broad range of tools that it is difficult to understand why anyone would immediately believe (or doubt) that it would improve education. COULD technology become a resource to support student learning? Of course. Could it also thwart student learning? Absolutely. It’s all in which forms of technology, how they’re being used, by whom, and when. The questions we need to pose involve not whether technologies are useful, but which technologies, and under what circumstances.

In this paper I discuss a framework for considering the potential of particular forms of technology for supporting student learning, drawing on work that I have published that has focused specifically on particular forms of student engagement. I then share some of my own work on designing and studying immersive videogames, and leverage this framework as a means of making sense of the potential of these technologies. In so doing, my goal is not to take a position about whether technology is useful for supporting mathematical learning or not, but instead, to suggest a way of thinking about technology that would better allow us to evaluate whether, when, and why leveraging technology would make a difference for mathematics learning.

A Theory of Context

In 2008, Jim Greeno and I published a chapter that focused on considering “opportunities to learn” as a set of *affordances* for learning. Drawing on Gibson’s (1979) theory of perceptual affordances as a framework, this paper described human activity as a co-constructed set of possible actions defined by: 1) the affordances of the environment for a particular action; 2) the intention of the agent to take up those affordances; and 3) the effectivities of the agent to actually

realize those affordances (c.f. Barab, Cherkes-Julkowski, Swenson, Garrett, Shaw, & Young, 1999; Gresalfi, Barnes, & Cross, 2012). *Affordances* refer to the set of actions that are made possible by a particular object; for example, a chair affords sitting, but a door does not. Of course, the extent to which an affordance can be acted on has to do with one's *effectivities*, an individual's ability to realize those affordances. To return to the earlier example, because of their quite different effectivities, a chair affords sitting for a human, but not for an alligator. And finally, the extent to which an affordance is realized depends on the dynamic *intention* that emerges among the elements of the system; just because one could sit on a chair does not mean that one does so, depending on the situation. Thus, understanding what happens at a particular moment requires unpacking what was afforded, and how those affordances were recognized and realized. It is important to note that something can only be afforded if it can be recognized and acted on; what makes an affordance actionable is inherently a dynamic relation between the environment and person (Greeno & Gresalfi, 2008). This focus on interaction acknowledges that a particular task might make something possible, but doesn't make it obligatory.

This framework helps to suggest a way of documenting "situated learning," that is, the assumption that what someone is able to do and, eventually, learn, is inextricably tied to the particular context and circumstances in which one is acting (Greeno & MMAP, 1998; Lave & Wenger, 1991). Situative and sociocultural theories of learning receive a lot of attention in educational research in general, and in mathematics education in particular. However, it is one thing to claim that learning is an interactional accomplishment; it is quite another thing to demonstrate how that is so. Much of the research in this area has relied on micro-level analyses of discourse and gesture, painstakingly demonstrating that what one person does cannot be separated from the other things that were done to/for/around that person at the same time (Engle & Conant, 2002; Esmonde & Langer-Osuna, 2013; Esmonde, Takeuchi, & Radakovic, 2011).

Despite their quality, these kinds of analyses are not scalable; they cannot be applied to large numbers of students with the goal of understanding whether and why a particular intervention worked as it did. Instead, for those large-scale questions, we often rely on assessments that target one particular moment in time and offer that moment (or a difference between two such moments) as evidence of learning. Condensing an entire learning activity to a single moment is problematic, but even more problematic is that these assessments are themselves part of an interaction (that also includes the learner), and thus understanding what someone demonstrated that they "know" cannot be reasonably accomplished without a careful analysis of what or how the assessment itself contributed to that moment of knowing. This kind of analysis is rarely, if ever, done, and thus we are left in a situation wherein we can offer detailed, textured analyses of learning as an interaction among a small number of students, but can offer no such analysis with larger numbers of students. What is missing, it seems, is a theory of context that could be leveraged in a scalable way such that larger studies of learning could also functionally offer analyses of learning as an interaction.

The Strength Of Affordances

In my own work, I have considered context as having affordances that make it more or less likely that someone will act in a particular way. In documenting what people do in relation to these affordances, we come to understand someone more nuanced about moments of knowing. In Gresalfi (2009), I proposed a framework for characterizing the nature of affordances that focused on the relative strength an affordance offered to enact a particular activity (see also Gresalfi, Barnes, & Cross, 2012). Specifically, an affordance was characterized as "strong" when it

required students to comply or when not complying would have been a social or rule violation. In contrast, a weaker affordance provides an opportunity for something to happen, but does not make it imperative that the affordance is realized. A videogame can offer a nice example of the contrast. In some videogames, the player is encouraged to collect particular objects as they play, which can be used for some purpose in later game play. In some contexts, the players are merely encouraged to be on the lookout for these objects—picking them up, or not, is voluntary. In that case, the affordance that is offered is relatively weak; picking up objects is possible, but not mandatory. In contrast, much stronger affordances could be offered if the game prevents a player from moving on if objects have not been collected. In this case, the affordance for collecting objects is very strong, because not doing so prohibits continued engagement with the game.

In this work, characterizing the strength of affordances, that is, attempting to make progress on a theory of *context*, allowed a different understanding of *individual* performance. As an explanation of why, consider a paper written by Ilana Horn in 2008 about “turnaround” students. Horn followed seven students at two schools, all of who had initial experiences of mathematical success despite having previous histories of lower mathematical achievement. She labeled this year their “turnaround” year, and then followed these seven students through the rest of their high school career. Four students were in a school that offered tracked mathematics classes, while the other three students were in a school that did not offer tracked mathematics classes. Horn (2008) offers great detail about the differences between these schools in terms of the *figured worlds* (Holland et al, 1998) that they offered to students; a very brief summary is that the tracked school offered a narrative of success being possible for some, but not all, while the de-tracked school offered a narrative that anyone could be successful if they put forth the effort. Horn documented that, in their “turnaround” year, all seven students were in classes that supported a shift in their identities to consider themselves to be good at math. However, in subsequent years, in the absence of this class structure, students in the tracked school reverted to their earlier mathematical performances and self-perceptions, ultimately enrolling in remedial math or dropping out of math altogether. In contrast, students in the de-tracked school all enrolled in college preparatory math, and completed, at a minimum, the courses they needed for college acceptance. Horn’s (2008) goal was to document how structure can influence identities, and the particular importance of curricular structure and teacher beliefs in shaping the kinds of figured worlds that allow identities to develop and persist. However, equally interesting is the question of what was similar about students’ “turnaround” experiences, and how that was related to what happened in later courses.

One could characterize this finding as illuminating the difference between students when they are offered strong opportunities to engage with mathematics through the activity of meaning making, in 9th grade, and then later only weak opportunities. Specifically, students in the tracked school were working with an extremely skilled teacher who was awarded the Presidential Award for Excellence in Mathematics and Science Teaching. She has been the focus of significant analyses for her skill at maintaining rigor while broadening participation and collaboration (Staples, 2007). In short, she was a very skilled teacher who created strong opportunities for students to experience success in her classroom. However, when moved to classrooms with teachers who arguably offered weaker opportunities to engage, these students, for whatever reason, failed to take up those opportunities. In contrast, the students who continued to be successful worked both in their 9th grade year *and* throughout high school with a department of math teachers who shared an approach to teaching that intentionally welcomed diverse forms of participation, leveraged heterogeneous grouping practices, and believed that all students could

succeed (c.f. Boaler & Staples, 2008; Staples, 2008). Like the students in the tracked school, students in the detracked school were offered strong affordances for students to engage deeply with mathematics and experience success. However, the students in the detracked school *continued* to be offered strong opportunities to engage in these meaningful ways. Suddenly these students are not “bad at math,” but rather need stronger opportunities to engage in order to be successful. But crucially, if offered those opportunities, they *are* indeed successful. This recasting offers a clear way of conceptualizing performance as a shared accomplishment between person and environment, and helps us to recast failure and success as something that lies not just on the shoulders of teachers, or students, but both.

To summarize, the potential of building a theory of context is that it helps us to understand what individual behavior is responding to. A theory of context in no way guarantees that we will know what people will do, but it gives us a working hypothesis about what could happen. It is for this reason that I find this framework compelling not only as an analytic tool, but also as a design tool.

Affordance to do what? One challenging thing about creating a theory of context that focuses on potential is that it is possible to do many different things in any given moment. One could think about an affordance to comply or not, to try or not, to work with others or not (the list is endless). For that reason, when attempting to design a theory of context, it is imperative to select a specific focus. In my work, I have focused on the nature of student engagement with content. This is a meso level focus; a macro focus might look at participation versus non participation or opposition (see Hand, 2010), while a micro focus might look at the nature of what, specifically, students understand about mathematical content (see Empson, 1999). A meso level looks at the ways that students are engaging with mathematics itself, characterizing that engagement in terms that could be applied across mathematical content. Specifically, I have considered four types of engagement: procedural, conceptual, consequential, and critical (Gresalfi & Barab, 2011; Gresalfi, Barab, Siyahhan, & Christensen, 2009). Procedural engagement involves using procedures accurately (Rittle-Johnson, Siegler, & Alibali, 2001); conceptual engagement additionally involves understanding why an equation works the way it does. Consequential engagement involves recognizing the usefulness and impact of disciplinary tools; being able to connect particular solutions to particular outcomes. For example, students are engaging consequentially when they not only solve a problem by calculating the mean, but also explain what the mean tells them about the situation. Finally, critical engagement concerns agency in problem solving, and involves explicitly choosing particular tools and interrogating their impact on a solution. This way of engaging builds on the others; students can engage conceptually or consequentially without making an explicit choice about how they are solving a problem; realizing that their approach to a problem impacts its solution is a significant shift in engagement.

To illustrate the difference between these levels, consider the problem below (Figure 1), taken from Henningsen & Stein (1997), which they characterized as having the potential of offering high cognitive demand. At the macro level, this problem (assuming it is offered in the context of a classroom activity, with a grade attached to its completion), offers a strong affordance to attempt to complete the task. Not attempting to complete the task, in that context, would generally represent a significant breach of classroom expectations. At the micro level, this problem offers strong affordances to make connections between fractions, percents, and decimals, simply because the question includes the same question in these three terms. What’s more, the problem offers a strong affordance for students to consider what $.725$ of the rectangle

actually looks like, as they are asked to shade this area in the rectangle. This task is made more challenging because the rectangle contains 80 squares, rather than 100, so students will have to do some actual mathematical work to determine what $.725$ of 80 actually is, and what it looks like in the representation.

1. a. Shade $.725$ of the area of this rectangle.
- b. What fractional part of the area is shaded?
- c. What percentage of the area is shaded?

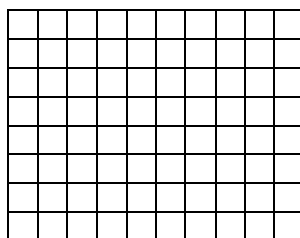


Figure 1: Sample Problem Offering High Cognitive Demand

At the meso level, we start to think about the nature of students' engagement with the problem—how they could go about solving it and what implications that might have for how they would understand it. A student could answer the question without considering the figure, as the fractional equivalent of $.725$ can be calculated procedurally, as $725/1000$, and the percentage could also be calculated procedurally, by moving the decimal point two places to the right. These are standard procedures that are commonly taught to students to enable them to determine equivalency between fractions, decimals, and percentages. Thus, this problem offers strong affordances for engaging procedurally. However, the addition of the figure, and the request to shade the figure, creates opportunities for students to consider what $.725$ actually *looks like*. If the students were given a 10×10 square, this would be relatively easy. But because the square is 10×8 , more calculation and thought is required. There are many ways that students could approach this task, but ultimately, successful completion requires both conceptual and, possibly, consequential engagement.

Conceptually, figuring out what $.725$ of 80 looks like requires moving beyond the rote execution of a procedure. The value of $.725$, or 72.5% , is not a quantity that is typically worked with. Students could solve the problem by figuring out what 75% of 80 is (60), and then estimating that it would be a bit less. Depending on the classroom context, this kind of solution might be acceptable and would surely demonstrate the students were taking up the affordance to think conceptually about what it means to take $.725$ of something. If a more exact solution was required, students might multiply 80 by $.725$. In either case, there is a strong affordance for students to move beyond simply using a procedure, either through estimating or calculating the area. As a brief note, the context of the classroom plays a significant role in the ways these affordances might be realized (Henningsen & Stein, 1998; Gresalfi, Barnes, & Cross, 2012).

Consequentially, the addition of the grid in this problem offers another strong affordance, which is to consider whether the solution that a student might come up with actually makes sense. Students often have trouble remembering the “rules” for decimal multiplication (for various predictable reasons, c.f. Martinie & Bay-Williams, 2003; Rathouz, 2011). Thus, the “simple” task of multiplication could yield a variety of solutions, few of which match onto the grid that the students are asked to shade. In this case, being asked to shade the area offers a

potential source of feedback for students, as it is impossible to shade 58000, 5800, or even 580 squares (the correct answer is 58). To make the strength of this affordance a bit clearer, contrast this task with one that asks to *determine* how much $.725$ of 80 is (without offering the grid). From a micro analysis (above), the two problems would offer identical affordances. From a meso analysis, however, the problems are quite different, as one contains a strong affordance to reason consequentially by testing out whether the calculational result actually makes sense.

Translating to Technology

Although the analysis of tasks is a well-worn area in mathematics education, the addition of technology is much newer. Technologies offer their own affordances, with the potential to support or thwart deeper forms of engagement. Potentially more problematically, most forms of technology offer not just different affordances for engagement, but also, more. Tools such as geometer's sketchpad or Tinkerplots create opportunities not just for productive engagement with mathematical content, but also offer a sandbox-like space that also allows for play and experimentation that can distract students from engaging with key ideas. More open technologies, such as video games, have the potential to offer even more affordances, both productive and not. In this context, with new tools that offer so many spaces for engagement, it is particularly important that designs be subjected to some kind of theory of context, with the goal of understanding what forms of engagement are truly likely to result.

Below, as an illustration, I offer an example of design changes that I made in the context of designing an educational videogame, which focused specifically on increasing the strength of an affordance to engage critically with content. These changes were made to the overarching dilemma of the game, which changed across three implementations to offer increasingly strong affordances to engage critically with content. As a reminder, critical engagement with content involves being reflective about the impact of a mathematical move on a designed context; in the case of statistics, one is engaging critically when they are able to discuss how the choice of a particular analytic tool (such as the mean) impacted what one could see in data, and how that might be different if a different analytic tool (such as the median) was used.

Because the goal of this paper is to focus on design, I do not share detailed information about the implementations themselves. A much more extensive discussion of the evolution of the unit, details about design-based research, and specifics about the implementations can be found elsewhere (Gresalfi & Ingram-Goble, 2008; Gresalfi, in review).

Evaluating Technology in Terms of Affordances for Engagement

The curriculum discussed in this paper takes place in the context of an online, interactive and immersive videogame called Quest Atlantis (QA) (www.QuestAtlantis.org). QA is an educational videogame that uses a 3D multi-user environment to immerse children, ages 9-12, in meaningful learning trajectories across multiple disciplines. The QA virtual environment involves engaging a core dilemma that requires leveraging disciplinary content for its resolution. In some ways, these dilemmas provide a framework much like a well-developed problem or project might. Specifically, in designing these games, our goal is to *situate the learner* as having the intention of making decisions that impact the dilemma, *situate the content* as information that is legitimately required to resolve the dilemma, and *situate the context* as modifiable and responsive to learner choices (Barab, Gresalfi, & Ingram-Goble, 2010).

Interactive, immersive educational games create several opportunities for student engagement that are frequently lacking in traditional schooling. In particular, the immersive

context can embed students' inquiry in contexts that can have consequence, thus transforming targeted concepts from rules to be remembered to tools that can be applied. In addition, the fantasy of online videogames support pedagogically useful tropes that can serve as legitimate feedback for students' thinking; rather than having to look to a teacher for whether one's solution is accurate, the situational context can change in response to students' decisions, thus serving to push back on student thinking and solutions. In QA, students take on particular roles and engage various plot lines, which can evolve and change based on the choices that they make. In this way, students are central players in an unfolding storyline that requires participation in academically meaningful activities, either in the real or simulated world. Although students are "playing," their actions have more consequence than their "real world" interactions in schooling. In this way, well-designed videogames hold the potential to position students in relation to content in particular ways by affording different kinds of engagement, while simultaneously setting out a trajectory of participation that has implications for more enduring relationships with content and a new way of seeing the world (Gee, 2003; Lee & Hoadley, 2007; Shaffer, 2006).

Ander City

The statistics unit discussed here, called Ander City, was created to support students' engagement with the following content standards (NCTM, 2000):

- Select and use appropriate statistical methods to analyze data.
- Describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed;
- Use measures of center, focusing on the median, and understand what each does and does not indicate about the data set;
- Compare different representations of the same data and evaluate how well each representation shows important aspects of the data.
- Develop and evaluate inferences and predictions that are based on data.
- Propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions.

The specific and somewhat unusual goal of the unit was to understand the role of statistical tools in allowing one to reveal—or hide—information. Students were afforded opportunities to consider how particular tools allowed some stories to be told, and not others. Although the exact storyline changed through cycles of design, as will be discussed below, the overarching issues remained consistent. Students were asked to make three crucial decisions for the city, all of which involved comparing two data sets that represented different companies or approaches. As an example, the first activity that students completed in all three revisions of the unit involved deciding which brand of bicycle the city should offer for rental in the park, based on which was the safest. All dilemmas were designed to support different conclusions depending on the statistical method leveraged for analysis (e.g. students might make a different recommendation if they calculate the mean of a data set than if they consider the mode or look at the distribution of the data set). As students help to redesign the city by making arguments for or against particular decisions, they engage with increasingly advanced statistical content, and are challenged to offer increasingly sophisticated explanations and justifications of their decisions.

To summarize briefly, looking across the iterations of the design (see Figure 2), we found that as we increased the strength of the affordance to engage critically with content, more students demonstrated critical engagement in their written recommendation.

Figure 2: Average total scores for Procedural, Conceptual, Consequential, and Critical Engagement across the Implementations.

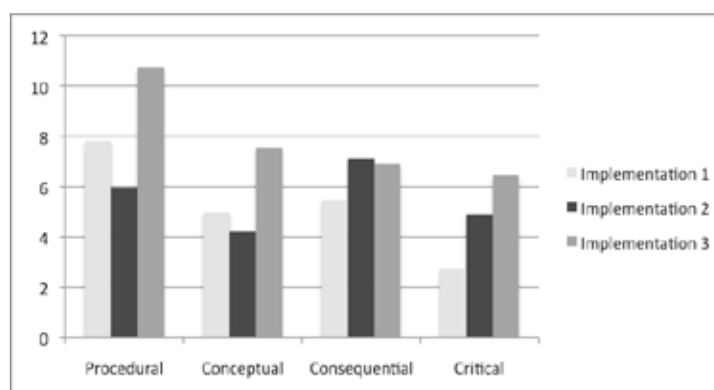


Table 1 summarizes the overall design decisions that were made. Below, I analyze the first and last of these decisions in terms of their affordances and share outcome data, particularly with respect to how those outcomes shaped our revisions for future designs.

Table 1: Overview of changes in narrative, intentionality, and outcome

Design	Narrative	Intention	Outcome
1	Mayor Enoch needs help making good decisions for the city.	Help Mayor Enoch	None
2	Mayor Enoch is being accused of deception, his opponent claims that he is making biased decisions that don't really benefit the entire city.	Defend Mayor Enoch against skeptics	Help Mayor Enoch keep his position
3	Mayor Enoch and Mr. Grant BOTH claim that they are making the right decision. They disagree with each other, although they both have data to support their positions. Is someone lying?	Determine which candidate is making better recommendations; or who might be lying.	Determine who will ultimately be elected mayor.

First Design

The storyline of the first version of the statistics unit centered on the fictional character of Mayor Enoch, mayor of Ander City, who claimed to need help. He was concerned that the citizens of the town were unhappy and were leaving the town. He felt that he had an obligation to make sure that he was making decisions that would make the city a better place to live, and would ensure that its citizens stayed. He thus needed assistance in making three crucial decisions: the first involved which brand of bikes they should offer for rental in the park, the second involved which brand of swings they should install, and the final issue involved the kinds of snacks they should make available in the snack cart that was adjacent to the playground in the park. These contexts became the decision points for students; they were invited to take a look at the data and make a decision to help Mayor Enoch based on their analysis of those data. The specific dialogues that occurred between Mayor Enoch and the player can be seen in Figure 3.

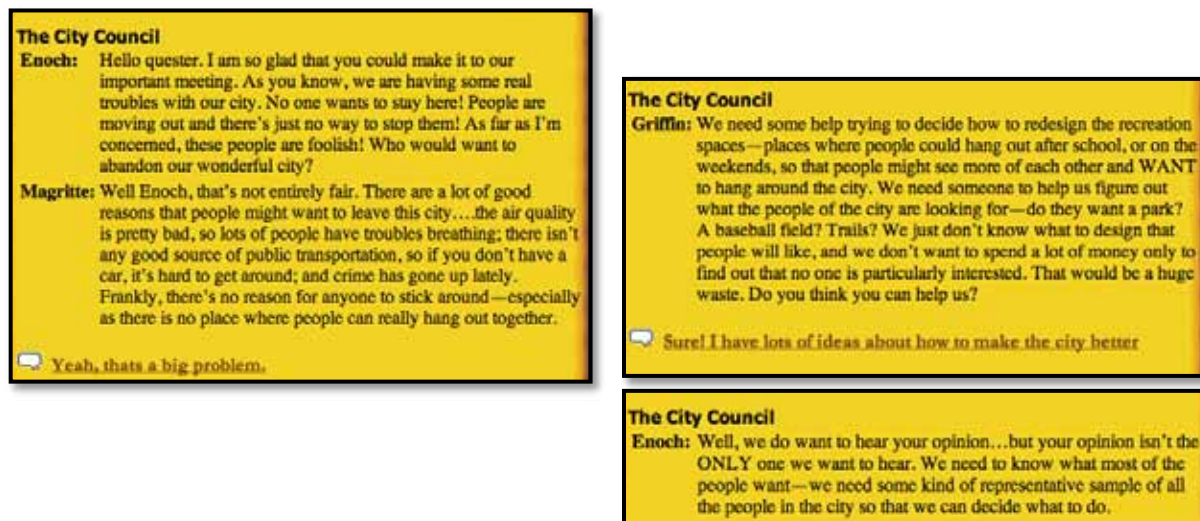


Figure 3: Narrative Framing for Design 1.

The narrative was designed with the goal of attuning students to the importance of making decisions that were *defensible*; decisions that could be justified with data. In creating a scenario wherein the Mayor of the city was in somewhat of a crisis, our initial conjecture was that the emotional salience of the situation would create a strong affordance not just to analyze data, but to be intentional about *how* those data were analyzed (critical engagement). We conjectured that this external audience (the citizens of Ander City) would raise the standards for students' recommendations, as they would take their activity to be about *convincing* rather than simply *complying* (Engle & Conant, 2002).

Uptake of Affordances. In this first implementation, we found that while students' critical engagement improved over the three decisions they made, there was still very little critical engagement overall. Specifically, students became accomplished at selecting one way to analyze the data they were given (the analytic method was always left open to the students), and then explaining what their analysis meant in terms of the situation (what we called consequential engagement). However, very few students seemed to realize that the way they analyzed the data impacted what they were ultimately able to say about those data. This suggested that what we had originally considered to be a strong affordance for engaging critically was in fact a fairly weak one; this was evidenced by the fact that so few students engaged in this way. This lack of uptake was in spite the fact that, in whole class discussions, they were able to compare and contrast solutions and therefore could clearly compare different solutions. In other words, we did not believe that there was a problem in students' effectivities for engaging these affordances.

Final Design. In our redesign, we attempted to raise the salience for the need for justification and considering both sides. We conjectured that the largely invisible audience of the citizens of Ander City did not provide a sufficiently strong affordance for engaging. Thus, we changed the narrative so that the mayor was running for re-election against a character who was maligning the mayor and his decisions. In the new scenario, students were asked to decide which candidate was making better choices for the city, and in particular, if one candidate was lying. We conjectured that shifting the role of the skeptic to the students themselves would serve to shift the agency for being critical directly to the students, as it would be impossible to consider who was

lying without evaluating both perspectives. In this way, the affordance for engaging critically was much stronger than in the initial design, as NOT engaging critically would constitute only a partial solution. The revised unit began with the following letter from another student in the town, seen in Figure 4.



Figure 4: Narrative Framing for Final Design

Uptake of Affordances. In the final design, many more students engaged critically with the content, offering explanations of not only what they did, but also what they might have found if they had done something different. An example of an excerpt from a student's recommendation is below; the student explains that she used two different tools (total first, and then range), and justifies why she believes range is a better measure for the situation under consideration.

"I analyzed the data by making a bar graph then added each 10 skids up and came out with speedy spokes for the answer as which is most reliable bike to stop soon enough before wrecking into something. But I just noticed the that the Speedy Spokes skid can be anywhere from 30 in. to 54 in. and the Rollin Steady skids were from 43 in. to 55 in. I would much rather be on a Rollin Steady bike because it is more consistent to what we are looking for here in Ander City so you don't run into the cars at stop sign. The reason why I chose range for the analysis is because I just thought that with a more consistent stopping bike that you wouldn't be worrisome about you stopping before the traffic or in the traffic this is why I chose the range for my analysis."

This student made it clear that she had considered the data in two different ways, each of which revealed something different about the data. Specifically, one way of looking at the data (total stopping distance) suggested that one brand of bike tended to stop sooner, but another way of looking at the data suggested that the other brand of bike was more predictable. These are both reasonable ways to think about safety, and thus it was left for the student to decide which analytic frame made more sense. Importantly, this student clearly felt that she had the agency to determine how to analyze the data, and was able to reason about what particular numbers (total stopping distance, range) actually meant in terms of the situation. Although not all students engaged critically even by the final implementation (the average score for critical engagement was 6.2/12, which is only over 50%), there was certainly a marked increase in the number of students who were able to approach data analysis in this more empowered, critical way.

Discussion

The examples above were intended to demonstrate how, in considering the strength of affordances, we can make sense of what students do in relation to what they have opportunities to do. In the case of the first design, very few students engaged critically in their written responses despite the fact that every single decision they were asked to make had two very plausible different solutions. One could imagine an analysis that suggests that students aren't able to consider two perspectives simultaneously, or, more insidiously, that students aren't thinking but are instead just following rules. Both explanations assume that there is a problem with students' interest or ability. Considering how engagement changed in light of the shift in the strength of the affordance to engage critically, a different interpretation is possible; specifically, that students *are* able to engage critically when they have a reason to do so—when the affordance is sufficiently strong that taking it up is required to successfully solve the problem. The shift in engagement that we saw across implementations clarifies that there wasn't a problem with students' effectivities, but rather, that a stronger affordance was required in order for students to recognize it and realize it.

An important question to be posed of this framework is whether or how one might actually measure the strength of an affordance. Indeed, the design trajectory shared above demonstrates compellingly that it is difficult to know whether students will take up an affordance, and thus initial determinations of something as being “strong” might ultimately be mischaracterizations. To be sure, there are parameters that we can use to offer initial conjectures about the potential strength of an affordance, but of course this can only be seen by examining a task or tool in interaction. This might seem a bit of a tautology: an affordance is strong if someone takes it up, and someone takes it up if it is strong. But this is not the case. In understanding what someone does in relation to what they have opportunities to do, we cannot simply examine a single individual separate from the classroom context, nor can we examine a single moment in time. If only one or two students act on an affordance, we can reasonably assume that the affordance was weak. Therefore, initial considerations of the strength of affordances must be grounded in small-scale analyses that document patterns in participation. Following this close analysis, this framework can begin to be used at larger scales, allowing for consideration of how the performance of one group, or many groups, differs from other groups, and to what we can attribute that difference. It is in this way that this framework can begin to bridge the large gap between case studies and “big data.”

All learning theories, despite their vast differences, define learning as a change of some kind. Considering the opportunities to learn framework, learning can be considered as a shifts in

students' sensitivities to affordances such that, over time, strong affordances for engagement are not required. A student who had had multiple opportunities to work on statistical data analysis by engaging critically might become increasingly sensitive to opportunities, thus requiring only weak opportunities to engage in this way (Gresalfi, 2009). Reconsidering learning in this way offers a different interpretation of success and failure, and, equally importantly, offers immediately actionable recommendations for students who are failing to engage mathematical content as we would like them to. Specifically, as demonstrated above, by subjecting a task, technology, or even small design decision to a consideration of the strength of its affordance, we can begin to develop conjectures about what students will do. Following data collection and analysis of student participation, we can develop a clearer understanding of how the context is contributing to what we ultimately see students doing, knowing, and learning.

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