EXTENDED NETWORKS: MATHEMATICS CLASSROOM COLLABORATION WITH MOBILE DEVICES

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This paper explores the potential of mobile devices to support novel forms of mathematics classroom activity. In particular, I consider two salient aspects of contemporary practice with mobiles—networking/communication, and picture taking—and explore technology designs that might exploit their potential for supporting related forms of mathematical practice. I present data from two cycles in a design-based research project focused on collaborative learning in classroom networks, one featuring graphing calculators, and the other using iPods.

Keywords: Algebra and Algebraic Thinking, Classroom Discourse, Modeling, Technology

Introduction

Modern mobile computing devices—smartphones, tablets, and the like—have rapidly emerged as central tools in the daily practices of many American youth. Moreover, they increasingly prevalent in the hands of k-12 students at school. According to a recent report, more than half of all high school students now bring a smartphone to school (Grunwald, 2013), and Los Angeles Unified, the nation's second-largest school district, plans to give an iPad to every one of its 650,000 students by the fall of 2014.

But what might this proliferation of new mobile digital tools mean for teaching and learning mathematics? Handheld computational devices, in the form of four-function, scientific and graphing calculators, have been commonplace in mathematics classrooms for years. So for math, the potential for instructional novelty more likely lies in the newer features unique to the latest generation of tools—internet connectivity, communication and information-sharing resources, photo and video capture capabilities, customized apps. Indeed, many forms of digital practices with mobile devices popular with today's teens have clear analogs with aspects of mathematical practice highlighted in current standards (White, Booker, Martin & Ching, 2012).

In this paper, I present results from an ongoing design-based research (Brown, 1992; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Design-Based Research Collective, 2003) program focused on using classroom networks of handheld devices to support student interaction and collaboration in secondary mathematics classrooms. In particular, I focus on two of these distinctive capabilities of contemporary mobile devices—communication and information sharing via wireless networking, and camera-capturing photo and video images—as resources for designing tools and learning activities that might have the potential to integrate informal digital and formal mathematical activity.

Communicating and Collaborating in Classroom Networks

Perhaps the most essential purpose of mobile devices, and the fundamental reason for their near-ubiquity among today's youth, is as tools for communication: talk, text, email, video chat, and social media interface. As such, these devices have the potential to facilitate learners' participation in critical aspects of mathematical activity. Recent national mathematics education frameworks such as the NCTM and Common Core practice standards have stressed the importance of providing students with opportunities to organize and express their own mathematical thinking as well as critically examining that of others. In this spirit, promoting student participation in mathematically rich classroom discourse has been a central theme in mathematics education research and practice over the last two decades (Ball, 1993; Lampert & Blunk, 1999; Yackel & Cobb, 1996). Often, instructional activity in this vein takes the form of teacher-facilitated whole-group conversations in which mathematical meanings, arguments and standards of evidence are established collectively (see, e.g., Forman, et al., 1998; Staples, 2007). In other instances, students work in pairs or small groups on collaborative problem-solving tasks, and thus have opportunities to discuss ideas and strategies, negotiate and coordinate interpretations, and provide peer tutoring (e.g., Barron, 2000; Boaler & Staples, 2008; Moschkovich, 1996; Leikin & Zaslavsky; 1997).

Recently, several innovative research and design projects have begun to map out ways that handheld devices connected to local computing networks might support and enrich these forms of communication and discursive interaction in mathematics classrooms. Classroom networks can support students' agency and participation in collective mathematical activity (Ares, Stroup & Schademan, 2009), attention to and identification with dynamic mathematical representations (Hegedus & Penuel, 2008), and opportunities to draw on diverse cultural and linguistic resources for participating in classroom discourse (Ares, 2008). Likewise, networked handheld devices in small group collaboration can facilitate greater communication, coordination and negotiation among peers (Zurita & Nussbaum, 2004), and expand and enrich avenues for active participation in joint problem-solving activity (White, 2006; White, 2009; White & Pea, 2011).

Contemporary mobile devices offer the potential to extend these networks beyond the classroom to connect students, teachers, and community members even as they move across contexts. Networked devices offer new possibilities for interaction; they present ways of rapidly distributing information, exchanging ideas and constructing shared artifacts. Just as in-classroom device networks have powerfully demonstrated the potential for merging peer communication with dynamically linked mathematical representations (Hegedus & Moreno-Armella, 2009; White & Pea, 2011), connections between mobile devices that extend the sharing of mathematical objects and ideas beyond the schoolyard represent unique opportunities for blending learners' informal digital activity with conventional forms of mathematics classroom discourse. In particular, photos, videos and other artifacts or data collected outside the classroom using mobile devices may form a particularly powerful resource for supporting learner's efforts to find conceptual coherence between real-world phenomena and school mathematics—especially if those captured artifacts or data allow learners to make mathematical meaning of personally relevant objects and experiences.

The *Graphing in Groups* Design

To explore these possibilities, this paper reports on two implementation cycles of a design for collaborative mathematics activities using a classroom network. The learning environment described in this paper is one among a family of designs greated according to the guiding principle that the *social* should be mapped to the *mathematical*—that collaborative relationships among students should be organized around the mathematical relationships that the learning activity seeks to help students understand (Stroup, Ares & Hurford, 2005). These designs have two primary pedagogical objectives: 1) to link each student participant in a small group with objects in the shared space of the classroom network to make important mathematical relationships salient, and 2) to encourage collaborative interactions among students by posing tasks and challenges that require participants to coordinate their individual contributions in order to jointly manipulate shared objects. We develop these activities using the NetLogo modeling environment (Wilensky, 1999) and HubNet network tools (Wilensky & Stroup, 1999) in concert with classroom sets of student devices–Texas Instruments graphing calculators or Apple iPod/iPads. This classroom network situates each student's device within a server-defined small group, and a screen projection from the server at the front of the classroom displays mathematical objects linked to both individual student devices and to student small groups. Exemplars of this approach include each member of the small group examining different dynamically linked representations of the same mathematical function displayed on their respective devices (White, 2006; White & Pea, 2011), transforming alternate sides of a shared equation (Sutherland & White, 2011), manipulating different vertices of a jointly constructed quadrilateral (Lai & White, 2010), or moving respective points in a shared graphing space in order to jointly manipulate a curve (White & Brady, 2010; White, Wallace & Lai, 2012).

In the present design, called *Graphing in Groups*, students are assigned to work in pairs. Each student uses the directional arrow keys on her calculator or iPod to adjust the Cartesian coordinate location of a point, which is displayed both privately on the student's device and publicly in a graphing window projected from the teacher's computer. Two pairs of students are assigned to each in an array of such graphing windows in the public space, as shown in Figure 1.

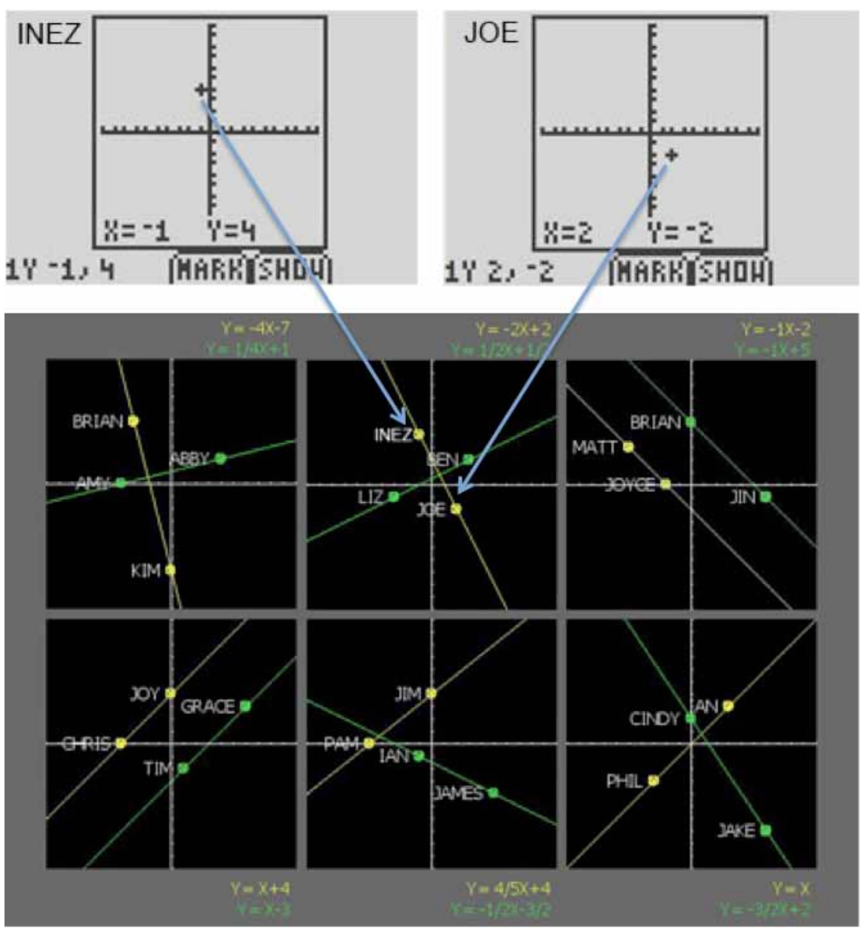


Figure 1: Two student Cartesian coordinate locations, on calculator screens at top, form a line drawn in a group window of a *Graphing in Groups* public display.

As each member of a pair marks a new coordinate location, the corresponding line between the students' respective points is dynamically redrawn and its slope-intercept-form equation updated in the public display. Classroom activities in this environment typically revolve around student pairs' successive efforts to construct lines with particular characteristics: a slope of three, an equation y=(4/3)x-2, x- and y-intercepts both equal to seven without either student placing her point on an axis, etc. In the iPod version of the Graphing in Groups design, students have the additional capability of sending photos taken using their devices up to the server, where they become the background for the group's graphing display (Figure 2).

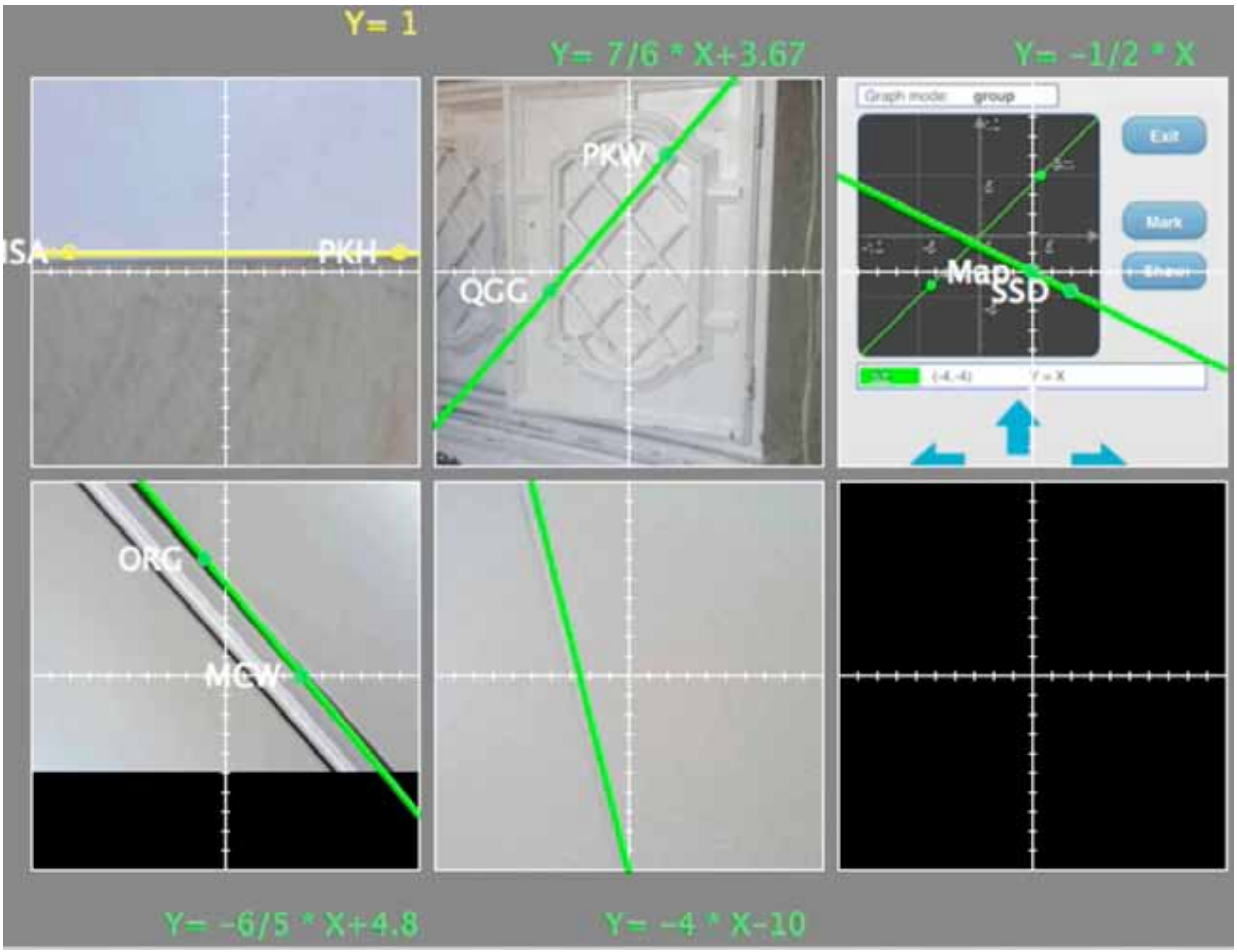


Figure 2: *Graphing in Groups* Display with Student Photo Backgrounds

Method

This paper presents data from two successive classroom-based design experiments using the *Graphing in Groups* environment at the same school site, first using the graphing calculator platform in the spring of 2010, and then again using the iPod Touch version in 2013. In the first cycle, six days of *Graphing in Groups* activities were part of a year-long project in which students participated in classroom network activities for a one-hour session each week as a supplement to their regular mathematics program. The author served as the teacher for all these class sessions, and sixteen 9th grade Algebra I students participated regularly throughout the year. The second cycle was conducted as a shorter instructional unit featuring 4 sessions with a different group of sixteen students in a mixed-age cohort spanning grades 7 to 9. Two other researchers shared teaching duties with the author in this second study. Two to three student pairs in each class were selected as focus groups and videotaped during all activities. All screen states of the public computer display were recorded as a video file for each class session, and an additional camera with a wide zoom setting captured this projected display along with the whiteboard at the front of the room, as well as whole-class discussions and other teacher moves.

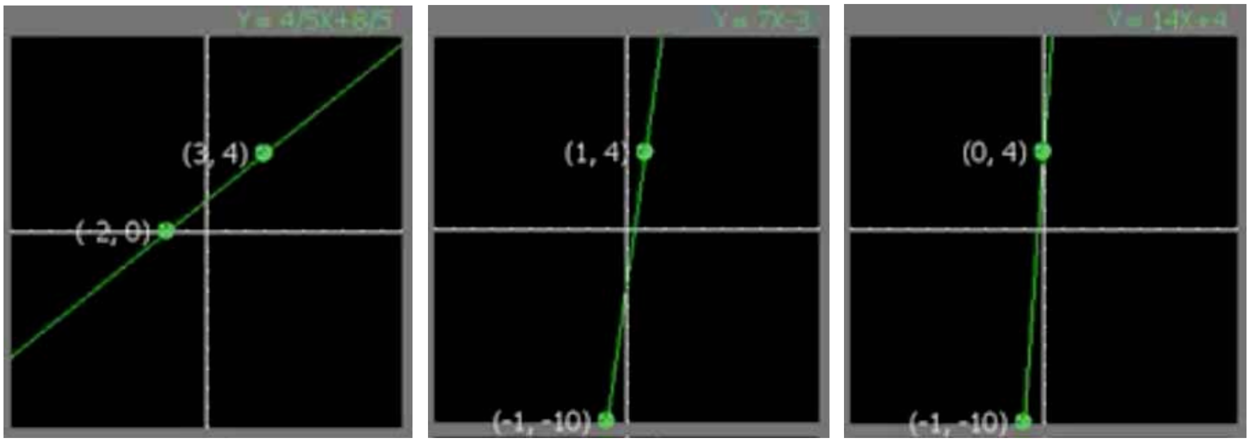
Results

To illustrate the kinds of mathematical activity supported by classroom networking tools, I will detail two brief episodes of classroom interaction as students worked together with the *Graphing in Groups* design. The first episode, taken from the study cycle featuring graphing calculators, highlights some of the complex interactions among students, teachers and mathematical objects typical of this environment. The second episode, from the iPod cycle, examines the potential for bridging these classroom mathematics activities with mathematical and digital experiences drawn from outside the classroom.

Episode 1: Classroom collaboration—supporting connections within and across groups

Below, I present a sequence of three consecutive segments of classroom dialogue during a line construction task in *Graphing in Groups*. Together, these segments span just over two minutes of classroom activity, as students worked in pairs to create lines with as large a slope as possible given the constraint of points confined to a graphing window with x- and y-min=-10 and x- and y-max=10. The first segment focuses on two students, Monica and Jamal, as they begin this task.

1. *Teacher*: Now, I want you to make the biggest slope that you can.
2. *Monica*: The biggest slope that we can?
3. *Jamal*: Oh, ok, I know how to do this. [From his initial location at (-2, 0) (Figure 3), begins moving his point to the y-axis and then down].
4. *Monica*: You know how to do this?
5. *Jamal*: Yeah. Alright, watch. [Continues to move down to (-1, -10)].
6. *Monica*: Where am I going? [Begins moving right from (3, 4)]
7. *Jamal*: Alright, hold on, go back, go back, go back, go back… [Monica moves her point back to (3, 4)]. Closer, closer… to the left. Go left… alright, go back. [Monica moves her point to (0, 4), then back to (1, 4)]. Yeah, right there. Stay right there!
8. *Monica*: Right there?
9. *Jamal*: Mark it. Yeah. [Both students mark points to form the line y=7x-3 (Figure 4)].
10. *Monica*: Seven? I think I can make it bigger than that. [Moves one unit left, to (0,4), and marks to form y=14x+4 (Figure 5)]. Fourteen!
11. *Teacher*: Group 3 has a slope of fourteen.
12. *Jamal*: Yup. [Sets his calculator down on the table in front of him.]
13. *Teacher*: That's pretty big.
14. *Jamal*: I think that's the biggest, cause…
15. *Teacher*: You think that's the biggest you can make? Let's see if anybody can make it bigger.



Figures 3, 4 and 5: Successive Lines Generated by Monica and Jamal

In this excerpt, Jamal opened with the assertion that he “knows how to do this” (line 3), and quickly began to move his point down to the bottom of the graphing window, presumably seeking to maximize the Δy between their points. When Monica began moving right, he directed her back to the left, likewise decreasing the Δx (lines 6-7). When both students marked their new points to form a line with slope m=7, Monica proposed that she could “make it bigger than that,” moving left one unit to form a line with slope m=14 (lines 9-10). As the teacher announced to the class that their group had formed the steepest line yet (line 11), the pair appeared satisfied with their efforts (lines 12-14) until challenged by the teacher to “make it bigger” (line 15).

Successfully completing tasks posed in the *Graphing in Group*s environment typically involves students’ establishing and maintaining a shared understanding of a solution strategy involving how to move their respective points to jointly construct the desired line. Several common characteristics of student actions and peer interactions in this environment (some of which are elaborated in greater detail in White, Wallace & Lai, 2011) are salient in this opening segment. First, ‘inhabiting’ this mathematical space (Noble, Nemirovsky, Wright and Tierney, 2001) takes embodied and multimodal form; students working on these tasks tend to rely on the coordination of spoken utterances with electronic actions in the shared graphical space (as when Jamal invites Monica to “watch” as he moves his point downward in line 5, and Monica likewise asks “where am I going?” even as she begins moving her point to the right in line 6).

Second, students often accomplish this coordination through careful negotiation and sequencing of their respective moves, rather than simultaneous action. Thus Jamal narrates his own problem-solving efforts in lines 3 and 5, then directs Monica’s movements “back,” “to the left,” and “back” again in line 7, then prompts them both to “mark” in line 9; Monica likewise announces her own successive strategic move in line 10.

Finally, the public nature of the group’s graphing display allowed the teacher to join them in assessing the product of their work, both by pointing out the value of their slope to the rest of the class (line 11) and prompting Jamal and Monica to consider whether the slope could be steeper. The next segment explores this public, whole-class dimension of the activity in greater depth. To do so, we widen our analytic lens, zooming out from one student pair (Monica and Jamal, Group 3) to include two other groups (Group 1, Juan and Miguel, and Group 4, Felix and Byron) working on the same task in parallel. In Table 1 below, the utterances of the teacher and the simultaneous dialogue within each pair are synchronized over a 32-second interval. Figure 6 shows the state of the whole class display projected at the front of the room as the next segment begins, immediately following the Teacher's invitation in line 15 above.

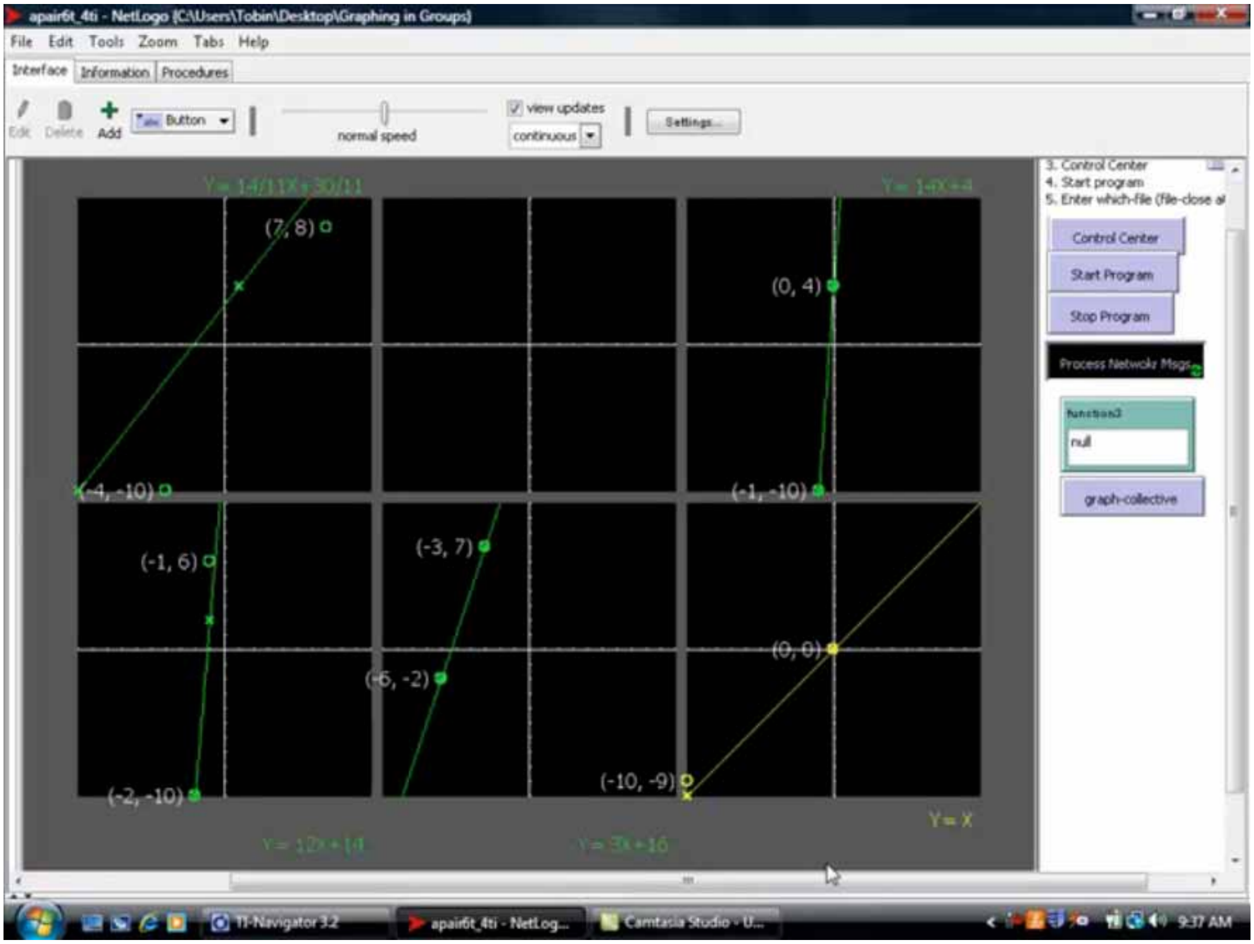


Figure 6: Graphing in Groups Whole-class display. Group 1 is in upper left corner, Group 3 in upper right, group 4 in lower left.

Table 1: Simultaneous Transcripts from Three Classroom Groups

Time (sec)	Teacher	Group 3	Group 1	Group 4
1	Group 4's got a slope of twelve.	*Monica*: [moving left to (-1, 4) as J moves right to (0, -10)]. Let me see, hold on, don't move.		[Byron marks at (-1, 10) to form y=20x+30]
2	Ooh, group 4's got a slope of twenty.	*Monica*: [as both students mark their new points] That ought to be a little bit better.		
3				[Byron punches the air in triumph]
4		*Monica*: Really then.	[J marks at (2, -10)] to form y=-14x+18]	
5		Go over to the, 10…	*Miguel*: Aww!	
7		[J begins moving to the left]		
8	Group 1, negative fourteen.		[J marks at (1, -10) to form y=N/A]	

9				
10	Group 3, negative fourteen.	[J marks at (-10, -10) to form y=(7/6)x+5/3]	[J marks at (0, -10) to form y=14x-10]	*Byron*: That's the lowest you can get.
11				
12			*Juan*: Go up. [M moves up one unit, to (7, 9)]	
14		[M marks at (2, 4) to form y=7x-10]	*Juan*: [Starts moving up] Go to the same one,	[F moves left and marks at (-3, -10)]
15			just go a little higher. [M begins moving left]	Byron: Just stop moving.
16			Same one you had.	
17		*Jamal*: Alright, hold on. Go all the way to the right corner.	*Miguel*: Oh, like right here [moves to (1, 9), directly over last mark	[F moves back to (-2, -10) and marks again]
18		*Monica*: [moving right to (7,4)] That's not going to be a very big slope, though.	at (1,4)] but higher? [moves up to (1, 10). J stops at (0, 2)]	
19		[M begins moving back to the left]	*Juan*: [M moves to (2, 10)] Yeah, right there.	
20		*Monica*: I have to be, like, closer to you.	[M moves back to (1,10)]	
21			*Miguel*: There.	[B begins drumming the desk with his thumb]
22		Jamal: [moving up] Oh, hold on, hold on. I've got an idea.	[J starts moving down]	
23			Juan: Wait, hold on, hold on.	
24			Yeah, right there.	
25		[J marks at (-8, -5) to form y=(9/10)x+11/5]	*Miguel*: Oh, do I mark it?	
26		[M marks at (-9, 4) to form y=-9x-77]	*Juan*: [still moving down] Yeah, mark it.	
27		*Monica*: That's only 9. [Begins moving right]	[M marks at (1, 10) to form y=20x-10]	
28		*Jamal*: [moves right] Alright, hold on. Go to the [waves finger to the right] right [points again] side.	*Juan*: [moving back down Yeah, you can't get higher than 20.	
29	Group 1's got 20.	[J marks at (-6, -5)]		
32		[M marks at (-3, 8) to form y=(13/3)x+21]		*Felix*: (to Group 1) Copycatters!

In the first two seconds of this segment, Monica and Jamal both tried further reducing Δx to increase the slope, producing a line with m=-14 even as the teacher announced Group 4's creation of a line with slope m=20. This news prompted different reactions from all three groups, as Byron thrust a hand forward in triumph (3s) while Monica mused "really then" (4s) and Miguel simply groaned (5s). Monica and Jamal then initiated a series of unsuccessful attempts at further increasing the slope of their own line, Jamal moving to (-10, -10) (10s) and directing Monica to the opposite corner (17s) even as Monica asserted that they should move closer together (20s). Meanwhile, Juan appeared to draw inspiration from Group 4's success, repositioning his own point at the bottom of the screen and then directing Miguel into a configuration that matched Byron and Felix's in order to likewise forma line with slope m=20.

This segment illustrates two additional characteristics common to *Graphing in Groups* classroom activity. The first is that the public display serves as a resource for the teacher not only to track the progress of all student groups as they work in their respective graphing windows, but also to comment on and provide feedback regarding their progress. The second is that the display also allows groups to attend to one another, sometimes modifying their own solution strategies based on their observations of others', as Juan appeared to do based on Group 4's work, and Jamal may well have likewise been attempting when he went to the lower left corner and directed Monica to the upper right after seeing a similar line constructed by Group 6 (lower right window in Figure 6).

In the moments that followed, Jamal and Monica continued to experiment with their own strategies until they converged on a solution:

16. *Monica*: I am just going to keep going until it makes it bigger. [Moves to the right]
17. *Jamal*: [Moves up and right, marks at (-5, -4)] Alright, hold on, let me go this way. [Moves to the left, marks at (-8, -4) as Monica marks at (-1, 8) and then (0, 8)] Holy crap. [Marks at (-9, -4), (-8, -4) again, and then at (-7, -4) as Monica marks at (-2, 10) to form y=(14/5x+78/5] Alright, fourteen…
18. *Monica*: [Moves left and marks, in quick succession, at (-3, 10), (-4, 10) and (-5, 10).] Don't move. [Marks at (-6, 10) to form y=14x+94, then moves to mark at (-7, 10) just as J marks at (-10, -6), forming y=(16/3)x+142/3] Why are you moving?
19. *Jamal*: Oh, sixteen…
20. *Monica*: [Marks at (-8, 10) to form y=8x+74] Eight.
21. *Jamal*: We had sixteen.
22. *Monica*: [Marks at (-9, 10) to form y=16x+154] Wait, now, sixteen.
23. *Teacher*: Sixteen, good, getting bigger.
24. *Monica*: [Marks at (-10, 10) just as J moves right and marks at (-9, -7) to make y=-17x-160] Stop it! Stop it!
25. *Jamal*: Seventeen, [marks at (-9, -8) to form y=18x-170] eighteen, [marks at (-9, -9) to form y=-19x-180] nineteen…
26. *Monica*: Twenty! (laughs) [J marks at (-9, -10) to form y=-20x-190] Twenty.
27. *Jamal*: Negative twenty, alright.
28. *Monica*: I think 20 is the biggest you can make it.
29. [Jamal, Miguel and the teacher have a brief exchange about the y-intercept]
30. *Jamal*: [marks at (-10, -10) to form y=N/A] Alright, hold on. Go right one
31. *Monica*: What did you do? Oh my God, Jamal. [moves right to (-9, 10), marks to form y=20x+190]
32. *Teacher*: Alright, there's another 20. Can anybody get bigger than 20?
33. *Monica*: I don't think it's possible.
34. *Jamal*: No, I don't think so.
35. *Teacher*: I don't think so either. Why don't you think so?
36. *Jamal*: Cause, that's the height [holds both hands in front of him, then waves right finger up and right, left finger down and left as if to mimic the respective positions of Monica's and his points (Figure 7)] like, the graph doesn't go any bigger.
37. *Monica*: The graph doesn't get any bigger.

Figure 7: Jamal's Gestural Depiction of the "Height"

Monica's opening comment sums up the approach she adopts in this last excerpt, moving two steps to the right and marking after each, and then reversing direction to systematically move and mark at every point from (-2, 10) to (-10, 10) (lines 17, 18, 20, 22, 24)—keeping on "going until it makes it bigger" (line 16). This kind of incremental variation represents another common strategy students use when trying to construct lines in *Graphing in Groups*, gradually tinkering with a line and feeling their way to a solution when they could not see a more direct analytic approach. Jamal, however, was not initially in synch with her efforts, instead moving and marking more haphazardly (lines 17, 18, 24) without consulting Monica. This failure to coordinate their efforts clearly frustrated Monica, who repeatedly asked Jamal to stop moving (lines 18, 24). By the time she reached the end of her leftward path, Jamal appeared to have recognized the progress she was making as he, Monica and the teacher all commented (lines 21-23) on their arrival at a slope m=16, their largest yet. Indeed, Jamal then took up the same iterative and incremental approach, moving to (-9, -7) and marking, then repeating the sequence for each step down to (-9, -10) as he and Monica called out the increasing (negative) slopes.

This segment thus highlights important additional aspects of action and interaction in *Graphing in Groups*; the embodied sense in which students act on these graphical objects through electronic point movements facilitates both individual exploration of the mathematical space, and nonverbal communication, as peers observe, react to and take up one another's patterns of motion. Moreover, the interdependence of the shared line on their respective

Cartesian points imposes constraints on individual action; students' common discovery that failure to coordinate their point movements undermines their individual problem-solving efforts and investigations, and thus builds a need for coordinated collaborative interaction into the learning environment.

Finally, Jamal's gestural activity during this episode further illustrates the embodied dimensions of learners' meaning-making in this virtual graphical setting. In the previous excerpt, Jamal's successive waving of his finger twice to the right in precise synchronization with the words "right" and "side" (28s) as he sought to orchestrate Monica's graphical movement highlighted the "semiotic bundling" of utterance, gesture and representation common to mathematical activity (Arzarello, Paola, Robutti & Sabena, 2009; Radford, 2003; 2009). Likewise in this segment, Jamal's carefully synchronized movements of each hand as he sought to explain why the line's slope cannot be greater than 20 while the points remain in the present graphing window (line 36) provide a visual rendering of the graphical situation, his hands abruptly stopping at the ends of their up- and downsweeps as if confined, just like their points, to the "height" of the available display (Figure 6). In this sense, these gestures with his hands at once appear to encapsulate the sequence of point movements necessary to construct the line, enact the relative positioning of his and Monica's points, and summarize the argument that the maximum slope has been achieved. That they are articulated via embodiment of two points, rather than a pitched hand or forearm to display the resulting line, suggests that the salient features of student action in this particular designed space provide specific resources for student reasoning about the corresponding mathematical space.

Episode 2: Extending Networks Beyond the Classroom

This section examines an excerpt from the work of another student pair in the *Graphing in Groups* environment, this time using iPods. This episode finds these students on their third day in the study unit. In the previous two days, they had been introduced to the iPods and the graphing tools; at the end of the previous session, the teacher gave a homework assignment in which he asked them to use their iPods to take photos of lines with different slopes to be shared and investigated when they returned to class. The excerpt below begins just after the teacher asked each pair to select one of their photos to upload to the network for display in their group's graphing window, and focuses on two students, Olivia and Jane, as they start this task:

1. *Olivia*: Okay, so these are the photos that I have [shows her iPod to Jane]. I have um…that…[opens Figure 8 on her iPod]
2. *Jane*: That would probably be good.
3. *Olivia*: That…[swipes through photo album to display Figure 9]
4. *Jane*: Is this stuff in your room?
5. *Olivia*: And that [swipes again to show Figure 10]. No, it's my sister's room. So which one do you wanna use, the one with the [holds hand up and makes a pitched line] stripes?
6. *Jane*: Yeah. Then we have different [trace a crisscross with her index finger] lines to go to. [A cropped version of the cabinet door from Figure 9 appears in Jane and Olivia's graphing window in the public display (Figure 11)].



Figures 8, 9, and 10: Candidate Photos on Olivia's iPod

In lines 1-6, we get some insights into the ways students made sense of this photo assignment. Olivia captured three images—one of a grid that might be a tile floor (Figure 8), one of a cabinet door (Figure 9), and one of a decorative string of globe lights hanging from wires at diagonals to one another (Figure 10). In each case, Olivia seems to have made a point of seeking out not just a single line to photograph, but multiple lines in parallel, perpendicular and/or skewed sets. In other words, she chose scenes that allowed her to fulfill the requirements of the assignment (find lines with different slopes) within a single image. She and Jane then further applied these same criteria to select the second image because it had the most pronounced slanting and "different lines to go to" (line 6).

As students' photos began appearing in the public display, Jane and Olivia proceeded to construct a line that matched one of those on the cabinet door:

7. *Jane*: [begins moving her point up and down along the y-axis, from the origin]
8. *Olivia*: What do you think the great…? [Jane pauses her point at (0, 3), right at the intersection of two lines in the photo image. Olivia begins moving left, approaching one of those two lines] Let's do the biggest one right [moves left again to (-4, -2), so that her point is now directly over the same line as Jane's] there.
9. *Jane*: This one?
10. *Olivia*: Like that?
11. *Jane*: Mark it?
12. *Olivia*: You can go higher if you want. [Olivia moves her own point up one unit] You see—you see where I am? Go on that line. You should go probably up two [Jane moves her point up two units to 0, 5) and then across, like…I don't know…one, two,…
13. *Jane*: [moves to the left] Like that?
14. *Olivia*: No, across the other way.
15. *Jane*: [laughing at the picture that just appeared in Group 3's window, moves right to (1,4)]
16. *Olivia*: One more. Go up.
17. *Jane*: [moves over and up to (2, 5), then back down] Ah!

18. *Olivia*: It’s okay. [Takes the iPod from Jane and moves her point to (3, 7), hovering just above and beyond the tip of the line on the cabinet door in a position that matches Olivia’s own relative to the other end of the same line]. There we go. [returns the iPod, then picks it up again to mark on it and then on her own to form y=(8/7)x+3.57 (Figure 11)]

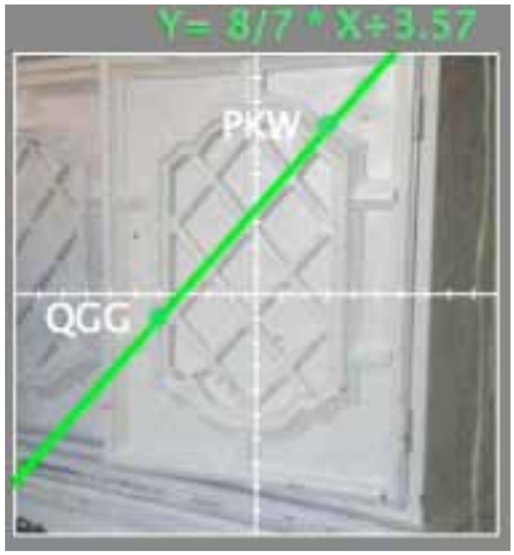


Figure 11: Olivia and Jane’s Graphing Window with Cropped Photo Background

Beginning in line 7, we can observe how these students set about making mathematical sense of the images they collected. In particular, they were quick to identify one of the longest line segments in their selected image (line 8), and to use it as a frame of reference for forming their own *Graphing in Groups* line. Importantly, their efforts in this segment do not reveal substantive reflection on or interpretation of the slope of the line they constructed or its relationship to the photographed image. Moreover, and in contrast to the more open-ended and more collaborative process through which Jamal and Monica sought to make the “biggest slope” they could in the previous episode, this segment depicts a relatively closed mathematical task, and a cooperative process in which one student—Olivia—took most of the initiative and did most of the work, directing Jane where to move and mark (lines 12, 14, 16), and even taking her iPod to complete the final steps herself (line 18).

Discussion and Conclusion

This paper offers a detailed look at the nature of classroom interaction and collaboration in the context of a local network of mobile devices, as well as a more cursory glance at the possibilities for extending that classroom network to incorporate and support mathematical investigation of material such as photos. The first episode illustrates several key elements of student participation in the *Graphing in Groups* environment: that accomplishing line construction tasks in this environment involves coordinated action between participants, that achieving that coordination can require careful and complex orchestration of utterance, gesture, and electronic action, and that shared constructions dynamically displayed on a public screen affords additional layers of both teacher orchestration and peer interaction. The second episode reveals some of the possibilities and the challenges associated with integrating student-generated images into this environment: they clearly yield personally relevant contexts to which learners bring their own aesthetic and mathematical lenses, and they clearly also require carefully constructed tasks that scaffold students' mathematical interpretations of those images and their collaborative interactions around them.

While the calculator version of this classroom networking design has been extensively studied and implemented in a variety of classrooms over several years, the iPod version and its photo and other extensions are still in a pilot phase. Our research group at UC Davis is actively developing these and other tools and accompanying classroom learning activities. The photo activity described here, for example, represents what we see as a bridging activity to begin connecting students' engagement with linear graphs with their experiences with linear phenomena in a variety of settings; the next and more substantive phase in this learning sequence involves students capturing video of phenomena that vary linearly over time, and then use tools on their mobile devices—and mathematics—to analyze that variation in greater depth.

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