RETHINKING MATHEMATICS INSTRUCTION: AN ANALYSIS OF RELATIONAL INTERACTIONS AND MATHEMATICS ACHIEVEMENT IN ELEMENTARY CLASSROOMS

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The research examined how teacher-student relational interactions affect students' mathematics achievement and learning. Namely, the paper highlights the different ways that mathematics teachers relate to their students and the subsequent impact on outcomes for mathematics success. Analyzing 7 second and third-grade classrooms in a large urban district within a low SES community, the study found that relational interactions explained a significant portion of the variance in mathematics achievement, even when controlling for prior achievement. The results speak to a needed reconceptualization of mathematics instruction as both an academic and social mechanism affecting equitable opportunities. A call for future research is made to investigate how student demographics (i.e. ethnicity, gender) warrant varying forms of relational interactions in the mathematics classroom.

Keywords: Equity and Diversity, Elementary School Education, Instructional activities and practices

Introduction

While the lack of quality of instruction in urban mathematics teaching has received much attention (see Ladson-Billings, 1997), other mechanisms that affect students learning in mathematics have not (Lubienski, 2002). One mechanism that the broader educational literature points to as a critical feature of classrooms is teacher-student relationships (Jerome, Hamre, & Pianta, 2009; Pianta, La Paro, Payne, Cox, & Bradley, 2002; Pianta, Nimetz, & Bennett, 1997). Scholars outside of mathematics education have found that teachers' interactions, particularly with students of color, can result in students' disengagement, misbehavior, or dropping out (Feagin, Vera, & Imani, 2001; Solórzano, Allen & Carroll, 2002). By looking at relational interactions as they play out in mathematics classrooms, research should be able to document the quality of these relational interactions between students and teachers in better understanding them as mechanisms that influence mathematics learning. These relationships go above and beyond content instruction in conveying messages about who is mathematically able, whose mathematical contributions are valid, and whose cultural and linguistic practices are legitimized in mathematics classrooms. This paper reports on a study of relational interactions across 7 second and third-grade classrooms in one urban district to understand their impact on mathematics achievement and learning for students of color in a low SES community.[i]

Literature Review

A number of scholars have documented mathematics instructional practices that differ by socioeconomic status (SES) or ethnicity in elementary grades. They have found that mathematics teachers of children of color and lower SES communities were more likely to disconnect taught procedures from students' thinking, teach fragmented or unexplained procedures, decontextualize mathematics vocabulary, assess students based on following steps rather than

mathematical reasoning or even correct/incorrect answers, and use less resources such as manipulatives even when available (Anyon, 1981; Ladson-Billings, 1997; Lubienski, 2002; Means & Knapp, 1991). As the forms of mathematics instruction available to students of color and those in poverty are often impoverished, they serve to limit the relationships that these students develop with the field of mathematics.

A separate literature documents the often-limited relationships that students of color develop with their teachers. Research on teacher-child relationships has found them to be a critical component of both psychosocial and academic development for students (Jerome, Hamre, & Pianta, 2009; Pianta, La Paro, Payne, Cox, & Bradley, 2002; Pianta, Nimetz, & Bennett, 1997). This work has compared teacher and student ratings of closeness and conflict for white, African American, and Latino students. Teacher ratings of their closeness and conflict with white students' ratings are very accurate. However, teacher ratings of relationships with African American and Latino students are decidedly inaccurate (Murray, Waas, & Murray, 2008). Jerome, Hamre & Pianta (2009) determined that teachers rate relationships with African American and Latino students as more conflictual than the students. These authors also note in this longitudinal work (kindergarten through 6th grade), that teachers rate their relationships with black students as becoming more conflictual over time in comparison to white students. So while many teachers do not have accurate notions of their relationships with African American and Latino students, they perceive these relationships as more conflictual, and this tends to worsen as students of color continue their schooling. This work however, was developed through questionnaires rather than direct observation and is not specific to mathematics content.

Within mathematics classrooms, the field is only beginning to understand the development of teacher-student relationships and their impact on the quality of instruction and the mathematics learned. In prior work, we found that the quality of instruction, as measured by an observational protocol, did not directly relate to the quality of relational interactions as determined through observation of classroom behavior (Battey, 2013; Battey & Neal, under review). The observation protocol measured commonly-held values about what quality instruction looks like in mathematics classrooms such as cognitive depth, classroom discourse, the nature of explanation and justification, and the types of problem solving in which students were engaged (Stecher et al. 2005; 2007). What this speaks to is that the relational interactions that are being measured are outside what the field typically considers to be quality mathematics instruction.

However, what we do not yet understand in this developing work is the impact of relationships on student learning and achievement. Lubienski (2002) makes the case that the field understands little about classroom mechanisms other than instruction that not only provide a lower quality mathematics learning environment, but also negatively affect achievement outcomes for students of color and those in poverty. While good mathematics instruction is requisite, it might not manifest itself in learning if teachers do not develop meaningful relationships with students in mathematics classrooms.

Theoretical Framework

We define relational interactions as a communicative action or episode of moment-to-moment interaction between teachers and students, occurring through verbal and nonverbal behavior that conveys meaning and can mediate student learning (Battey, 2013). These interactions can be either positive or negative in nature and can range in intensity. In conceptualizing this in the realm of mathematics, previous work documented five dimensions of relational interactions: addressing behavior, framing mathematics ability, acknowledging student

contributions, attending to culture and language, and setting the emotional tone (Battey, 2013). We briefly review each dimension below.

Addressing behavior captures the ways in which teachers respond to various forms of student behavior. *Framing mathematics ability* is more specific to mathematics. Mathematics ability can be framed as innate or changeable and can pass on various messages about students' capabilities to succeed mathematically (Battey & Stark, 2009). *Acknowledging student contributions* entails an episode between teachers and students that responds directly to the mathematical thinking of students. While a teacher's decision to address the mathematics within a student contribution would typically fall under the category of content instruction, the form of acknowledging, valuing/devaluing, or praising/disparaging shows relational aspects outside this realm. *Attending to culture and language* denotes the inclusion or omission of cultural and linguistic practices in mathematics instruction. It can be an important communicator of whose ideas are valued, what forms of mathematical thinking are acceptable, and who can participate in formal mathematics. These interactions are typified by how student language is dealt with by teachers or how a teacher draws on cultural resources, designs mathematics instruction to integrate home practices, or embeds students' home language. *Setting the emotional tone* doe not have to be a specific response to a student contribution. This dimension speaks to teachers establishing expectations of what it means to do math. For instance, it might be a teacher sharing a personal story about struggling in mathematics or an interaction that sends a message that students just need to rotely learn the mathematics content.

These five dimensions of relational interactions in the mathematics classroom serve as the framework used to operationalize the in the moment behaviors of students and teachers. While the interactions have been established and related to the quality to instruction in prior work, we do not have a sense of their relationship to students achievement and learning in mathematics. The study explored two research questions: 1) How do relational interactions explain variance in student achievement in elementary mathematics classrooms? and 2) How do relational interactions differ based on demographic variables such as sex, English language proficiency, and ethnicity?

Methods
Participants
Four second-grade and three third-grade elementary mathematics classrooms (137 students) from one large, urban district in Southern California were the focus of this research study. The district served students who are predominantly (99%) African-American and Latino (82% are Latino), receive free or reduced lunch (93%), and many were enrolled in an English Language Learner program (52%). The district, in its 2nd year of new leadership, had a history of poor performance and a long-standing sense from those outside the district that it would never do well. According to the state's ranking system and standardized test scores, it was one of the lowest performing school districts in California (lowest decile ranking, 1 out of 10, on California's Academic Performance Index during data collection) bringing the school district to be ranked as one of the lowest performing in California. At the start of the research study, only 57% of the teachers in the school district held credentials and 30% of the teachers were in their first or second year of teaching.
Teacher Professional Development
Prior to the research study, administrators and teachers acknowledged the importance of elementary mathematics students' engagement with algebraic reasoning. This led to the seven

observed teachers' participation in a large-scale study and on-site professional development series regarding students' algebraic reasoning skills and its affordances in meaningfully understanding arithmetic. The professional development intervention was based on *Thinking Mathematically: Integrating Arithmetic and Algebra in the Elementary School* (Carpenter, Franke & Levi, 2003) with a specific focus on *relational thinking* including concepts such as (i) understanding the equal sign as an indicator of a relation, (ii) using number relations to simplify calculations, and (iii) generating, representing, and justifying conjectures about fundamental properties of number operations. This intervention included school-based workgroup meetings and on-site support measures for the establishment of a learning community among classroom teachers, mathematics coaches, and professional development facilitators (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). All seven teachers were observed for approximately 2 hours each month for 8 months after the end of the professional development intervention.

Data Sources

Each teacher's classroom was videotaped on two occasions within a one-week period. To capture video and audio for as many students as possible (12 in each classroom), we used two cameras (to record 8 students) and six audio recorders (to record an additional 4 students). Each video camera had two audiofeeds connected to flat microphones (four flat mics in all), so that 8 students could be recorded simultaneously. Each flat mic was positioned between a pair of students. For the students audiotaped only, in addition to the flat mic between 2 students, each student had an individual lapel mic (each attached to a different audio recorder). Triangulating the recordings from the flat mic and the individual mics helped identify the speaker for students who were audiotaped only.

In addition to the video and audiotaping, the data includes the state standardized tests, namely the California Standards Test (CST) and the California Achievement Test (CAT6). The CST is initially taken in second grade (allowing for a control of prior achievement in third grade only) while the CAT6 is taken starting in third grade. Briefly, the CST is based on the state standards while the CAT6 is a norm-referenced examination. Scaled scores were used for this study to compare students uniformly across grades based on state means. Additionally, we had current grade level scores reflecting student performance on each of the district's benchmark quarterly assessments. This allowed for a measure of mathematics learning rather than achievement as in the case of the state standardized tests. Finally, the dataset included demographic data on sex (for both grades), as well as English language proficiency and ethnicity for third graders alone.

Analysis

Relational interactions were coded in four layers on the video specific to each student. First, we identified relational interactions and coded their dimension as discussed in the theoretical framework. Second, we coded forms of emphasis to determine intensity. Forms of emphasis referred to both verbal and nonverbal communication that accentuated an interaction such as stressing one's voice or gesturing. Next, we identified interactions as positive/negative based on the tone of the interactions and the message it sent about student thinking or behavior. Fourth, we coded the intensity of the interaction (low-1, medium-2, high-3). If an interaction did not contain any form of emphasis, it was coded low. If an interaction contained one or more emphases, we classified it as medium or high depending on the extent of the emphasis. Inter-rater reliability was 92% in identifying relational interactions, but the researchers came to agreement on 99% of coded episodes. Interactions without agreement were eliminated.

After coding the relational interactions, we calculated a relational interaction rate for each dimension for each student. This consisted of summing the interactions (as positive and negative), multiplying by their intensity, and dividing by the minutes of instruction across the two-videotaped lessons. The rates were then multiplied by 100 to make the decimals more manageable. After examining descriptive statistics for each relational interaction dimension, we entered them into linear regression models to see the amount of variance they explained with respect to mathematics achievement and learning to respond to the first research question. Finally, we ran T-Tests and one-way ANOVAs to examine the quality of interactions with respect to demographic variables in light of the second research question.

Results

The following results first examine descriptive statistics for relational interactions and their subsequent impact on all students' mathematics achievement on the CST. The results are then parsed to explore the impact of relational interactions for third grade and second grade students separately. Finally, the results end by examining the relationships of demographic variables to the quality of the relational interactions.

Relational Interactions: Descriptive Statistics

Dimension Rate	Mean	Standard Deviation	Minimum	Maximum
Addressing Behavior	-6.32	13.11	-46.60	8.55
Framing Mathematics Ability	1.66	1.52	0.00	7.34
Acknowledging Student Contributions	9.38	7.32	-1.20	31.20
Attending to Language and Culture	0.04	0.25	0.00	2.40
Setting the Emotional Tone	4.96	4.16	0.00	13.10

The most common, negative, and intense dimension of relational interactions was addressing behavior across classrooms. The other dimensions skewed positive with acknowledging student contribution as the second most common type of interaction. Attending to language and culture was very uncommon in these classrooms.

Students' Mathematics Achievement

We entered all of the relational interactions rates and sex (the only available demographic variable for both grades) as independent variables with the CST score as the dependent variable into a linear regression. Across all of the students, the only significant relationship was Acknowledging Student Contributions (F = 21.57, $p < .01$). It explained 13.4% of the variance in students' mathematics test scores.

Third Grade Mathematics Achievement and Learning

Linear regression was used to analyze third grade CST scores employing all of the relational interaction rates, sex, English language proficiency, and ethnicity. Additionally, the prior school year's CST scores were included as independent variables. Prior CST scores accounted for 61.4% of the variance of students' subsequent CST score and were significant. The only other variable that accounted for a statistically significant part of the variance was setting the emotional tone, accounting for 12.6% of the variance in CST scores. The effect size for the model was high ($F = 63.63$, $p < .01$).

Additionally, we ran a linear regression on CAT6 scores for third grade. We first ran the regression with the relational interaction variables, sex, language, and ethnicity. Addressing behavior significantly accounted for 13.8% of the variance on CAT6 scores ($F = 8.96$, $p < .01$). No other variables were significant. We then added CST scores from the previous school year to the regression model as a measure of prior mathematics achievement. When that was included, it significantly explained 56.5% of the variance in third grade CAT6 scores. Setting the emotional tone also significantly accounted for 11.2% of the variance in CAT6 scores with no other variables explaining a significant part of the variance ($F = 47.06$, $p < .01$).

Finally, we ran a regression on students' fourth-quarter scores on the district mathematics assessment (dependent variable) as a measure of mathematics learning rather than achievement. Again, all of the relational interaction rates, sex, language, and ethnicity were entered along with the prior three quarterly assessment scores as measures of prior mathematics achievement. The third quarter assessment accounted for 68.1% of the variance ($F = 104.63$, $p < .01$) while a second model included setting the emotional tone, which explained an additional 3.0% of the variance ($F = 58.75$, $p < .05$). Both were statistically significant though no other variables were.

Second Grade Mathematics Achievement and Learning

Since second grade did not take the CST in first grade, we used the first-quarter district assessment scores as measures of prior mathematics achievement in addition to relational interactions and sex. The first quarterly assessment accounted for 58.3% of the variance in CST scores ($F = 102.97$, $p < .01$). A second model was significant that included acknowledging student contributions, which explained an additional 7.7% of the variance in CST scores ($F = 66.85$, $p < .01$). However, no other variable was significant in the model.

Lastly, we ran a regression on the fourth-quarter district assessment, similar to that executed for the third grade, as an assessment of mathematics learning. The first quarter assessment accounted for 57.7% of the variance while the third quarter test accounted for an additional 7.2% of the variance ($F = 63.07$, $p < .01$). These were the only significant findings of the analysis.

Relationships between Demographic Variables and Relational Interactions

We ran *t-tests* for sex and ethnicity since there were only two ethnic groups in the data – namely, African American and Latino. We also ran one-way ANOVAs for language and sex by ethnicity. However, whenever ethnicity and language were part of the statistical analysis, only third graders were included due to the available demographic data. The only analysis that found a significant relationship was the T-Test on sex. Teachers engaged in acknowledging student contributions at a higher rate for girls than boys across both grades ($p < .0$). While this was the only significant finding, we should note that the gender by ethnicity analysis only included 7 African American girls and 9 African American boys. While not statistically significant, there were notable differences in addressing behavior (advantaging Latinas (mean = -8.49) and disadvantaging African American girls (-18.54) and acknowledging student contributions (advantaging African American girls (14.83) and disadvantaging African American boys (6.03). This result raises the need for future research with more students to see if this pattern is sustained.

Discussion

Acknowledging student contributions and setting the emotional tone both explained statistically significant portions of the variance for both state standardized tests and district assessments, even when controlling for prior mathematics achievement. These variables accounted for between 6 and 15% of the variance that was unaccounted for by prior achievement. This suggests that relational interactions are measuring a classroom mechanism not explained through prior achievement and learning in mathematics.

Addressing behavior accounted for a significant portion of the variance on CAT6 scores for third grade students. When prior achievement was factored into the analysis however, this relationship was no longer significant. This suggests that addressing behavior is related to prior mathematics achievement scores in this dataset. One way to interpret this is that lower achievement or prior negative framing of ability is leading students to resist or engage in non-compliant behaviors to current mathematics schooling. A longitudinal analysis would be needed to look at the relationship of behavior, framing ability, and achievement across multiple grades to see the social construction of noncompliant behavior.

It is not surprising that attending to culture and language and framing mathematics ability did not produce any significant results. This was probably due in part to two possible sources: (a) the overall infrequency of these relational interactions and (b) the teachers' tendencies to engage in these dimensions of relational interaction with the whole class rather than with individual students, which results in a lack of variance for the students in the study. Additionally, since framing mathematics ability correlated highly with setting the emotional tone, it was unlikely that it would explain more of the variance in achievement.

Additionally, teachers acknowledged the mathematical contributions of female students in the classrooms more frequently and intensely than boys. This raises concerns about whether the mathematical contributions of Latino and African American boys are missed in classrooms.

Conclusion

We think these results point to the need for future research in understanding relational interactions. In our prior work, we found that more positive relational interactions do not directly relate to higher quality mathematics instruction (Battey, 2013; Battey & Neal, under review). In this analysis, both setting the tone and acknowledging student contributions significantly explained variance in mathematics achievement and learning. Further research is needed to see if this relationship exists outside of the classroom contexts and students in this study.

Additionally, it is important to understand if students of color are treated differently than their white counterparts. This would mean that relational interactions are a mechanism through which students are treated differentially in mathematics classrooms and could be a factor in producing racial differences in achievement. The research literature has found that teachers perceive their relationships with students of color as more conflictual than their relationships with white students. They also rate their relationships as more conflictual than their African American and Latino students. Taken together, this prior research could explain the negative bent to addressing behavior in the current study. Future work could examine the relationship between perceived and observed behaviors of teachers and students.

The relational interaction framework used here seems to be an important facet of instruction in understanding student achievement and learning. We think the findings of this study provide

evidence that relational interactions are an important construct to measure in future research in mathematics classrooms.

References

Anyon, J. (1981). Social class and the hidden curriculum of work. *Journal of Education, 163*(1), 67-92.

Battey, D. & Neal, R. A. (under review). Understanding the Association between Mathematics Instruction and Relational Interactions in Urban Classrooms. Submitted to *Mathematical Thinking and Learning*.

Battey, D. (2013). "Good" mathematics teaching for students of color and those in poverty: The importance of relational interactions in instruction. *Educational Studies in Mathematics*, 82(1), 125-144.

Battey, D. & Stark, M. (2009). Inequitable classroom practices: Avoiding ability attributions from misconceptions in mathematics. In C. Malloy (Ed.), *Mathematics for All: Instructional Strategies for Diverse Classrooms* (pp. 167-177). Reston, VA: National Council of Teachers of Mathematics.

Jacobs, V., Franke, M., Carpenter, T., Levi, L., & Battey, D. (2007). A large-scale study of professional development focused on children's algebraic reasoning in elementary school. *Journal for Research in Mathematics Education, 38*(3), 258-288.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating algebra and arithmetic in elementary school*. Portsmouth, NH: Heinemann.

Feagin, J. R., Vera, H., & Imani, N. (2001). *The agony of education: Black students at White colleges and universities* (2nd ed.). New York: Routledge.

Jerome, E. M., Hamre, B. K., & Pianta, R. C. (2009). Teacher–child relationships from kindergarten to sixth grade: Early childhood predictors of teacher-perceived conflict and closeness. *Social Development, 18*, 915–945.

Ladson-Billings, G. (1997). It doesn't add up: African American students' mathematics achievement. *Journal for Research in Mathematics Education, 25*(6), 697-708.

Lubienski, S. T. (2002). A closer look at Black-White mathematics gaps: Intersections of race and SES in NAEP achievement and instructional practices data. *The Journal of Negro Education, 71*(4), 269 - 287.

Means, B., & Knapp, M. S. (1991). Cognitive approaches to teaching advanced skills to educationally disadvantaged students. *Phi Delta Kappan, 73*(4), 282-289.

Murray, C., Waas, G. A. and Murray, K. M. (2008), Child race and gender as moderators of the association between teacher–child relationships and school adjustment. *Psychology in the Schools, 45*, 562–578.

Pianta, R. C., La Paro, K. M., Payne, C., Cox, M. J., & Bradley, R. (2002). The relation of kindergarten classroom environment to teacher, family, and school characteristics and child outcomes. *The Elementary School Journal, 102*, 225–240.

Pianta, R. C., Nimetz, S. L., & Bennett, E. (1997). Mother–child relationships, teacher–child relationships, and school outcomes in preschool and kindergarten. *Early Childhood Research Quarterly, 12*, 263–280.

Solórzano, D. G., Allen, W. R., & Carroll, G. (2002). Keeping race in place: Racial microaggressions and campus racial climate at the university of California, Berkeley. *Chicano-Latino Law Review, 15*(23), 15-111.

Stecher, B. M., Borko, H., Kuffner, K. L., Martinez, F., Arnold, S. C., Barnes, D., Creighton, L., & Gilbert, M. L. (2007). *Using artifacts to describe instruction: Lessons learned from studying reform-oriented instruction in middle school mathematics and science* (CSE Technical Report 705). LA, CA: National Center for Research on Evaluation, Standards, and Student Testing (CRESST), Center for the Study of Evaluation (CSE), & The Regents of the University of California.

Stecher, B. M., Wood, A. C., Gilbert, M. L., Borko, H., Kuffner, K. L., Arnold, S. C., & Dorman, E. H. (2005). *Using classroom artifacts to measure instructional practices in middle school mathematics: A two-state field test* (CSE Report 662). LA, CA: National Center for Research on Evaluation, Standards, and Student Testing (CRESST), Center for the Study of Evaluation (CSE), & The Regents of the University of California.