TEACHER CANDIDATES' PERCEPTIONS OF MATHEMATICAL, COGNITIVE AND PEDAGOGICAL FIDELITY OF THE "FILL&POUR" VIRTUAL MANIPULATIVE

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Virtual manipulatives as cognitive tools, dynamic/interactive, Web-based representations and/or technology-based renditions, allow users to engage in mathematical meaning making. This research investigated teacher candidates' perceptions of the mathematical, cognitive, and pedagogical fidelity of Fill and Pour virtual manipulative. Findings suggest that the degree to which the mathematical entity is faithful to the essential mathematical properties of that item in the virtual environment has been granted by teacher candidates to virtual manipulative designers automatically, without closer examining of the mathematical, cognitive, and/or pedagogical fidelity. Further qualitative probing was carried through to better understand the nature of such assumptions.

Keywords: Mathematical Knowledge for Teaching, Teacher Education-Preservice, Technology, Problem Solving,

The ongoing challenge of reshaping mathematics education with integration of technology tools (NCTM, 2000; Ball, 2003; Alagic, 2003, 2004) has led to consideration of pedagogical, mathematical and cognitive fidelity of virtual tools as well as concern about teacher preparation to utilize quality math based technology (e.g., Dick, 2008; Bos, 2009). This study examined teacher candidates' perceptions of the mathematical, cognitive, and pedagogical fidelity of a certain problem solving virtual manipulative tool. The study is an illustration of challenges that we face in teacher preparation programs related to use of technology in mathematics classrooms.

Virtual Manipulatives: Mathematical, Cognitive and Pedagogical Fidelity

This section provides a brief review of the research literature related to representational and cognitive characteristics of virtual manipulatives and their mathematical, pedagogical and cognitive fidelity as they relate to quality teaching and learning mathematics.

Virtual Manipulatives as Cognitive and Representational Tools

Virtual manipulatives (VM) are typically designed as Java or Flash applets. They are often modeled after existing manipulatives such as geoboards, tangrams, base ten blocks, fraction bars,…They allow learners to relate concrete models to abstract mathematical concepts. Dorward (2002) defined virtual manipulatives as "computer based renditions of common mathematics manipulatives and tools" (p. 329) while Moyer, Bolyard, & Spikell (2002) defined them as "... an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). Therefore virtual manipulatives must be web-based and they must afford users to interact/manipulate these dynamic objects which shows potential for learning and teaching mathematics interactively (Moyer-Packenham, Salkind, & Bolyard, 2008). Other names for virtual manipulatives include interactive math applets and Mathlets (JOMA Web Site 2006).

As cognitive tools, virtual manipulatives support, guide, and extend the thinking processes of their users. They are based on the principle that learners need to make their own meaning of new

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concepts. Jonassen (1992) defined cognitive tools as "generalizable tools that can facilitate cognitive processing" (p.2) and "make effective use of the mental efforts of the learner" (Jonassen, 1996, p.10). Virtual manipulatives have properties that go beyond their counterparts, physical manipulatives. They provide learners a way - often both visual and verbal including hints, feedback with pop-ups and help features - of representing their understanding of a new concept/phenomena and how it relates to their existing understanding of the same idea (Derry, 1990; Alagic & Palenz, 2004; Moyer, Niezgoda, & Stanley, 2005; Zbiek, Heid, Blume, & Dick, 2007).

When learning complex new mathematical ideas it helps to interact with multiple representations (Cox and Brna, 1995). Virtual manipulatives as *external representations* may help learners' ability to transfer among multiple representations, to extend what has been learned in one context to new contexts, developing representational fluency (NRC, 2000; Zbiek, Heid, Blume, & Dick, 2007). Virtual manipulatives help develop *representational fluency* by linking symbolic, pictorial and concrete representations. Not making connection between different representations may even inhibit learning (Ainsworth, Bibby & Wood, 2002). *Representation standard* (NCTM, 2000) articulates representations as crucial components in facilitating learners' conceptual understanding of concepts and relationships. The term representation applies to both processes and products. Furthermore, the same term is used for product/process that are observable externally (external representations) as well as to those that ensue internally (mental models). In this context, virtual manipulatives contribute to richness of representations in a unique way, as they often comprise multiplicity of representations – visual, dynamically visual, symbolic and verbal (Goldin & Shteingold, 2001; Alagic, 2003).

The literature review of the existing research indicated that learners using virtual manipulatives demonstrated improvements in mathematics understanding and achievement (e.g., Bolyard, 2006; Moyer et al., 2005; Lee, Silverman, & Montoya, 2002; Lee & Jung, 2004). A study described by Reimer & Moyer (2005) about third graders using virtual manipulatives during a 2-week long unit on fractions revealed a statistically significant improvement in students' conceptual knowledge. Some research suggested that use of virtual manipulatives might have a positive effect on student engagement and developing procedural and conceptual understandings (e.g., Moyer, Niezgoda, & Stanley, 2005; Raphael & Wahlstrom, 1989). However, mathematical meaning is not necessarily explicit in use of manipulatives and VMs cannot be expected to improve learners' understanding; to be effective, virtual manipulatives use must involve active cognitive processing by learners (Ball, 1992; McNeil, 2007; Roberts, 2007; Smith, 2009). Many believe that virtual manipulatives can be particularly helpful to students with language difficulties, including English language learners (Moyer, Niezgoda, & Stanley, 2005).

As illustrated, there are a number of studies about teachers' and pupil's use of virtual manipulatives demonstrating the unique characteristics of these tools for developing conceptual understandings and teaching mathematics. However, there is no much research about teacher candidates' understanding of both potential and pitfalls in using virtual manipulatives.

Mathematical, Cognitive and Pedagogical Fidelity of Virtual Manipulatives

The mathematical fidelity of a virtual manipulative refers to faithfulness to the defining properties of a mathematical concept or a phenomenon that manipulative is attempting to represent. Simply, we ask, is a representation provided via virtual manipulative true to the mathematical concept that it is trying to represent? "In order to function effectively as a representation of a mathematical "object," the characteristic of a technology-generated external representation must be faithful to the underlying mathematical properties of that object" (Zbiek, Heid, Blume, & Dick, 2007, p. 1174). For an example, the answer to a division problem using the calculator is truncated, but the calculator is giving you the most feasible answer for the place values it allows. "Technology's limitations are a constant concern for mathematical fidelity …" (Bos, 2011, p. 4404).

The cognitive fidelity of virtual manipulatives is related to user's cognitive engagement while developing patterns and making connections that were only possible in one's mind. A familiar examples are those applets that allow change of a parameter over time (using a slider) resulting in changes of a graph, allowing a mental process recognizing a resulting series of graphs. In other words, cognitive fidelity leads to question, Is the concept better understood due to the user's capability of acting on the related virtual object? (Bos, 2008). Cognitive Information Processing Theory as well as Dual Coding Theory (DCT) is based on the premise that two interconnected systems and their sets of codes (visual and verbal) are the base of information processes and storage. These collections of codes include both visual and verbal codes, which can represent letters, numbers, or words. According to these theories, facilitating learning with functionally independent both visual and verbal codes has cumulative effects on their recall (Clark & Paivio, 199; Moyer-Packenham, Salkind, & Bolyard, 2008).

Table 1: Pedagogical, Mathematical, and Cognitive Fidelity Chart to Determine Degree of Fidelity (Adapted for VM from Bos, 2009, p. 526.)

	Low Fidelity	Medium Fidelity	High Fidelity
Pedagogical	VM interactivity is not obvious; not intuitive, confusing to use. Not appropriate for the concept being represented. VM hard to access.	Manipulation/interactivity is not intuitive, but after reading the directions it is doable. May be easier to do without the technology.	VM is appropriate for activity. Mathematical manipulation is doable, encourages active involvement, and requires little or no training.
Mathematical	Math concepts behind VM either too simplistic or too complicated. Patterns are not revealed. Not real-life related. Leads to rote memorizing rather than conceptual understanding.	VM patterns lack predictability. Mathematical significance is minimal. Application of mathematics unclear.	VM is mathematically correct. Maximizes the use of patterns. Believable and livable use of mathematics.
Cognitive	VM static with no opportunities to formulate and test conjectures. Patterns do not make sense. Difficult to relate to prior knowledge; confusing and unyielding.	VM provides limited opportunity to explore and test patterns. Patterns require either minimal or too much manipulation to make sense of the mathematical concept behind it.	VM can be used to construct and deconstruct, test, and revise to understand the patterns and structure of concepts. Interacting leads to the patterns and greater depth of understanding.

The pedagogical fidelity of virtual manipulatives refers to the degree to which a learner believes that a virtual manipulative affords her to act mathematically in ways that correspond to

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the nature of mathematical learning via discovery. Zbiek, Heid, Blume, & Dick (2007) define *pedagogical fidelity* as "the extent to which teachers (as well as students) believe that a tool allows students to act mathematically in ways that correspond to the nature of mathematical learning that underlies a teachers practice…" (p. 1187). It is about allowing learners to learn mathematics by doing – facilitating the creation of objects, acting on objects, explicating evidence - without being distracted by low quality of the applet or other technology limitations (Bos, 2009; Dick, 2008).

Degree of mathematical, cognitive and pedagogical fidelity may vary for virtual manipulatives used in mathematics. The Table 1 is a VM adaptation of a chart for determining degree of fidelity that Bos (2009) used in the study of mathematical, cognitive and pedagogical fidelity for mathematics related websites. Furthermore, Bos (2008) described a study that suggested technology high in mathematical and cognitive fidelity lead to greater student mathematical achievement. Selecting a virtual manipulative for instructional purposes requires careful consideration of the mathematical, cognitive, and pedagogical fidelity of the virtual manipulative as well as its externalized representation will affect mathematics learning and teaching (e.g., Zbiek, Heid, Blume, & Dick, 2007).

Threshold Concept and a Critical Incident: *Fill and Pour* Virtual Manipulative

The course *Mathematics Investigations* (Alagic, 2006) is designed to investigate demands of digital technologies integration and inquiry-based approaches to teaching and learning of mathematics, while bridging the gap between two traditional courses: Mathematics for Elementary Teachers and Instructional Strategies in Elementary Mathematics. The main thrust of the course are three Problem Sets assignments, each focusing on one big mathematical idea/concept developed around an open-ended, real-life related and challenging problem. The problem set consists of 6-7 additional problems scaffolding "down" the main concept. Each student is designing a unique collection of problems and submitting their work individually. However, students are encouraged to discuss collaboratively their work. Each problem set utilizes technology tools in an essential way. At the end of the problem set, a required metacognitive reflection reports about students' thinking during the process of the problem set design.

A *threshold concept* refers to realizing a new and previously unreachable way of thinking about a certain concept or phenomena. It represents a transformed way of meaning making or interpreting something relevant to learner's progress in understanding (Land, Meyer, &Smith, 2008). In the context of virtual manipulatives it refers to the fact that "Students do not necessarily see on the screen what is "evident" [to the software designer and maybe the teacher] (Dreyfus, 2002, p. 23). The following is an example of what I consider threshold concept for the students in Mathematics Investigations class because it lead to deeper understanding of the need to consider fidelity of virtual manipulatives in general.

Critical incident. During class activities teacher candidates were asked to investigate virtual manipulative *Fill and Pour* (Figure 1), with problems of the following type:

> You have a soda fountain but only two unmarked containers (one 5 ounces and one 9 ounces) that can be filled or emptied or poured back and forth as needed. Your goal is to get precisely the target amount (7 ounces) in one of the containers.
> (http://matti.usu.edu/nlvm/nav/frames_asid_273_g_3_t_4.html).

In a matter of minutes, every teacher candidate in the class was "filling" and "pouring". Some candidates quickly got answers, others struggled. As some candidates were losing their patience, a candidate offered a hint, "Just fill and pour in the same direction, eventually you will get an answer". Very quickly, the idea spread around, everyone was showing off their cherries and ducklings (Alagic, 2006).

Teacher candidates were asked to record the process in some way. Most of them described in long paragraphs the process they followed. Some used some kind of algorithmic representation to capture the process. One of the students neatly captured the following:

Let's mark containers with L (left) and R (right). To solve this problem we can
Fill L => (transfer to) R
Fill L => R
Empty R
Transfer L (1oz) =>R
Fill L => R
Fill L => R
Empty R
Transfer L (2oz) =>R
Fill L => R
Fill L => R
L has 3oz

The same candidate provided a description of the opposite process in a similar manner.

So, correct solution is available through a sequence of automatic "fill" and "pour" steps without a deeper consideration how the problem should be solved and what kind of mathematical reasoning needs to be involved. This example was an inspiration for the study of mathematical, cognitive and pedagogical fidelity of *Fill and Pour* virtual manipulative.

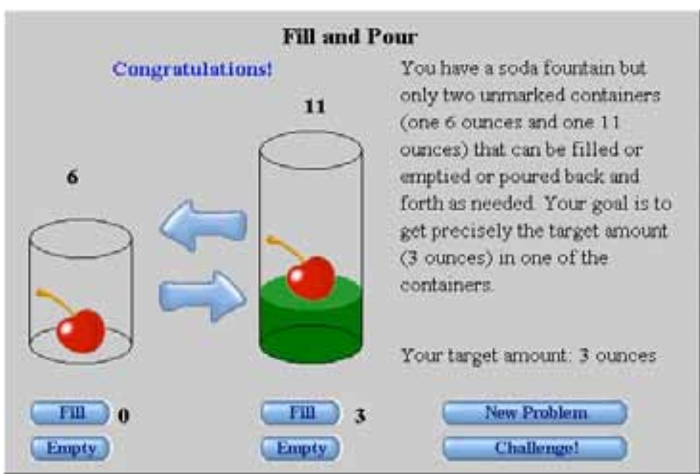

Figure 1: Virtual Manipulative Fill and Pour applet from NLVM (2006)

Mathematical, Cognitive and Pedagogical Fidelity of the *Fill and Pour* Virtual Manipulative: Teacher candidates' Perceptions

Mode of Inquiry and Analysis

Across 6 semesters, total of 224 teacher candidates voluntarily participated in this case study. Participants were elementary teacher candidates enrolled in the Mathematics Investigations class. Background information on the participants included their prior use of manipulatives and technology. All participants had prior experience with physical manipulatives and some experience with basic virtual manipulatives, such as virtual base-10 blocks, geoboards, pattern blocks, and tangrams as these were utilized in the required class preceding Mathematics Investigations. The critical incident described above was an inspiration for investigating students discovery and understanding of the fidelity of VMs in the following 6 semesters.

Participants were asked to solve problems posed in *Fill and Pour* in two ways, using (a) paper-pencil method, and (b) virtual manipulatives. Half of the participants would first complete paper-pencil method and follow by using virtual manipulatives. Other half would work first with virtual manipulatives. This assignment was followed by metacognitive reflections required in order to better understand how students make meaning of fidelity based on this experience. Those metacognitive reflections represented data analysed for this study. Furthermore, students were given the chart (Table 1) without fidelity terminology and asked to identify one cell in each row that corresponds to their understanding of how virtual manipulative *Fill and Pour* can be characterized.

Results. 75% of participants discovered that there are three types of problems in the *Fill and Pour* virtual manipulative: (i) following the pattern of fill and pour works in both directions (from smaller to larger or larger to smaller container – the required amount can fit in either container); (ii) following the pattern of fill and pour works only in one direction (required amount cannot fit in the smaller container); and (iii) impossible problems (required amount odd number and containers hold even number of ounces). Qualitative probing of those participants that did not reach this conclusion showed that either they did not know they needed to figure that out or that all the problems they tried were of the similar nature.

Out of these 168 (75%) participants, 120 were in the groups interacting with VMs first, before attempting paper and pencil approach. The following table illustrates percentages of students identifying mathematical, cognitive and pedagogical fidelity based on only descriptors provided in the Table 1 (no fidelity terminology used, yet).

Table 2: Fidelity Chart with Number of Participants Selecting Pedagogical, Mathematical, and Cognitive Fidelity Descriptors Based on Their Interaction with *Fill and Pour* Virtual Manipulative

Related to	Low Fidelity	Medium Fidelity	High Fidelity
Teaching using *Fill and Pour*	14	10	200
Understanding Mathematics behind *Fill and Pour*	5	39	180
Recognizing problem solving patterns in *Fill and Pour*	40	27	157

Limitations. More attention is necessary to understand challenges that student have about paper-pencil approach and bridging the two representations– paper-pencil approach vs. virtual manipulative *Fill and Pour* approach to problem solving of this type of problems.

Discussion and Conclusions

This study provides an initial examination of participants' perceptions of mathematical, cognitive and pedagogical fidelity based on provided descriptors. The findings illustrate that for significant majority of participants, perceptions that using virtual manipulatives to solve problems is advantageous regardless of mathematical, cognitive and pedagogical fidelity. For 80% of participants using virtual manipulative to solve problems of the type *Fill and Pour* is a better choice (high fidelity) regardless of the fact that process is a akin to rote memorizing rather than to conceptual understanding. 90 % of participants would teach such problems using virtual manipulatives rather than paper-pencil method. Some participants commented in their reflections that "paper-pencil" is too complicated when they know to follow the pattern "fill and pour in the same direction, eventually you will get an answer". In terms of cognitive fidelity, reflections and qualitative probing seem to identify patterns as types of problems based on (i) the required amount can fit in either container; (ii) required amount can fit only in the larger container; and (iii) impossible problems. However, it is important to notice that this reasoning is coming from somewhat automatic play with virtual manipulative and not from patterns recognized in paper-pencil problem solving.

Follow up classroom discussions helped clarify some of the misconceptions inherent in mathematics thinking and learning related to problem solving utilized via *Fill and Pour* virtual manipulative. This provides further confirmation that simply using manipulatives without follow-up conversations may lead to deepening some misconceptions; not making connection between different representations may even inhibit learning (Ainsworth, Bibby & Wood, 2002).

Implications and Further Research. This study, although focused on only one VM, illustrated potential challenges in using dynamic/interactive web-based tools in terms of mathematical, cognitive and pedagogical fidelity for three types of learners– students, teachers and instructional designers.

There are many follow up questions to be studied, some of which the author is already pursuing. This study can be considered as a pretest for understanding students conceptualization of of mathematical, cognitive and pedagogical fidelity for a specific virtual manipulative. How this threshold concept might be used to sharpen teacher candidates' inquiry into fidelity of virtual manipulatives in general? What is the effect of teacher candidates' knowledge of problem solving on their ability to conceptualize mathematical, cognitive and pedagogical fidelity for virtual manipulatives?

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