CHARACTERIZING PIVOTAL TEACHING MOMENTS IN EXPERIENCED MATHEMATICS TEACHERS' PRACTICE

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This report presents the findings of a study that was designed to characterize the pivotal teaching moments (PTMs), as defined by Stockero & Van Zoest, (2012), faced by experienced mathematics teachers. To better understand how experienced mathematics teachers identify the PTMs and appropriately respond to them, the mathematics teaching videos from Annenberg Learner's multimedia resources were analyzed. Implications for teacher education are discussed.

Keywords: Algebra and Algebraic Thinking, Classroom Discourse, Mathematical Knowledge for Teaching

Introduction

Students' mathematical achievement is influenced by many factors. One such factor is the instructional practice of their teachers. Practices such as engaging students in rich mathematical tasks, encouraging students to explain their thinking processes and building students' mathematical understandings from prior knowledge have been reported to increase students' mathematical achievement. (Henningsen and Stein 1997; Stein and Lane 1996; Superfine 2008).

This study examines what Stockero and Van Zoest (2012) call Pivotal Teaching Moments- "interruptions in the flow of a lesson [which] provide an opportunity to modify instruction to improve students' mathematical understanding" (p.3)-observed in videos of algebra instruction. This report shares the findings of that study, offering implications for teacher education.

Literature Review

Many researchers have documented how teachers could implement these recognized instructional practices in their classroom to advance students' mathematical learning. Chi, Leeuw, Chiu and Lavancher (1994) claimed that encouraging students to generate more self-explanations promotes greater learning and understanding of new knowledge. Cengiz, Kline and Grant (2011) explored the teaching of six experienced elementary school mathematics teachers and identified their individual instructional actions to extend student thinking. They emphasized that the first step in extending student thinking is to recognize the potential of a particular situation which requires careful listening to student thinking and having clear goals about the mathematical ideas and concepts they are to pursue. Van Es (2011) refers to classrooms as complex settings in which all kinds of interactions take place at one time and teachers need to decide on what to pay attention to and how to respond to the events and interactions. Ball and Cohen (1999) also suggested that teachers should learn to carefully look at or think about a situation and then decide how to act from moment to moment. Van Es and Sherin (2002) propose that the skill of noticing for teaching consists of three main aspects: identifying what is important in a teaching situation; using what one knows about the context to reason about a situation; and making connections between specific events and broader principles of teaching and learning.

How can teachers be supported in learning to recognize the most important elements in the classroom practice? Stockero and Van Zoest (2012) analyzed videos of beginning secondary school mathematics teachers' instruction to identify and characterize what they called *pivotal teaching moments* (PTMs) in mathematics lessons. PTMs are hi gh-leverage moments that can significantly impact student learning. They defined a PTM to be an instance in a classroom lesson in which a student-generated interruption in the flow of the lesson provides the teacher an opportunity to modify instruction in order to extend or change the nature of students' mathematical understanding. In their exploratory study, they identified five circumstances that led to PTMs: (a) extending: when students make a comment or ask a question that is grounded in, but goes beyond, the mathematics that the teacher had planned to discuss; (b) incorrect mathematics: when incorrect mathematical thinking or an incorrect solution is made public; (c) sense-making: when students are trying to make sense of the mathematics in the lesson; (d) mathematical contradiction: when two different answers to a problem that clearly should have only one answer are provided or two competing interpretations of a mathematical situation; (e) mathematical confusion: a student's expression of mathematical confusion.

This initial PTM framework is of potential usefulness to the teacher educators as well as teachers to improve mathematical noticing ability and decision-making. However, Stockero and Van Zoest (2012) also suggest that in order to fully understand PTMs and teacher decisions in response to them, experienced mathematics teachers' practice need to be included into the data set. Therefore, this paper is supposed to fill this gap. The following two research questions were explored:

1. What are characteristics of PTMs faced by experienced secondary school mathematics teachers during classroom instruction?
2. What types of decisions do they make when a PTM occurs during their instruction?

Perspectives

In considering the above questions, the researchers turned to previous work on students' mathematical thinking. The National Council of Teachers of Mathematics (2000) calls for mathematical instruction that builds on children's mathematical thinking and many research studies show that such practices result in a richer instructional environment and greater student achievement (Carpenter, Gennema, Peterson, Chiang, & Loef, 1989; Sowder, 2007; Wilson & Berne, 1999). However, to learn and practice such instruction is complex and difficult (Ball & Cohen, 1999). In many cases, teachers either missed the opportunities to use student thinking to further their mathematical understanding or did not properly act upon them (Peterson & Leatham, 2009; Stockero & Van Zoest, 2012).

An important first step in helping teachers capitalize on important mathematical moments is to recognize that such moments exit (Stockero & Van Zoest, 2012). Providing a framework for teachers to use helps them to pick up easily mathematically valuable moments that occur during instruction and appropriately act upon them. This paper aims to characterize the PTMs faced by experienced mathematics teachers and the types of decisions that they make in response to those critical moments.

Methodology

In this exploratory, descriptive study of PTMs, the researchers focused on experienced teachers who had more than five years teaching experience in secondary-school mathematics.

Data Collection

For this study, videos from Annenberg Learner's multimedia resources (http://www.learner.org/) were used. Annenberg Learner's goal is to use media and telecommunications to advance excellent teaching in American schools. The statement that "All Annenberg Learner videos exemplify excellent teaching" (http://www.learner.org/about/) implies the mathematics teachers demonstrating their teaching in those videos are expert teachers. Hence, their existing body of video data best served the purposes of this study.

The mathematics video resources featuring real classrooms on the Annenberg Learner website are kept in four series: Teaching Math: A Video Library, K-4; Teaching Math: A Video Library, 5-8; Teaching Math: A Video Library, 9-12 and Insights Into Algebra I: Teaching for Learning. Each series contains videos that vary in length (some as short as 15 minutes) and number. "Insights Into Algebra I: Teaching for Learning" offers the most in-depth look at classroom practice, containing 8 one-hour video programs. This set of videos features nine experienced middle and high school mathematics teachers teaching 16 topics found in most Algebra I programs. Each of the eight sessions contains two half-hour videos displaying effective mathematical teaching strategies (http://www.learner.org/resources/series196.html). Because the longer videos provided more opportunities to examine effective teaching practices, the videos from this series were selected for this study.

Data Analysis

This analysis was made up of two stages. The first stage was to reduce the video data to potential episodes for PTMs. The second stage was to identify the PTMs and characterize them.

Stage one: video reduction. The PTM definition provided by Stockero and Van Zoest (2012) was used to pick out PTM episodes. In the definition there are three important components. First, the definition emphasizes "an interruption in the flow of the lesson" (p.3), which means that it is an unanticipated event such as a student's question or comment. Second, the definition offers an opportunity for the teacher to "modify instruction" (p.3). The third component is that there is potential to "extend or change the nature of students' mathematical understanding" (p. 3). Accordingly, the researchers paid close attention to students' questions or comments that provided teachers a chance to extend or change their mathematical understanding. The two researchers then individually watched the videos and marked the time when a PTM occurred. Through discussion that aimed toward consensus, 29 episodes remained in the data base and were transcribed for further analysis.

Stage two: characterizing PTMs. After identifying all those PTMs episodes ,the researchers used Stockero and Van Zoest (2012)'s characteristics of PTMs in beginning mathematics teachers' practice as a guide to characterize the PTMs in the experienced mathematics teachers' classroom. During this phase of analysis, the researchers aimed at two things: to identify the PTM type, and to code the teacher decision action. If the identified PTM is beyond the framework created by Stockero and Van Zoest (2012), the researchers used open coding (Carspecken, 1996) to label the feature. Each code was discussed between the researchers until an agreement was reached.

Results

The coding process resulted in three PTM types: incorrect mathematics, sense-making and confusion. The following four teacher decision actions were identified: extend/ make connections, pursue student thinking, emphasize mathematical meaning and wait to allow student explore first (see table 1).

Pivotal Teaching Moment Characteristics

Incorrect mathematics. Out of the 29 PTMs in the data base, 7 were classified into incorrect mathematics. It could be student's incorrect mathematical thinking or an incorrect solution. For example, when a teacher asked a group of students “what is zero divided by zero”, that group of students immediately answered “zero” without any hesitation. The students’ incorrect mathematical understanding provided an opportunity for the teacher to clarify why zero can’t be a divisor.

Table 1: Summary of PTMs Identified in the Data (modified from Stockero and Van Zoest, 2012, p. 10)

Pivotal Teaching Moment	Teacher Decision
Incorrect mathematics (7)	Extends/connections
	Pursue student thinking (2)
	Emphasize mathematical meaning (2)
	Wait to allow student to explore (3)
Sense-making (12)	Extends/connections (2)
	Pursue student thinking (2)
	Emphasize mathematical meaning (7)
	Wait to allow student to explore (1)
Confusion (10)	Extends/connections (2)
	Pursue student thinking (3)
	Emphasize mathematical meaning (1)
	Wait to allow student to explore (4)

Sense-making. Aligned with the findings of Stockero and Van Zoest (2012)**,** PTMs occur most when students are trying to make sense of the mathematics. There were twelve sense-making PTMs in our data. For example, when a teacher asked students how to graph the function $y = -(x-3)^2 + 4$ from $y = x^2$, they talked about how -1, -3, and 4 affected the graphs. The teacher sketched the graph, but the graph passed through the origin, which was incorrect. One student commented “So it doesn’t matter how wide the parabola is?” This comment showed that the student was trying to make sense of graphing quadratic equations from parent functions. It provides a chance for the teacher to highlight the critical aspects of the mathematics at hand.

Mathematical confusion. Ten PTMs occurred when students expressed that they were confused about a mathematical idea. For example, when a teacher wrote down the general form of a parabola $y = a(x - h)^2 + k$, he asked students what they noticed when they graphed quadratic equations on their stations. One student said that when h is positive, the graph shifted to the left; when h is negative, the graph shifted to the right. Another student put up his hand and said, “I know it is right, but I don’t understand why it is right? Because, like, if it is positive, shouldn’t it go to the right? If it is negative, shouldn’t it go to the left? That’s what we’ve been taught.” This gives the teacher an opportunity to revisit vertex form, the graph and other important mathematical ideas related to it.

Teacher Decisions in Response to PTMs

There are four types of teacher decisions made by the experienced teachers: pursue student thinking, emphasize mathematical meaning, wait to allow student to explore first, and extend/make connections.

Pursue student thinking. One of the decisions made by the teachers in the data base is to pursue student thinking. The teacher tried to understand the meaning of what a student had said by asking the student to provide more information about their thinking (Stockero & Van Zoest, 2012). This occurred seven times in the data base. One example happened when the teacher asked how many tiles she would need if she wanted to tile the border of a 9 x 4 pool. One student answered 26 tiles. By asking the student to explain how she got the answer 26 tiles, the teacher helped students to clarify that while 26 tiles was the perimeter of the 9 x 4 pool, it was an insufficient number of tiles to completely surround the pool, leaving the corners untiled.

Emphasize mathematical meaning. Another response to a PTM is to emphasize the mathematical meaning behind the subjects. There are ten such instances in the study. One of them happened when the teacher asked students to provide some real-world examples that show indirect (inverse) proportion. One student answered, "The farther you drive, the less gas you have. So you get more miles, but you have less gas." The teacher grasped this opportunity to emphasize that "In a true indirect proportion, you never could touch the x-axis", but in this student's case, it is likely that the car would completely run out of gas. This led the class to seek a better illustration of an indirect proportion, coming up with the relationship between the speed driven and the time taken to drive a set distance.

Wait to allow student to explore first. This type of response happened eight times in the study. When a PTM occurred, the teacher didn't immediately offer an answer to the student. Instead, he/she waited for a few seconds to allow students to explore first. During this process, two possibilities might happen. One is teacher-teach-student and the other is student-teach-student. The teachers in this study chose to use this method nine times. One instance happened when students got "error" on their calculators while operating zero divided by zero, the teacher didn't immediately pointed out that 0/0 is undefined. Instead, from the point she posed the question "why do you think that is?" at 15:25, she waited for 26 seconds before she actually explained it. During these 26 seconds, she allowed students to struggle with this question. Schoenfeld (2011) claimed that the simple act of waiting after asking a question made it clear that the questions are not rhetorical but are meant to provoke student responses.

Another instance happened during a group of students' presentation. These students were trying to demonstrate how they solved the question, "how many hot dogs must be sold in order to raise at least $250?" They set up an inequality $250\geq .50h - 450$. The teacher waited patiently until this group finished presenting their method. Then she asked the whole class, "Are there any questions for this group?" This gave observing students some time to make sense of the group's method, and one pointed out the mistake – choosing the wrong inequality symbol. Schoenfeld (2011) wrote, "Giving "the answer" prematurely can deprive students of the opportunity to do sense making on their own, and perhaps even of the confidence that they can do it" (p. 138). This instance shows how students can take advantage of classroom discussions to make sense of complex concepts.

Extend/make connections. This type of response happens when teachers encourage students to reflect on their ideas and further make connections between their prior knowledge and their claims (Stockero & Van Zoest, 2012). The teachers in this study chose to use this method four times. In one instance, the class had been working on using their equations to predict what the area would be for an oil spill of one liter. One group came up with the answer 150,000 square centimeters. When they were asked to convert that to square meters, they responded "divide by 100" and "divide by 1,000". The teacher asked them to think about how many square centimeters are in one square meters and encourage them to draw a picture to help them. In this way, the teacher provided students with an opportunity to develop connections among mathematical concepts and to move beyond their existing mathematical knowledge.

Discussion and Limitations

Pivotal Teaching Moments can occur in any classroom if teachers make an effort to make student thinking public. They offer teachers intense opportunities to deepen students' knowledge, positively impacting their achievement in mathematics. As mathematics teacher educators, it is our feeling that helping teachers improve their abilities to recognize and effectively respond to pivotal teaching moments will offer great rewards in the classroom. Aligned with Stockero and Van Zoest (2012), the findings of this study display a better understanding of PTMs and teacher decisions in response to them. These findings can be used by teacher educators to help teachers capitalize on PTMs during classroom instruction, especially in the case of novice teachers who have less experience and knowledge to rely on as they encounter such moments (Stockero & Zoest, 2012).

One limitation of this study was that the videos that were used were professionally edited videos. The purpose of those videos was to display effective instructional strategies; therefore, some parts of the classroom practice may be ignored or deleted. Future studies should be designed to observe real-time classroom practice, in order to determine a clearer picture of what all of the students in a classroom are grappling with, and what factors influence the teacher's decisions that are observed. Including interviews with the teachers about his/her decisions when planning and executing lessons would also add depth to the study and inform the practices of teacher educators.

Continued research on Pivotal Teaching Moments is warranted as long as mathematics teacher educators are concerned with developing teachers' abilities to take advantage of 'teachable moments' in the classroom. By examining teaching practice through the lens of PTMs, pre-service and inservice teachers can learn to recognize and act on these moments with their own students, with the aim of increasing their abilities to positively impact students' academic achievement.

Acknowledgements

The authors would like to thank Dr. Jennifer Chauvot for her valuable feedback at various stages of writing this manuscript.

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[i] The student participants in this research are Latino and African American, located in an urban center, and of lower socioeconomic status (SES), all of these terms will be used to reference them. Additionally, since Latinos and African Americans are considered non-white populations, the term "students of color" is also used to refer to both groups simultaneously. While this depicts whites as not a "color" (and therefore at times cultureless) and could also refer to Asians, Asian Americans, as well Arabs for instance, the term is specifically focused on Latinos and African Americans in this paper. There are problems with this term as with all groupings of diverse groups of people, but currently students of color represents a common reference for these student groups.