

## MIDDLE GRADES TEACHERS' ENACTMENT OF KNOWLEDGE OF SAMPLING

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*In this study, we examined the ways in which teachers' statistical knowledge is enacted during instructional practice. Using classroom video recordings from three middle-grades' teachers, we qualitatively analyzed the conversations around a task designed to develop students' knowledge of statistical sampling. Our analyses showed how teachers' selection and use of contexts during instruction impacted students' opportunities for statistical reasoning. Our research extends other literature that has begun to identify the statistical knowledge needed for teaching.*

**Keywords:** Instructional Activities and Practices, Mathematical Knowledge for Teaching, Classroom Discourse

### Purpose of the Study

Holding the view that mathematics and statistics are different fields, statistics education researchers (e.g. Groth, 2007) suggest that the knowledge needed for teaching mathematics is different from the knowledge needed for teaching statistics. As such, Groth (2007) has called for researchers to work on mapping the structure of the knowledge needed to teach statistics. To help frame our understanding of what teachers need to know to teach statistics in K-12 settings, we draw on the existing research from the Learning Mathematics for Teaching Project (Ball, Hill & Bass, 2005; Hill, Schilling & Ball, 2004). Specifically, we examine teachers' enactment of specialized knowledge for teaching (SKT), common content knowledge (CCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). Our goal is not to provide a framework that encompasses the broad spectrum of knowledge referred to as statistical knowledge for teaching. Rather, in this paper, we focus here on one concept that lies within the "collecting data" component of statistical investigation (as outlined in the GAISE report (Franklin et al., 2007)): sampling. We describe what knowledge is needed to teach this concept and examine how this knowledge is enacted in practice in three middle grades classrooms. We focus on answering the following questions: (i) What knowledge of sampling do teachers need to possess to orchestrate productive statistical conversations? and (ii) In what ways is teachers' knowledge of sampling enacted in practice?

### Theoretical Framework

Creating high-quality learning experiences requires more than simply combining practices and curricula; rather, quality teaching encompasses meaningful engagement of students in activities that foster mathematical and statistical thinking (Franke, Kazemi & Battey, 2007). The ability to support this kind of engagement requires teachers to use deep mathematical content and pedagogical knowledge (Ball, Thames & Phelps, 2008).

### Mathematical Knowledge of Teaching

Much of the knowledge and skills needed to teach effectively has been encapsulated under the umbrella of mathematical knowledge for teaching (MKT). Teachers with strong MKT know the best ways to represent mathematical ideas to support students' understanding, know how algorithms are developed and why they work, anticipate students' misconceptions and plan

instruction to avoid or resolve them, and understand the typical developmental sequences of students' mathematical thinking (Ball et al., Hill, et al., 2004).

MKT encompasses four main domains of knowledge – common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). CCK refers to mathematics knowledge that is common to people across professions. SCK represents subject matter knowledge and skills that are unique to the work of teaching, and therefore is not commonly used in those ways by most other professions (Ball et al., 2008). KCS represents a combination of knowledge of the subject matter with knowledge about how students engage with that content. It comprises, but is not limited to, knowledge of common misconceptions, or what mathematical topics are particularly challenging for students. The fourth knowledge domain, KCT, refers to knowledge of the content combined with knowledge of ways to teach that content (Hill, et al., 2008). This involves knowing which problems are good for developing understanding of concepts, how to sequence problems, and what representations work best to support learning. Although MKT integrates these four domains, in this study we focus on highlighting the ways CCK and KCT are enacted in practice.

### **Knowledge Needed for Teaching About Sampling**

To describe the knowledge needed for teaching sampling, we adopt Groth's (2007) approach by aligning the GAISE framework with the domains of MKT relevant to this study. The *Collecting Data* component of the GAISE framework primarily involves designing a plan to collect data and employing that plan (Franklin et al., 2007). This encompasses both mathematical and non-mathematical activities. Sampling in particular, involves primarily non-mathematical activities (Groth, 2007), such as designing data collection instruments including surveys, establishing quality control measures on data collection procedures, and determining the research design that will best answer the research questions. While much of this knowledge falls under common knowledge, research suggests that students have difficulty grasping key ideas necessary to carry out these activities successfully.

Watson and Moritz (2000) have described a hierarchy to describe the three tiers students typically move through as they develop knowledge about statistical sampling. In the first tier, students draw on their prior personal experiences of samples, such as something you get for free or a small piece, including samples of food or medicine. In a second tier, students are beginning to distinguish the difference between a sample and population and how using random or stratified sampling methods can impact results. Finally, in the third tier students understand the sample as being representative of the population, and students can recognize bias and interpret data based on the sample size and selection method used. Without quality instruction, students struggle to understand the process of sampling as described in tier 3. In a related study, Jacobs (1999) also documented the interesting ways students interpret sampling scenarios. For example, students expressed surprise if survey results contradicted what they expected; others evaluated a sampling method based on its decisiveness, citing that sampling methods that produced results in which one choice resulted in a large preponderance of the data was more valid than those in which the data might be close to a 50-50 split. Other research has shown that students are sometimes reluctant to advocate for a randomized sample. For example, when students were asked to compare three sampling methods (random, convenience, and a method where participants self-selected), 64% of students chose the self-selection method (Shaughnessy, 2007).

### **Knowledge Enacted in Practice**

It has long been thought that a teacher's ability to instruct is strongly connected to what she knows. For several decades, mathematics education researchers have sought to empirically

substantiate the relationship among teachers' mathematical knowledge, instructional practice and student learning. From the results of various studies (see Ball, 1990; Fennema, Franke, Carpenter & Carey, 1993; Lampert, 2001), there is now strong empirical evidence to show that "...there is a powerful relationship between what a teacher knows, how she knows it, and what she can do in the context of instruction" (Hill et. al., 2008, p. 496).

Teachers' MKT is perhaps most visible in practice when they orchestrate instruction in ways that students are engaged in the intended mathematics and the richness of the mathematics is sustained throughout (Sleep, 2012) - what Sleep describes as steering instruction toward the mathematical point. To specifically examine how teachers utilize their knowledge in the act of teaching, researchers have identified characteristics of teaching through which they are able to determine the quality of instruction. Hill et al. (2008) identified six themes classified as *Elements of the Mathematical Quality of Instruction (MQI)*. These include (a) mathematical errors, (b) responding to students inappropriately, (c) connecting classroom practice to mathematics, (d) richness of the mathematics, (e) responding to students appropriately and (f) mathematical language (see Hill et al. (2008) for a full description). Although not an exhaustive list, we found that these elements aligned well with our own observations of the ways teachers' knowledge are enacted in instruction. In the next sections, we highlight how teachers implemented a task that focused on interpreting the limitations of data with bias caused by a non-representative sample.

### Methods

The task (Fig 1., adapted from Rossman and Chance, 2008) is designed to have students examine how the sampling method used can impact the data collected. The intent of this task is to develop an understanding that the inferences made from data depend on how that data was collected. Students should understand that to make valid inferences, they should look at data with critical skepticism, which is an important statistical disposition (Wild & Pfannkuch, 1999).

1. On the 12<sup>th</sup> anniversary of the death of Elvis Presley, a Dallas record company sponsored a national call-in survey. Listeners of over 1,000 radio stations were asked to call a 900 number (at a charge of \$2.50) to give their opinion about whether or not Elvis was really dead. It turned out that 56% of the callers felt that Elvis was alive.

1. Identify the population of interest and the sample actually used to study that population in this example.

Population:

Sample:

2. Do you think that 56% is an accurate reflection of beliefs of all Americans on this issue? If not, identify some of the flaws in the sampling method and suggest how it could have been improved.

**Figure 1. The Elvis Task**

### Participants

The participants included three middle grades teachers and approximately 110 students from one elementary and one middle school in a predominantly white, suburban school district in the Midwest. Mr. Jackson was a sixth grade teacher while Mr. Horn and Ms. Ottey taught middle school mathematics to seventh- and eighth-grade students.

### Data Sources

We used transcripts of video recordings of classroom instruction and field notes to investigate how teachers' knowledge of sampling was enacted in practice. Three cameras were used; one recorded actions of the teacher, and the second and third focused on individual groups of students for the entirety of a lesson. The videotaping was supplemented with observational

field notes taken by a member of the research team.

### Analysis

Members of the research team watched the classroom videos and read the transcripts of lessons where the relevant task was discussed. The transcripts were open coded and examined for statements that captured the teachers' instructional practices. Given our research questions, we developed two categories of codes – discourse codes and statistical knowledge codes. Discourse codes (e.g., revoicing, questioning, telling) aligned most closely with elements (b), (c) and (e) of the MQI, while the statistical knowledge codes (e.g., Knowledge of Sampling) aligned with elements (a), (d) and (f).

Each type of code was counted for each teacher, then the percentage occurrence of each code per category was found to determine dominant discourse types. We also used statistical knowledge codes to do a second round of data analysis. We identified all the segments of the transcripts that had a statistical knowledge code and examined the discourse around each code to describe the ways this knowledge was drawn on during the act of teaching. From the analyses we found different ways in which the teachers' knowledge of sampling were enacted in practice.

### Findings

In this section, we contrast how Mr. Horn, Ms. Ottey, and Mr. Jackson introduced the Elvis task. All three teachers made explicit attempts to connect the task to students' prior experiences, but not all teachers were able to engage students in discourse that would allow them to think statistically. Prior to introducing the Elvis task, Mr. Horn asked students to identify the favorite type of music among people in their town. Several students shared, then Mr. Horn followed by asking, "how can we figure this out?" Students suggested advertising in the local newspaper or collecting data online. Calling in to radio stations was also suggested. Students then made a list of different radio stations in their local area. After documenting these stations on the board, he asked students to consider the music genre of each station. The following conversation ensued:

*Mr. Horn:* You guys had some great ideas about getting a survey out and kind of asking the people of Oakville to respond in a survey. If we did it through the radio, let's say we picked one of these stations.

*Student:* 100.3

*Mr. Horn:* 100.3; if we had on the radio, if we had 100.3, that station perform this survey for us and asked the people of Oakville, okay call in and tell us what is your guys' favorite radio station. We had that station list that for us, what do you think would be the most people? The most response to 100.3? What would be the favorite of Oakville? Jeremiah?

*Jeremiah:* It would, the station that advertised it

*Mr. Horn:* The station that advertised it so 100.3 plays mostly country right, why would they get more country people?

*Students:* That's the station.

*Mr. Horn:* Awww, good say that a little.

*Student:* That's the station.

*Mr. Horn:* That's the station that plays country music so people who naturally like country music are going to listen to 100.3. If we only had that radio station put out our survey?

*Student:* You would only get country people.

*Mr. Horn:* We'd only get the country people responding right?

*Drew:* You pick one from each...

*Mr. Horn:* Awww, Drew had an excellent idea. In order to make sure that we kind of get everybody within Oakville, why don't we kind of pick one from each little group, if we had a classic rock station, we had a country station...an oldies station.

This excerpt documents Mr. Horn helping his students understand the importance of samples being representative of a population. In particular, he wanted students to understand that when data is collected from a homogeneous group of people, the results may not be generalizable to the entire population. By choosing the context of the radio station, Mr. Horn posed questions to students situated in a context to connect their personal experiences of listening to music on the radio to a larger statistical concept. In addition, Mr. Horn prepared his students to engage in the Elvis task, a problem that also involved responses of radio listeners, which was not representative of the population for other reasons. Thus, Mr. Horn's instructional choices both built on students' lived experiences and prepared students for engagement in a future task.

Ms. Ottey also found ways to connect statistical ideas to her seventh-grade students' prior experiences. During one class session, Ms. Ottey introduced the Elvis problem, and immediately engaged students in a discussion to ensure they understood the two new statistical terms – population and sample. Originally, she asked students, “What is the population of this room?” The students responded by saying twenty-eight until a student mentioned that it should really be thirty, because they had failed to include Ms. Ottey and the researcher in the room. She then asked about whether there were different sub-groups in the room (e.g., girls, adults, brown-haired people, thirteen-year olds), and noted that all of these make up the population. The excerpt below documents the discussion that followed.

*Ms. Ottey:* ...Are you clear on population? [pause] What is a sample? If I wanted to take a sample of this group, who would I sample?

*Jamie:* Me! All of us!

*Ms. Ottey:* That would be the whole population. That would be like saying I'm going to give my breakfast survey to every person in the whole school.

*Dacia:* Three tables.

*Ms. Ottey:* Oh wait, wait!... What were you going to say?

*Dacia:* Well, I was just, what was your question?

*Ms. Ottey:* My question is: What would my sample be? If I were going to take a sample of this room, what would a sample be? [pause] Yes? [pointing to Blake] Oh, yes?

*Blake:* A person?

*Ms. Ottey:* I could take one person. That might be a sample. Maybe there's a big survey and I'm, maybe it's like student council. Let's see, who's your student council members? Okay, Jackson, Erica, Kevin, Ariel. So, those four are a sample of our class. And we use those to represent our class, right? Okay, so that would be a sample. Does it include girls? Yeah, but it includes boys, but there's a specific number. There's four. Umm, so we wouldn't have an eighth grader and have them represent our team or our class, right? Cause there are no eighth graders here. So, a sample is a small part of the population.

As Ms. Ottey helped her students to distinguish population and sample, she drew on her knowledge of her students and statistics to make several important teaching moves. For example, she recognized that these terms might be difficult for her students to distinguish, so she asked directed questions to ensure the students had a collective understanding of the terms.

By situating the discussion in terms of the student council members, the students could see how a sample is contained within a larger population. Furthermore, the members of the student council represent the members of the class, much like the data from a statistical sample are used

to represent a population. She extended this conversation by helping students to recognize the importance of considering the membership of the sample by emphasizing the need to include particular subgroups (girls) and exclude those that would not be representative of the class (eighth graders). Although the metaphor of the student council was helpful for understanding the purpose of a sample, it did have limitations. Typically, the students in a class elect their representatives on the student council. However, in statistical studies the population does not typically decide which of their members will constitute a sample. One might argue that Ms. Ottey should have disclosed this difference to her students.

As the examples of Mr. Horn and Ms. Ottey have demonstrated, using contexts that students understand can be a useful tool in helping students to think statistically and learn statistical concepts. Contextualizing a problem can also be used as a motivational tool. Mr. Jackson attempted to help his sixth-grade students recall who Elvis Presley was by playing his music.

*Mr. Jackson:* ...raise your hand if you know who Elvis is? This guy here. Right? [plays Elvis Presley's *Hound Dog*] ... Right? [Some students begin moving arms as if they are dancing in their seats.] And Chad and Karl like to dance to it. And Bradley too. Okay? So, if we read this problem, if we read this problem: the twelfth anniversary of Elvis Presley, Bradley. On the twelfth anniversary, a Dallas record company...

Mr. Jackson introduced the Elvis problem in a way that would help students recognize Elvis Presley. For some students, the music might provide motivation to further understand the problem. However, playing music did not provide a means to engage students in statistical discourse. As evident in the excerpt, while the music played, students were dancing to the music. After giving the students several minutes to talk about the Elvis task in small groups, Mr. Jackson debriefed the discussions as a whole class.

*Mr. Jackson:* Do you think that 56% is an accurate reflection of beliefs of all Americans on this issue?

Gabe: Yes!

*Mr. Jackson:* And if not, you need to tell us some of the flaws in this method and suggest how could they improve it, how could the radio station actually improve it, okay? But before we answer that question, let's go and answer the first one. What is the population of interest? What's the population of interest? Bradley?

*Bradley:* People around the state.

*Mr. Jackson:* So you think people just around the state? Okay, what do you think?

*Lanie:* Everyone around the United States of America.

*Mr. Jackson:* Okay, Lanie thinks the population of interest is everyone around the United States of America.

*Lanie:* Unless it's a local radio stat—

*Mr. Jackson:* Unless it's just a local radio station, right, Nick?

*Nick:* Yes.

*Mr. Jackson:* But what was the sample that they actually took? Jordan?

*Jordan:* People who listen to the radio station.

The students' misunderstandings about sample and population are prevalent in this discussion. First, Gabe's comment suggesting that such a high percentage of Americans would believe Elvis was still alive suggests he doesn't have a clear understanding of the context. Bradley incorrectly identified the population as "people around the state." Finally, Jordan misunderstood what the sample was, identifying all people who listened to the radio station, rather than those listeners who paid \$2.50 and called in.

Mr. Jackson's students had been given ample time to discuss and critique the task in small groups, so why did these misunderstandings persist? One possible explanation is that Mr. Jackson, unlike Mr. Horn and Ms. Ottey, had not framed the context and launched the task in a way that really allowed students to make sense of the important statistical ideas in the task and connect it to their personal experience in a meaningful way,. Furthermore, Mr. Jackson's primary discourse move in this excerpt was to repeat students' responses. He did not react to either correct or incorrect thinking in constructive ways.

### Discussion and Conclusions

From our data, we observed that Ms. Ottey and Mr. Horn had clear understandings of population (set of entities to which inferences will be made) and sample (representative subset of the population), evident in their discussion with the students. However, to help students grasp the idea of "representativeness," having CCK of population and sample was insufficient. To build this understanding, the teachers framed the classroom discussions within meaningful contexts that connected to the students' experiences, demonstrating aspects of their KCT. In particular, by referring to the composition of the Student Council and foregrounding the need to collect data from fans of different music genres, the teachers made it clear that an appropriate sample should include all the types of people from the population. These contexts were conceptually-rich, allowing students to distinguish between population and sample and identify methods to reduce sampling bias. Mr. Jackson's choice of context was quite different; he chose to play and discuss an Elvis song prior to the discussion. While this prelude may have captured the students' attention, in contrast to the other classrooms the context did not help build meaning of the statistical concepts involved in the task. In fact, based on his interaction with the students, one could question whether Mr. Jackson's had CCK related to these concepts. Engaging students' statistical thinking requires that teachers understand the concepts (Hill et al., 2008), select appropriate contexts (Boaler, 1993; Sullivan, Zevenberger & Mousley, 2003), and know how to frame statistical questions in ways that allow students to build connections between their prior experiences and a task's embedded statistical concepts.

Teaching quality is related to how well teachers can draw on and integrate these domains of knowledge in the act of teaching to engage students in the intended mathematical (or statistical) work (Sleep, 2012). In this regard, maintaining the richness of the statistics (content), resolving misconceptions and responding (in)appropriately to students' statements were elements of practice that were enacted differently across classrooms. Although Mr. Jackson did continuously question the students, he did not deeply probe students' responses or use follow-up questions in ways that encouraged students to think critically about the ideas being discussed (Franke, et al., 2009). His interactions with students were primarily repeating students' statements, rather than fostering a discussion that could lead to the intended statistical idea. His missed opportunities to counteract students' incorrect statements served to perpetuate misconceptions held by the students. Conversely, Mr. Horn and Ms. Ottey were better able to guide the discussion towards the statistical point. They wanted students to develop a healthy skepticism of data and to acknowledge potential bias caused by non-representative samples. To do so, they drew on the experiences of the students, identified their current understanding of population, and used this information to craft questions and responses that guide the conversation towards to the intended statistical point.

As the field moves towards building a knowledge base for teaching statistics, important questions to frame this discussion include: what contexts best support students' understanding of

sampling, and statistical concepts more broadly? What are the affordances and constraints of different contexts? What contexts are effective for particular groups of students (different grade levels, rural vs. urban etc.)? What instructional practices are most effective to foreground the intended statistical ideas? The results of this paper add to this emerging body of knowledge by providing usable knowledge about ways to build meaningful contexts for statistical thinking around students' experiences. Also, we provide insight into what teacher moves work and how to deploy these moves to foreground the intended statistical ideas.

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