SMALL GROUP TO WHOLE GROUP SHARING OF IDEAS IN AN ELEMENTARY GEOMETRY CLASS UTILIZING TECHNOLOGY

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This research reports on data and analysis investigating the sharing of ideas and strategies from the level of a small group to the level of the whole class (and vice versa) in a fourth grade mathematics classroom utilizing digital technology. This research takes a sociocultural perspective positioning meaning and learning as shaped by mediational means which can drive or suppress the transition of ideas and strategies from the small group to the whole class and vice versa. The three potential mediational forces are: social, technological, and pedagogical. The digital technology of focus is a research version of Sketchpad® Explorer on the iPad. Using this specific technology, in conjunction with connectivity, students can interact with and manipulate dynamic constructions in a small group that can then be sent to a shared classroom space to be analyzed and discussed at the level of the whole class.

Keywords: Technology, Classroom Discourse, Elementary School Education

Introduction

This research paper seeks to investigate the transition of ideas and strategies across levels (from small group to whole class to small group) in an elementary mathematics classroom utilizing a research version of Sketchpad® Explorer for the iPad that supports collaboration with connectivity. Sketchpad Explorer is based on The Geometer's Sketchpad® (Jackiw, 1991, 2009) and allows the user to drag and manipulate mathematical sketches on the iPad. Using this specific technology, in conjunction with connectivity, students can interact with and manipulate dynamic configurations in a small group that can then be aggregated into a shared classroom space to analyze and discuss at the level of the whole class.

In the transition from small group work to whole class sharing and discussion, where the small group collaborative activity transitions to a whole class level, there exists the potential to share ideas and strategies that can potentially influence the future work of small groups. For this research, we investigate the role of mediation in this transition by focusing on mediational forces influencing what is shared or suppressed at the whole class level. This perspective on mediation assumes tools, peers, and the teacher are not neutral, rather the tools, interactions between students, and between student and teacher shape the mathematical meaning within the interactions and from the interactions. Three possible mediational forces of focus are: social, technological, and pedagogical.

Theoretical Framework/Background

From a sociocultural perspective, in a classroom, meaning and learning are products of social activity. In a small group collaborating on a task utilizing digital technology to create a mathematical object, the social interaction amongst the group members is fundamental to individual learning because “human action typically employs ‘mediational means’ such as tools and language, and that these mediational means shape the action in essential ways” (Wertsch, 1991, p. 12). Wertsch continues referring to the student(s) as ‘individual(s)-acting-with-mediational-means’ such as language and other psychological tools such as words, gestures, and diagrams, opposed to speaking only of the 'individual(s)'. Digital technologies used in the classroom can support students' exploration of mathematical ideas and can mediate their social interactions with both teachers and peers. A common pedagogical practice for small group work includes bringing the small groups back together for a whole class discussion of the task and artifacts created. In a connected classroom, this can take place quickly and with ease allowing multiple cycles of small group to whole class to small group work within a single class period.

Classroom technologies incorporating connectivity have shown to have great impact on learning and opportunities to learn in the classroom (Brady, White, Davis, and Hegedus, 2013; Hegedus and Kaput, 2004; Stroup, Ares, and Hurford, 2005). Classroom connectivity offers opportunities for both group and individual learning experiences in the classroom structured around participation. Embedded in the design and intention of software technologies such as NetLogo (Wilensky, 1999) and SimCalc MathWorlds® is the recognition that a student in a classroom is not learning or building knowledge in isolation, rather alongside other students with varying mediators at play. As such, the more recent designs and uses of classroom connectivity include pairs or small groups of students working together on a task or creating a mathematical object within a technological environment.

Goos, Galbraith, Renshaw, and Geiger (2003) found that the technology used in the classroom, graphing calculators, could facilitate communication and sharing of knowledge in both private and public settings highlighting that "learning becomes a process of appropriating tools that change the ways in which individuals formulate and solve problems" (p. 75).

Enyedy (2003) focused research around student small group work (pair) and the relations between private work on a computer and public work during whole class discussion time. This research positioned mathematical activity and mathematical understanding as a function of participation and called the individual computer simulation work and the whole-class discussions two social configurations of the classroom. Within the intersection of these two social configurations, Enyedy examined the ways in which individual and social processes are mutually constitutive. The analysis included two case studies in which examples were found tracing a student's reasoning back to the interactions with other students, the teacher, and the software used. Additionally, the author found that agreements within the pair setting potentially caused conflict with other pairs' work when presented at the whole class level.

This finding is consistent with findings from the SimCalc MathWorlds work in which small group work is aggregated at the whole class level via connectivity (Hegedus and Roschelle, 2013) and agreement is not always achieved at the level of the whole class. This resonates with work of Pea (1994) who notes that communications using multimedia-learning environments are open to multiple interpretations in how they express representations of phenomena and so there will be a need for interaction, negotiation, and repair amongst participants.

A difference between these research investigations exists in how and what student small group constructions are made public. For Enyedy, student verbalizations of strategies or findings are made public, whereas with connectivity, verbalizations of ideas or strategies along with artifacts of student constructions are made public. These public interactions have implications for everyone in the class as a learning opportunity for future mathematical investigations or revisions of strategies.

White, Wallace, and Lai (2012) developed collaborative activities for classroom networks to investigate the intersection of individual student engagement and collectively constructed artifacts that are made public via connectivity. The authors position their research within a largely social view of learning which takes words, gestures, and inscriptions of the small group as different modalities working together for social sense-making. Students in three ninth-grade Algebra 1 classes worked in pairs, each controlling a point on a grid space, and were asked to create specific linear functions. Analyses of the video data of the pairs focused on whether the strategies were implemented by a single student in the pair, or jointly by the pair of students. Two important findings from this work includes: 1) the authors' finding that the students shifted emphasis from asynchronous actions to synchronous actions in the shared graphical space, and 2) not all pairs in the class displayed the same degree of interaction and negotiation with their partner (White, Wallace, and Lai, 2012). This is somewhat due to the relations between the mathematical objects and social relations; each shape and are shaped by each other via the mediational forces. Our broader research aims to identify the mediational forces at play for various groups and how these forces drive the interaction between students and from small group work to whole class discussion.

As Moreno-Armella and Sriraman (2010) have pointed out, research into the nature of tool mediation is a crucial goal for the further development of the field of mathematics education, explaining that over time, tool mediation has become an integral part of human intellectual activity. With the addition of connectivity, dynamic mathematics, and the affordances of the iPad device, which includes multimodal interaction, the mediational role of this technology within the classroom in which students are working collaboratively in groups, human action (student action) will shape and be shaped by these mediational means.

Methods

Descriptions

The data presented in this report was collected as part of a larger study. The participants in the main study were three fourth grade teachers and their students (n = 61) which focused on teachers' implementing multi-modal activities focused on multiplication using Sketchpad® Explorer for the iPad. The three teachers collaborated with the researcher team and together the geometric activities using Sketchpad Explorer were designed.

This paper focuses on a single class of one teacher. The teacher is State certified and has been teaching for eleven years. She had never used an iPad in the classroom before but participated in the development of the activities and training workshops focused on implementing the activities. There were twenty students in her fourth grade classroom.

The design of the activities and the teaching sequence included the following types of activities: 1) introduction to the activity, 2) small group work in which students collaborate using the iPad to manipulate/ configure a mathematical object which will get sent to the teacher, 3) individual paper work in which students respond to questions and detail their strategies for configuring the mathematical object, and 4) whole class discussion usually initiated by the teacher.

Data Collection and Analysis

Data collection. The intervention activities were implemented in each classroom once a week for three weeks during which classroom video data and student work were collected. Three video cameras captured the lesson in each classroom; one roaming camera focused on students, one roaming camera focused on the teacher, and one stationary camera with a wide angle lens in a back corner to get a visual of the whole classroom. For the purposes of this paper we focus only on the classroom video data from week 3 which introduced connectivity.

Classroom activity. The overall main goal of the activity is an exploration into partial products. There are four activity tabs within the Sketchpad sketch (see Figure 1). Each tab represents a specific element in the sequence of the activity. The goal for the first tab is for students to create one shape from the partial products, a quadrilateral. The square and rectangular pieces in this tab are a fixed size so while the arrangement of objects can be different across groups, the total sum of the area will be the same.

In the second activity tab, the square and rectangular pieces are again a fixed size but the four pieces cannot be placed together to form a quadrilateral. Students must determine which partial product is not fitting and why. Students will write a multiplication sentence to represent the total area. While configurations across groups can be different, the total sum remains the same.

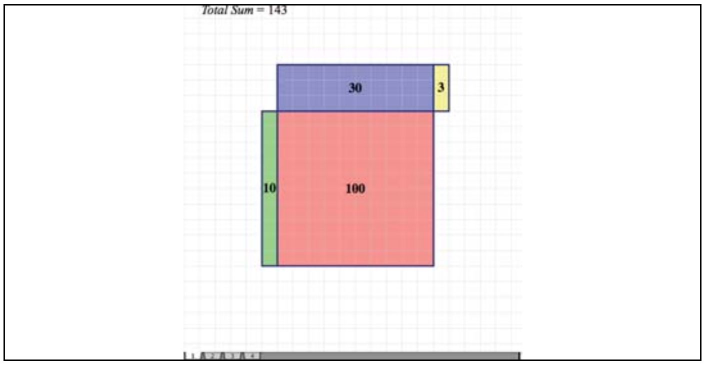

Figure 1: Screen Capture of the Activity 3 Window

In the third activity tab, students are asked to create a rectangle from the four pieces given; three fixed sized pieces, and one editable piece. The three fixed pieces are the same pieces from activity tab 2 and the one editable piece initially has the same area as in tab 2. Students will be writing a multiplication sentence to represent the total area and the total area will be the same for all students. Here, they are investigating the partial products of 11x13.

In the fourth tab, students manipulate and arrange a new set of partial products into a rectangular configuration. There are four pieces and only one is dynamic. Students are asked to write a multiplication sentence for their configuration. In this activity tab, it is possible for the total sum to be different across groups depending on how students configure the pieces; there are three possibilities. Students are asked to explain their strategy on a paper worksheet provided.

Over the four weeks the teacher of the class tried to support group work by physically rotating the position of each student in a group relative to the iPad. Students worked in groups of three (and one group of two), with one iPad per group. The iPad primarily remained on the center desk and the three students changed their seats several times during a single activity. This ensured every member of the group could reach the iPad with his or her hands/fingers. Of course the iPad did not *always* remain on the middle desks, students did pick up the device to show each other or move the device to make an edit. During this lesson incorporating connectivity the student seated at the center desk submitted the work to the teacher under his or her name.

Analytical methods. The analysis presented in this paper focused on the classroom video data from one classroom engaged in the fourth tab of the activity, described above. The aim of this analysis was to investigate the question: as student ideas and strategies are developed and shared within a small group, how do the mediational forces drive the transition to bring these ideas and strategies to the level of the whole class?

Video from both the teacher and student camera angles was transcribed. Included in the transcripts were utterances, pauses, noted gestures with descriptions, and a description of referents when known. The video of the class was watched alongside and in conjunction with the creation of the transcript and was watched during the process of coding the transcript. We coded completed transcripts for instances of transitions of an idea or strategy, 1) where a student idea or strategy was presented within the small group, and 2) where the small group work transitioned to the whole class during the whole class discussion. A second cycle of coding identified instances where the social, technological, and pedagogical mediational forces existed.

The following conventions were adopted in the transcription: (a) conversational utterances are numbed sequentially (as a result, portions of the transcript will not begin with utterance one); (b) non-verbal information including action gathered from the video is included in brackets and italicized; and (c) [inaudible] indicates utterances that could not be heard. Due to space restrictions, ellipses (…) were used to denote a break in the transcript.

Findings

In this section we wish to present a classroom scenario that illustrates an instance of how a group strategy from one small group is taken up by another small group and shared with the class. We refer to this as borrowing since the strategy taken up by a small group can potentially be returned at the level of whole class via connectivity and the sharing of student work. With this episode we illustrate: 1) the transition from the level of the whole class to the level of the small group, and 2) the transition from the level of the small group to the level of the whole class through the lens of mediational forces, which shape the action.

Small Group Borrowing of a Strategy

The following transcript pieces are centered on one small group (Sabrina, Pablo, and Nick) and the whole class discussion of their work. All names are pseudonyms. Sabrina is seated at the center desk with her group members on either side of her. In an interaction during small group work, the teacher asks the group about their strategy. Sabrina says the strategy for their configuration comes from an idea that Fred, a student in another group, had previously done. She says,

> Um, well, when I was making this, I thought of what Fred was doing, and, um, I was thinking, like, how I was gonna, like, build it. And I wanted it to be different from everybody else's 'cause I know everyone was building it upwards and I wanted... [horizontal].

Sabrina refers to how she wanted to configure the rectangle and her own specific strategy for this creation. Her language is very personal referring to how she was going to build it. The teacher interrupts Sabrina, but from her configuration in Sketchpad Explorer and her verbalization it seems she specifically tried to do something different from her classmates because Fred and his group had done something different from his classmates during a previous activity tab earlier in the class period. While it is unclear how Sabrina determined what her classmates did for this activity tab (in order to do something different), this strategy can be seen as both mathematical and social. It is mathematical because, for Sabrina, Fred and his group have illustrated through sharing their work at the whole class level, that there can be more than one mathematical solution, or configuration in this case. It can also be thought of as mathematical because Sabrina presumably would have had to determine what some of her classmates had done and how she can complete the task differently from the other groups.

Her rationale for this strategy may have been social: wanting to stand out from others in the class, or to receive special attention from the teacher. During the previous activity tabs the teacher focused the whole class discussion on only a few arrangements specifically looking for "something different", a phrase she used several times during the class period. When previously discussing the work of Fred and his group members during the whole class discussion, the teacher identified Fred as someone who "thinks outside of the box". However, completing the task with this strategy of finding something different from what other small groups are doing has mathematical underpinnings, even if Sabrina's configuration was mediated by social and pedagogical factors.

Small Group to Whole Class

After student work had been sent to the teacher, the teacher showed a configuration she herself had created and asked if students had the same total sum: 143. The teacher asked who has a sum smaller than her own and members from six of the seven small groups raise their hands. She asks who had a total sum larger than 143 and only Sabrina and Pablo raise their hands. Sabrina had succeeded in creating a configuration different from all her classmates with a total sum larger than the total sum of all configurations. The teacher directed attention to Sabrina's arrangement, both by referring to it as "interesting" (line 1501) and by putting the arrangement in the public space to be discussed with the whole class.

1489 *T*: Why would the total sum change a little bit depending on how you did the shapes?
1490 *James*: It's because you put that one bigger than all of...
1491 *T*: Is it bigger than all the others?
1492 *James*: No, it could be smaller and bigger. It's bigger than ours.
1493 *T*: Okay. Alright. Why else?
1494 *Haley*: Because you could move the blue, um, the purple shape anyway you want.
…
1499 *T*: How many people have a total sum smaller than that?
[Members from six of the seven small groups raise their hands.]
1500 *T*: Anyone have larger than that? *[Sabrina and Pablo raise their hands.]*
1501 *T*: Interesting. Okay. Let me bring yours up. I wanna take a look at it. Alright.
…
1508 *T*: Okay. *[Posts the work of Sabrina, Nick, and Pablo.]* This is what they have.
1509 *James*: One bigger. Are you serious? *[James turns and looks at Sabrina's group as he responds. He is referring to the total sum as being 1 value larger than the total sum of the teacher's configuration.]*
1510 *[Sabrina smiles.]*
1511 *T*: Why is it one bigger?
1512 *Pablo*: Um, because our, the blue, … is a sum of thirty-three.
1513 *T*: Okay. So, this whole thing here *[Points to blue rectangle on projected screen]* has thirty-three little squares in it?

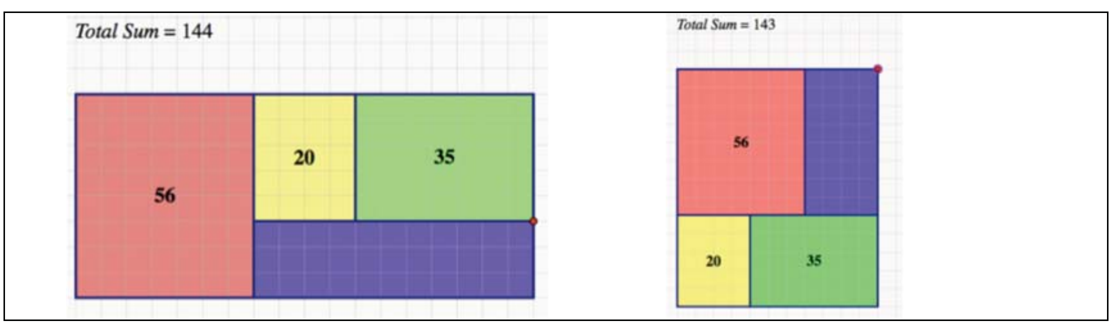

Figure 2: Sabrina, Pablo, and Nick's (Left) and Teacher's (Right) Configurations

Martinez, M. & Castro Superfine, A (Eds.). (2013). *Proceedings of the 35th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Chicago, IL: University of Illinois at Chicago.

With the affordance of connectivity and the teacher questions (lines 1493, 1499, and 1500), the technological mediational force and the pedagogical mediational force act in tandem to bring about this interesting mathematical instance. By making multiple groups' work public the class has unearthed these two configurations, which differ in total sum by 1 square unit. The teacher presses for justification by asking why the configuration is "one bigger" than the previous configuration. She presses for further explanation after a student has identified the editability of the purple piece as being responsible for the different total sums (line 1494).

The social force initially mediated Sabrina's action with this activity; she pulled on a strategy used by Fred to come up with an arrangement that was different from her classmates'. As Sabrina verbalized to the teacher, this difference originated in the shape of the configuration, to make a configuration horizontal while others were making the configuration vertical in appearance. The teacher acted with a mathematical rationale in bringing Sabrina's group configuration to the level of the whole class, this configuration was different, both in shape and in size. While it was likely not anticipated by Sabrina (or Pablo, and Nick) when creating their arrangement that it would have the largest total sum, Sabrina's borrowed strategy of trying to create something different from her peers led to an interesting mathematical learning opportunity. In the following transcription piece, Pablo is offering an explanation.

1527 *T*: Okay. So, it's one bigger because this is thirty-three. That's the only reason?
1528 *James*: Yup. Pretty much. No, wait. If you combine...
1529 *Pablo*: No, because that, the multiplication fact that goes with it is eleven times three, and that one hundred, and that for thirty-two would be, that would have a total sum of everything for one thirty, one forty-three. Because of eight times four.
1532 *T*: Okay.
1533 *Pablo*: Because if it has four...
1534 *David*: I don't get how it's like one bigger.
1535 *Sabrina*: Because it just wants to be. *[Sabrina turns her head and directs this response to David before looking back at the teacher.]*
1536 *T*: 'Cause it just wants to be? Okay. Alright. *[Sabrina laughs.]*
1537 *T*: Okay. Let's see. *[Posts a different group's work.]*

Pablo's explanation is not entirely clear although he does seem to reference his group's purple piece (with a size equal to 11 times 3), the teacher's purple piece (with a size equal to 8 times 4) and the total sum of the teacher's configuration (line 1529). It is not known whether Pablo's explanation is clear to the teacher, it is not clear to us nor is it clear for David who follows up on what Pablo says by asking why is it one [unit] bigger. Sabrina, the primary creator of this configuration, responds with "it just wants to be." This non-mathematical response does not resolve the mathematical question at hand but seemed to serve as a break in the activity as the teacher chose to move on to review the work of another small group. During this portion of class time the small group configuration remained static. The technology no longer played a mediating role in shaping the mathematical meaning; that ended when the students sent their work. The technology existed as a display space only at that point. The student interaction was mostly directed toward the teacher, and only a few students took part. At this point in the activity the interaction between the student(s) and the teacher via talk and reference to a static image was the only form of mediational means. This potentially can be limiting as could be argued in the above analysis, as the teacher moved the lesson forward by bringing up another groups' work.

Conclusions

In this classroom episode the strategy used by Sabrina, and the elevation of her group configuration was mediated by social, technological, and pedagogical means, which shaped the student action. A strategy was borrowed from another small group who manipulated their configuration to be different from others' in the class. The Sketchpad Explorer activity along with peers supported the small group exploration into configuring something different. An interaction between connectivity and the questions posed to the class by the teacher elevated the configuration of Sabrina's group to the level of the whole class. The interaction of mediational means played a critical role to get to that point which resulted in a rich learning opportunity for the class. Future work will focus on varying activities to explore the conditions under which such forms of mediation are suppressed or enhanced.

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