# A TOOLBOX FOR SUPPORTING EARLY NUMBER LEARNING IN PLAY: MOVING BEYOND "HOW MANY'?

Anita A. Wager
University of Wisconsin-Madison
awager@wisc.edu

Amy Noelle Parks
Michigan State University
parksamy@msu.edu

This paper explores the ways preschool teachers orchestrate instructional environments to promote mathematical play related to early number and, how they intervene during play to promote children's engagement with early number. We highlight these practices to identify resources for the growing numbers of early childhood teachers. This is important as many prospective and practicing teachers do not have access to the knowledge of teaching that supports young children's math learning because of the constraints of mathematics methods courses and the dearth of research on early childhood mathematics in mathematic education journals – particularly in-depth attention to early number and teaching mathematics in play.

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One of the most important challenges for young children in mathematics is gaining fluency with foundational concepts related to number, which include developing an understanding of the number word list, 1-to-1 correspondence, cardinality and related concepts like subitizing, and relationships of more, less, and the same (National Research Council, 2009). Generally, children develop understandings of these number concepts from ages 2 to 6 (Ibid.), meaning that the early years of public schooling are important to helping children master more advanced concepts, such as conceptual subsidizing (decomposing larger sets into easily identifiable subsets), and to helping children who come to school without mastery of early skills related to number quickly catch up. Increasingly, research has demonstrated that early and successful mastery of these early number concepts is predictive of later success in mathematics in particular and in schooling more broadly (e.g., Duncan et al., 2007; Jordan Kaplan, Locuniak & Ramineni, 2007). For example, in a largescale longitudinal study, Duncan and colleagues (2007) found that early math skills are better predictors of later school success than measures of reading, attention skills, or socio-emotional behaviors, with little difference across gender and socio-economic status. More specifically, in a study of low-income families, Ramani, Row, Eason and Leech (2015) found that caregivers' engagement of children in activities aimed at teaching about number and caregivers' use of advanced number talk predicted children's understanding of cardinality, ordinal relationships, and early arithmetic. Additionally, Ramani and colleagues found no relationship between demographic variables, such as parent education, home language, or family income and children's knowledge of the number core

Other research has shown that adults' intentional engagement of children around the number core, particularly in relation to constructs, such as cardinality and subitizing, improves children's understandings of these concepts. For example, Gunderson and Levine (2011) found that children's understanding of cardinality increased when adults counted sets and labeled them simultaneously (e.g., "1,2,3. That's 3.") in comparison with counting or naming sets in isolation. In a study of three-year-olds, Mix and colleagues (2012) found similar results. This body of work, generated primarily by developmental psychologists, demonstrates the importance of attending to early number and suggests that there are particular strategies that early childhood teachers could draw on to promote children's mastery of the number core. Yet, relatively little attention is paid to early number in

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mathematics education research, particularly in relation to teacher education practices for prospective elementary teachers (Parks & Wager, 2015).

Indeed, none of the studies cited above about the importance of early number concepts for predicting later mathematical success were published in mathematics education journals. In a recent review of 20 years of mathematics education and early childhood education journals, we found that only 28 of the 239 articles published about early childhood mathematics attended to the number core at all. In addition, the mention of early number concepts in many of these 28 articles was quite cursory – often an aside to a broader discussion. In fact, cardinality was explored in depth in only one article; 1-to-1- correspondence was explored in two articles, and subitizing was examined in four. (As a point of comparison, 66 articles mentioned fractions, 11 in some depth, and this was within a dataset of articles identified as attending to pre-K-Grade 3 contexts). In addition, only three articles concerned with early number concepts focused on strategies for helping preservice or practicing teachers support children's early number learning (Schack et al, 2013; Tirosh, Tsamir, Levenson & Tabach, 2011; Tsamir et al, 2013), which suggests a need for greater attention to developing strategies to help teachers promote early number understandings.

Recent increases in publically funded prekindergartens and staffing of these programs with teachers who have been certified in elementary education makes this need more pressing. In the 2012-2013 school year, 28 percent of four-year-olds in the US were enrolled in state-funded preschool programs (Barnett, Carolan, Squires & Brown, 2013). More than half of the states require that teachers in these prekindergarten (pre-K) programs have four-year degrees in education, which means they have likely learned to teach mathematics in university methods courses. Given current interest in universal pre-K (Barnett et al., 2012), these numbers are likely to increase, resulting in greater numbers of future pre-K teachers in elementary mathematics methods courses across the country. Additionally, given the lack of attention to early number in research (and anecdotally on methods course syllabi), future kindergarten teachers almost certainly also need more support in attending to early number than they have been getting in typical mathematics methods coursework.

In addition to requiring more in-depth attention to early number (and geometry), these future pre-K and kindergarten teachers also need strategies for helping their children maximize opportunities to learn mathematics in play settings in the classroom. Recommendations for best practices in early childhood settings almost universally include calls for time for play, which promotes cognitive development, language learning, and social growth (Copple & Bredekamp, 2009). Typical classroom play materials, such as blocks, games, and toy collections, offer potentially rich opportunities for children to learn and practice early number concepts (Wager & Parks, 2014; Seo & Ginsburg, 2004; van Oers, 2010); however, without teachers who intentionally intervene to mathematize play by attaching mathematical language to play, scaffolding more complex play, and directing student attention toward potential mathematics, children are unlikely to get the full benefits from their mathematically oriented play (Ginsburg, 2006; Graham, Nash & Paul, 1997). Yet, there is also little attention to teaching strategies for mathematizing play in the research literature. In the review described above, we found only one study (Eberly & Golbeck, 2001) that focused on understanding ways that practicing or preservice teachers learned about or implemented strategies to mathematize play.

In order to address these gaps in the literature, this study draws on a rich library of data from two broader studies in order to identify productive strategies that practicing preschool teachers used in order to promote mathematical play in their classrooms and to mathematize that play for children. The research questions guiding our analysis were: How do preschool teachers orchestrate their instructional environments to promote mathematical play related to early number? And, how do these preschool teachers intervene during play to promote children's engagement with early number concepts?

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#### **Theoretical Framework**

In this study, we take the perspective that early number learning develops not only as a result of maturation, but is significantly influenced by context and engagement with knowledgeable others (Vygotsky, 1978). Although Piaget is best known for his stage theories of development, which argue that children move through predictable stages of learning (e.g., going from more concrete to more abstract understandings of quantity) as they mature (Piaget, 1962), Piaget also did not believe that these stages operated independently of the environment or of social interactions with others (Piaget, 1964). Similarly, Vygotsky argued that adults can and should provide instruction that "marches ahead of development" (Vygotsky, 1962, p. 104) to foster children's learning.

This perspective is important because it means that rather than waiting until a child is "developmentally ready" for a particular mathematics concept, teachers must take responsibility for guiding children toward new understandings. In relation to early number, this means that primary grade teachers should not simply wait until understandings of the number core develop before beginning instruction in early arithmetic nor should they assume that children who come to school without competence with cardinality, 1-to-1 correspondence, and subitizing will develop these skills by engaging with instruction aimed at early arithmetic. Rather, they must both create instructional environments that foster children's growth in relation to these early number understandings and engage in intentional instructional practices that will move children along the developmental continuum (Parks, 2015). Both Vygotsky (1968) and Piaget (1962) saw play as an important context for these kinds of instructional interactions, both because play is motivating for children and because it encourages exploration with physical objects, which often leads to more sophisticated understandings of quantity.

# **Mode of Inquiry**

The analysis presented here is drawn from data collected in two broader studies, both aimed at understanding mathematical learning and teaching in early childhood classrooms. Both studies were designed within a tradition of interpretive research that values attention to meaning-making by participants and understanding the role of context in human interactions (Erickson, 1986; Graue & Walsh, 1998). The first study provided an intervention of sorts as teachers engaged with professional development explicitly designed to support their understanding of mathematics learning in playbased pre-K classrooms, whereas the second study observed existing phenomena – what math was happening. Thus, we have examples from teachers who were supported to think about math in pre-K and from a teacher who was not.

The first study involved a Professional Development (PD) program designed to promote culturally and developmentally responsive early number teaching. The project team designed, facilitated, and studied the PD for three cohorts of pre-K teachers who were teaching in a local districts' new 4-year-old kindergarten. Each cohort took four graduate courses over a two-year period. Data included audiotaped group discussions and artifacts from the course, interviews with teachers, and bi-weekly observations in a subset of participants' classrooms. For the current analysis, we focused on artifacts – in particular, 51 learning stories (narrative assessments of young children's learning, Carr, 2011) teachers wrote to identify what they noticed and how they responded to children's mathematics engagement in play.

The second study was a longitudinal examination of the mathematical experiences of a cohort of children as they moved from preschool to Grade 1 in a rural public school. Data included video of classroom events, assessment interviews, and out-of-school engagements, as well as interviews with teachers and parents. For the current analysis, we focused on video data collected in the preschool classroom of mathematics-related interactions in both formal and informal settings. The teacher in this pre-K was an experienced, white, female teacher with more than 15 years of experience in pre-K

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and kindergarten. The majority of children in the cohort were African American (13 out of 16) and all children in the cohort came from low-income families.

For this analysis, we identified episodes in both sets of data that had been previously coded as involving cardinality, 1-to-1-correspondence, and subitizing as well as adult interactions. We then coded these episodes as relating to organization of the instructional environment or intervention in the moment. Then, each of these sets of data was analyzed (by each researcher and through conversation and writing with each other) to identify strategies used by the teachers in both studies to promote students' understandings of early number.

## **Findings and Discussion**

We found that teachers engaged (or had the opportunity to engage) children in rich math interactions in three instructional spaces: (a) activities purposefully planned for whole class engagement in math; (b) play with math-like materials in the room or in specific math centers (e.g. board games, counting manipulatives) and; (c) during play that might not obviously be seen as mathematical (Wager, 2013). For each of these instructional spaces we provide vignettes from classroom observation (either ours or reported by the teacher) and identify strategies teachers used.

## Planned activities

Teachers in both projects planned activities that were purposefully designed to engage children in counting related activities, although teachers in the professional development project were more likely to design activities where children routinely got to handle objects as they practiced counting. In the classroom observed without intervention the most frequent counting activities involved reciting the number sequence, singing a song prompted with cards to practice number recognition, and counting during calendar time, when only the teacher or a single child handled objects while counting.

Activities when children could handle objects and count typically occurred during transitions, meals, or other routines such as morning meetings. Sometimes these were whole group activities while other times individual children 'led' the activity for the class. A common practice among teachers was to start each class period by having a child count how many children were present. In the following examples we see how a teacher supports a child who is still working on 1-to-1 correspondence and another who is moving toward problem solving. In Vignette 1, Betty reflects on her observation of Bob counting his classmates during circle time. The children had been doing this activity for about six weeks.

Vignette 1: When I asked [Bob] to count how many children were sitting in the circle, he was not exactly sure where to start. With a little guidance he started with the child that was sitting next to me. He counted with 1:1 correspondence tapping each child on the head, however, he skipped one girl and she said, "hey you missed me". He turned to me and looked confused. He started again and I encourage him to slow down. He skipped another child and then the aide assisted him so he could finish counting all the way around. (Betty, Counting Children)

In the next example, Marley shares how she supported Sam to make counting his classmates a more challenging activity. She had previously noticed that Sam would write addition problems on a dry erase board and ask, "how much is this?"

Vignette 2: I asked Sam to first count the girls and then count the boys as I wrote the numbers on the easel. I used this opportunity to also explain what the + and = signs mean. After I recorded the numbers I had all the children stand up to be counted showing that 5 girls plus 6 boys equals 11 children all together. (Marley, Math problems...)

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Snack time is another space that teachers engaged children in counting activities. Sadie had a snack helper in her class whose job was to count out 14 plates and put 4 orange slices on each plate.

Vignette 3: I helped Clara with the first plate, working to count out 4 orange slices. She put 3 slices on the second plate, then counted the number on the first plate and the number on the second plate and asked me, "that the amount?" When I asked her what she thought, she put on more on to make 4. Then on the 4<sup>th</sup> plate she put 5 slices and asked again. I helped her take one orange slice away, using the words more, less, and take away, until she discovered the plates had the same amount. (Sadie, Clara's Snack Task)

The teachers in these vignettes used a variety of strategies to support children's early number development. In vignette 1, Betty recognizes that Bob can count using 1-to-1 correspondence to about 3 or 4 before he switches to rote counting. Her strategy was to encourage Bob to slow down so he could match a number to each child that he taps, and ultimately have an adult count with him. In vignette 2, Marley modeled a joining problem by using a practice the children regularly engaged with in the class. In vignette 3, Sadie does not directly tell Clara whether she is right or wrong but rather asks her what she thinks. Sadie then modeled how to compare sets. Both Sadie and Marley modeled strategies for children using math vocabulary.

## Math games and materials

Preparing the environment for children to engage with learning is a critical aspect of any early childhood classroom, and certainly true for mathematics learning. Yet providing materials and organizational structures that encourage interactions around math is not enough. We found in both of our studies that teachers provided varying levels of support to children as they played with math games and activities. All the classrooms had several math games both teacher-made and commercial. In addition, teachers made decisions to highlight the mathematical nature of materials. For example, Sarah organized collections of small toys, such as cars and animals, on a shelf along with a balance. By placing these objects in proximity, she encouraged her students to engage with them in a mathematical way. Vignette 4 describes the impact of Sarah's decision to organize unifix cubes in sticks of 8, rather than loose in a bucket.

Vignette 4: Clay sat down among a pile of unifix cubes that other children had been playing with and began, without prompting to organize them into sticks of 8. He began by touching each cube as he counted it to ensure he had the correct amount. After he made a few sticks of 8, he started using his previous sticks as points of comparison, holding up a new stick and then removing two cubes when he noticed that it was too long. After watching Clay for a moment, two other boys began to make sticks of 8, touching each cube as they counted.

Sarah intentionally asked the students to keep the unifix cubes in sticks of 8 (a number chosen because it was the number of spaces on their bingo cards) because she knew it would provide opportunities to practice meaningful counting whenever the cubes were cleaned up; however, she did little to deepen students engage with these cubes across the year, such as changing the goal number for the size of sticks or asking students to articulate the strategies for composing sticks from smaller groups of blocks or comparing finished lengths. Sarah, who did not receive professional development around early mathematics and who reported that she'd had little preparation around the counting core in her preparation program, was not as adept as the teachers receiving PD as recognizing mathematics in play and intervening to highlight it for children, as demonstrated in the next vignettes.

In analyzing teachers' interactions when playing games with children or observing children play the games, we were particularly interested in those examples of teachers finding alternative ways to

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ask 'how many' or go beyond asking 'how many'. The ways in which teachers did this included general comments about sets that encouraged the children to count such as "wow, that is a lot" or asking which one has more when children had more than one set of objects. In vignette 5, Betty observes John at the math center working on a Counting Book.

Vignette 5: On the first page, he wrote the numeral 4 and then stamped 4 stamps. On the second page he wrote 1-10 saying each number to himself as he wrote. I asked him, "what did you write", he told me the numbers up to 10. I asked, "how many will you stamp" and he started stamping and counted each one up to 10. On the next page he stamped 7 stamps, counted them and wrote the number 7. Then he stamped on more and stopped. I asked, "what now". He crossed out the 7 and asked me how to write 8. I showed him a picture of the numeral 8 and he tried it. (Betty, How Many Stamps?)

In this next vignette, Chela describes how Kiki used the balance scale to count and compare various objects in the room. Kiki had filled the cups with different objects from the science table, putting insect and spider counters in one and one-gram yellow weights in the other. Then using a chart that Chela had made for children to compare the number of objects in different sets, Kiki lined up the insects on one side and weights on the other.

Vignette 6: In a very precise manor, Kiki had lined up the yellow grams and commented, "wow this is lots!" At this point I prompted him to also line up the insects and spiders, which Kiki thought was a great idea. Kiki had a difficult time knowing for certain which one had more, but thought that it was the yellow grams because there were lots. I asked him how he could know which was more and he counted each. (Chela, Measurement and Balance Activity)

In Vignette 5, Betty asks the question 'how many' but does so in a way that encourages action to count out rather than how many are already in a set. She also asks John 'what now' so that he could choose between crossing out a stamp or changing the numeral. In Vignette 6, the nature of the materials Chela has in class encourage the practice of counting. By asking the child how he could find out which set had more, she encouraged him to think of counting rather than directing him to do so.

#### **Mathematizing Play**

Perhaps one of the most challenging responsibilities that early childhood teachers face with respect to supporting math learning is mathematizing the activities children engage with in free play. To do this effectively, teachers must recognize the mathematics that children are engaging with and make in-the-moment decisions to support further learning. We found examples of situations in which teachers were able to recognize and respond to math in a variety of classroom environments. In Vignette 7, Sadie observes Clara in the housekeeping area as she was making food for her friends. Clara was the mom and her friends were her babies.

Vignette 7: They all started talking about birthdays and her babies said they needed 4 'cupcakes' because they were 4 years old, or 5 'cupcakes' because they were 5. She counted 4 cupcakes for one of her babies and 5 for the other. When I was invited over to share in the birthday festivities, I was given a plate and a pile of food. I asked how many cupcakes I had and Clara counted 8 of them and stopped (there were more pieces on my plate). When I said I want 2 cupcakes because I am 2, Clara removed some pieces of food off my plate and left me with 2 and said "there, now lets sing". (Sadie, Clara's Counting Kitchen)

In Vignette 8, Sadie observed Sam using 3D shapes to build "a huge castle". Although play with shapes could be coded as mathematical materials, in this situation we are examining the interaction

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around counting and thus we coded this as play. There was considerable conversation about the different shapes Sam used for his castle. Here we pick up where Sadie's reflection shifted to a discussion of counting.

Vignette 8: While the conversation continued we talked about the various heights of his towers. He counted with 1:1 correspondence up to 4, which was the tallest tower. He noticed that the lower tower was only 2 shapes tall and compared them saying "this one is shorter than that one." We talked about how the tallest tower did not have the most shapes. (Sadie, The Castle King)

Vignette 7 provides a wonderful example of how teachers can enter children's play to support mathematical thinking. Clara was already counting and showing evidence of 1-to-1 correspondence and cardinality but Sadie was able to push on this without disrupting the play by having Clara think about how to remove enough 'cupcakes' to leave Sadie with two. Sadie could have taken this further by asking how many cupcakes Clara took off her plate. In vignette 8, Sadie joined Sam in his castle building and raised questions about the height of his towers, which encouraged him to compare heights, count, and ultimately see that more blocks did not always mean a higher tower.

Not surprisingly we found more instances when teachers recognized the opportunity to mathematize but did not do so in the moment or did not engage with mathematics at all, despite the opportunity to do so. For example, students in Sarah's classroom routinely built tall towers of blocks and argued with each other about whose was the largest, but the teachers in the room never intervened to ask the children to find out. Similarly, while mathematical games were available, teachers did not intervene to teach children the rules of the games, which would have supported mathematical thinking, but instead let them play however they wanted, which often resulted in playacting with the pieces (intellectually valuable, no doubt, but unlikely to lead to development of the skills highlighted in the number core).

## **Implications**

The vignettes provide examples of how teachers who have had PD explicitly focused on early number were able to provide activities and materials, and respond to children's engagement in these activities, materials, and play in order to support early number development. In interviews and classroom discussion about the PD, each of these teachers shared that they would not have attended to math in these ways without the PD. The purpose of the above is not to argue for the effectiveness of the particular PD but to see the possibilities of early number engagement when teachers are supported to develop the skills for these interactions. Many of our prospective teachers do not have this opportunity because the constraints of programs are such that mathematics methods courses are intended to cover a broad grade band and the early counting is often left out. Without more research in mathematics teacher education journals on the practices that support early math and the content knowledge required, our future early childhood teachers will not be able to engage in these kinds of interactions and the children in their classrooms will not interact with the math thinking that research shows is important.

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