

PRE-SERVICE SECONDARY TEACHERS LEARNING TO ENGAGE IN MATHEMATICAL PRACTICES

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The Common Core Standards require students to learn content and mathematical practices, and so teachers must have content knowledge and be able to engage in practices themselves. This raises the question of how novice teachers learn to engage in mathematical practices. I investigate pre-service secondary teacher learning of mathematical practices following participation in a mathematics content course for teachers using a pre/post design. Four participants completed think-aloud interviews solving algebra tasks. All participants increased their engagement in mathematical practices and began to engage in them in more nuanced ways. Many changes in participants' practice engagement were related to opportunities to learn in the content course around making sense of problems, justification, and attention to precision. These results have important implications for teacher preparation and research on teacher learning.

Keywords: Teacher Education-Preservice; Mathematical Knowledge for Teaching; Teacher Knowledge; Problem Solving

What mathematical understanding is necessary for high school teaching, and when and how do teachers develop it? Recent research answers this question by considering mathematical knowledge for teaching (MKT) –mathematical knowledge entailed by the profession of teaching (Ball, Thames, & Phelps, 2008). This view of professional knowledge requires considering knowledge of school mathematics, knowledge of mathematics beyond the school curriculum, knowledge of how to unpack the mathematics of the school curriculum, and pedagogical content knowledge. Implicit in the definition of MKT is the relationship between knowledge *of* mathematics and engagement *in* mathematics. One way to describe the act of engaging in mathematics is through the lens of the mathematical practices. Thus, practices and mathematical knowledge for teaching must be considered jointly when investigating teachers' mathematical preparation. In this paper, I explore pre-service teacher learning of mathematical practices.

Teacher Engagement in Mathematical Practices

Mathematical practices describe the tools needed to *do* mathematics. They include making a conjecture, justification, and attention to precision (Common Core State Standards, 2010). Because mathematical practices are a key part of the Common Core, a focus on teacher engagement in them has become a particularly salient issue for current research. Despite robust literature on teacher knowledge, teacher engagement in mathematical practices has not been explicitly incorporated into commonly used definitions of MKT. "Conceptions of teacher knowledge have seldom considered the kinds of mathematical practices that are central to teaching. For example, rarely do teachers have opportunities to learn about notions of definitions, generalization, or mathematical reasoning" (RAND, 2003, p.21). Attention to teachers' engagement in mathematical practices matters for two reasons. First, teacher engagement in practices helps demonstrate what teachers *do* with the mathematical content that they *know*. Second, the ways in which teachers engage in practices themselves may affect how they teach students to engage in practices.

I draw on the mathematical practices identified in the Common Core Standards to describe ways in which secondary novice teachers *do* mathematics. I conceptualize teacher engagement in mathematical practices as being intertwined with MKT, just as the standards of mathematical

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practice are necessarily embedded within mathematical content (McCallum, 2014). Teacher engagement in practices as mathematical problem solvers themselves is connected with subject-matter knowledge. Similarly, understanding how to teach students to engage in practices is connected to pedagogical content knowledge.

Much of the existing literature on teacher engagement in mathematical practices emphasizes proof. Overall, research on teachers and proof has endeavored to (1) understand teacher beliefs about proof and its role in the classroom (e.g., Knuth, 2002; Staples, Bartlo, & Thanheiser, 2012) and (2) investigate teacher knowledge related to the analysis of specific types of proof (e.g., G. Stylianides, A. Stylianides, & Philippou, 2007). However, there remains a gap in the literature about how teachers actually engage in proof themselves. Given the emphasis on mathematical practices in the Common Core Standards(2010), it is critical to investigate teachers' engagement in proof, in addition to their knowledge about it.

Equally important is research that investigates practices more broadly, looking beyond formal proof alone. Mathematical practices rarely occur independently of one another, making it critical to look at them in concert with each other and within a variety of mathematical content domains. Investigating teacher engagement in mathematical practices more broadly will contribute a great deal to understanding teachers' MKT and its link to mathematical practices. Just as pedagogies of enactment support novice teachers in learning to teach (Grossman et al., 2009), teacher engagement in mathematics can support student engagement in mathematics. This makes it essential to conduct research that focuses on teacher engagement in mathematical practices connected with teacher content knowledge. If we argue that teachers need to develop MKT and need to engage in mathematical practices, what then does it look like for teachers to be learning to engage in mathematical practices?

Teacher Learning

Following (Lave & Wenger, 1991), I take a situative perspective on teacher learning; that is, learning is described as a change in the way a teacher participates in a community of practice. In the context of teachers learning math content and practices, evidence of learning can come from changes in the way teachers interact with one another in the context of solving a math problem, but it can also come from changes in the way they individually reason about a math task (Cobb & Bowers, 1999). Focusing on mathematical practices in particular, learning means looking at the way teachers take on, or appropriate (Moschkovich, 2013; Rogoff, 1990), the mathematical practices and how they transform their engagement in those practices within a community of mathematics teachers. For example, a pre-service teacher might appropriate the practice of justification by utilizing more mathematically appropriate proofs or explanations (such as using examples to motivate a generalized proof, rather than using examples as proof).

In this study, I consider the following question: What did participants in an abstract algebra course for future secondary teachers learn about mathematical practices? In particular, I examine the extent to which their engagement in mathematical practices changed from the beginning of the course to the end of the course.

Methods

To answer this question, I conducted a case study of an abstract algebra course designed for future secondary teachers. The course took place at the beginning of a yearlong preparation program. I selected this site for my case study largely because of the program's commitment to the deep mathematical preparation of future teachers. A mathematician taught the course; he tailored the course to attend to the needs of the teachers he was preparing. I observed all sessions of the abstract algebra course. The course met for ten weeks for three hours each session.

Participants and Data Sources

Four pre-service teachers participated in the study. Though these participants are not a representative sample of all future secondary math teachers, their mathematical preparation is consistent with that of typical pre-service secondary math teachers. The participants each had different trajectories into teaching and as such represent different specific features of novice teachers. It is valuable to treat each participant as an individual case study. Daniel entered the teacher preparation program after a long career in engineering and business. He had an undergraduate degree in engineering. Laura was a paraprofessional for several years before pursuing her math credential. She had a math major, but expressed a lack of confidence in her math ability. Sam served in the military after high school and then got an associate's degree in engineering. He later returned to school to finish his undergraduate degree in math and then complete his credential. Tim entered the teacher preparation program immediately after completing his undergraduate degree in math and physics.

At the beginning and end of the abstract algebra course, participants completed in-depth task-based interviews. Participants were asked to think aloud as they solved (Ericsson & Simon, 1980), and they engaged in "free problem solving" (Goldin, 1997). Though the interview tasks addressed high school content, the problems were non-familiar, that is, participants were unlikely to have seen any of the particular problems before. I investigate participant learning by comparing participant engagement in mathematical practices at both time points. Because of the think aloud structure, participants' solutions were both oral and written.

Data Analysis

I coded each interview for engagement in mathematical practices, using the Common Core practice standards as a framework and dividing each standard into sub-codes. For example, during the pre-interview algebra task, Laura decided to test some specific values to make sense of a more general algebraic statement. She said, "I'm just going to put real numbers in this for a minute". I coded this as trying a special case (MP1), because it was evidence trying a particular example while solving a general problem. Table 1 shows additional mathematical practice codes.

Table 1: Sample mathematical practice codes

Code name	Code description	Example
Connect representations (MP1)	Explain correspondences between equations, verbal descriptions, tables, and graphs	Sam (pre): Your slope is going to be somewhere in the middle, and then when you add these together $[(b + d) \text{ in the equation } (f + g)(x) = (a + c)x + (b + d)]$ the intersect is going to be somewhere in the middle <i>[indicates origin of the graph]</i> .
Test conjectures (MP3)	Make conjectures and build a logical progression of statements to explore the truth of their conjectures.	Daniel (post), after conjecturing that the slope must always be -1: So I will pick a point that's at (1, 2) <i>[plots (1, 2)]</i> . That's one of the points. The other point should be at (2, 1) <i>[plots (2, 1)]</i> . Nicely, we see this slope is now going to be negative <i>[connects points (1, 2) and (2, 1)]</i> .

Using a multiple case approach (Miles & Huberman, 1994), I describe participants' individual learning as well as investigate trends across participants. I used analytic memos to create problem-solving cases for each task. I looked across the four participants at each time point to identify similarities and differences in their content knowledge and practice engagement. Finally, I looked for change over time by comparing the pre and post interviews.

Interview Tasks

The algebra tasks used during both think aloud interviews required participants to prove a statement about linear functions. This makes them excellent sites to consider participant learning around proof and justification along with other mathematical practices. Both tasks focused on high school level content, though the particular tasks themselves were unfamiliar to participants. Both dealt with linear functions, a standard part of school algebra. Both tasks also required participants to prove a general statement about linear functions was true. The focus on the pre-task was on the sum of two functions at a particular point. The focus on the post-task was on the x - and y -intercepts of a particular line. Table 2 shows the pre and post algebra tasks.

Table 2: Interview algebra tasks and key features

	Pre-Interview Algebra Task	Post-Interview Algebra Task
Key features	High school level content	High school level content
	Linear functions	Linear functions
	Prove a general statement is true	Prove a general statement is true
	Focus on the sum of two functions at a particular point	Focus on the x and y -intercepts of a particular line
Tasks	Prove the following statement: If the graphs of linear functions $f(x) = ax + b$ and $g(x) = cx + d$ intersect at a point P on the x -axis, the graph of their sum function $(f+g)(x)$ must also go through P . (TEDS-M International Study Center, 2010)	Take a point (p, q) on the Cartesian plane. Reverse the coordinates to obtain a second point (q, p) . Prove that on the line between these two points, the x -intercept and the y -intercept are the sum of the coordinates.

Findings

As a group, participants engaged in many of the mathematical practices, and the practices they engaged in were directly related to the nature of the tasks. All participants worked to make sense of the problems (MP1), create representations (MP2), construct arguments (MP3), and attend to precision (MP6); three participants engaged in making use of structure (MP7). Nobody engaged in modeling with mathematics (MP4), for instance, because the tasks did not entail mathematical modeling. All four participants showed changes in their engagement in mathematical practices, but the nature of the changes varied across participants. In this section, I explore these changes in detail. Additionally, I connect some of the observed changes in math practice engagement back to the opportunities to learn present in the abstract algebra class.

Learning to Justify and Attend to Precision: Daniel and Tim

Daniel and Tim both did very well on the pre-tasks. Daniel produced a complete and correct algebraic argument, providing his rationale aloud as he talked through his solution. He supported his argument with graphical examples (see Figure 1a). Tim produced a complete algebraic argument that was nearly correct except for an imprecise use of mathematical notation (see Figure 1b). Tim did not show substituting a point P into the equation, though that seemed to be his intention based on what he said. So his final line reads as though the result were true for any x value, rather than for a particular value of P (he wrote $(f+g)(x) = 0$, rather than $(f+g)(x_0) = 0$). Based on their performance, it seemed as though there would be little opportunity to see growth on the post task. However, both showed growth across several mathematical practices.

(1a) Daniel's work

(1b) Tim's work

Figure 1: Daniel and Tim's written pre-task work

Daniel showed growth through more substantial and meaningful connections between his algebraic and graphical representations, and through his attention to precision. He took the same basic approach to the two problems, using a mostly algebraic approach with graphical examples on both. On the post-task, Daniel made stronger connections between the two representations (MP1) than he had on the pre-task in order to overcome an initial error. On the post-task, Daniel also carefully noted which variables were free and which were fixed. This attention to precision (MP6) enhanced the rigor of his final proof, giving him a more complete and detailed argument (MP3) than his work on the pre-task had been, even though both were correct results.

Tim also showed growth in his attention to precision (MP6), along with attending to the domain of his argument (MP3) and the way in which he communicated his conclusions. In particular, he distinguished between his preliminary scratch work and how he would develop a more formal proof. These changes allowed Tim, like Daniel, to produce a rigorous and detailed proof on the post-task that showed growth over his performance in the pre-task.

Learning to Make Sense, Persevere, and Justify: Sam and Laura

Sam and Laura struggled with the pre-tasks, though in different ways. Sam attempted to use an algebraic representation, but misinterpreted the problem and was unable to construct a complete argument. Laura tried using a special case, but due to not attending to all the conditions of the problem, chose an example that led her to believe the statement she had to prove was false, even though participants were told to prove the statement was true. After conjecturing that the statement was false, Laura did not attempt to justify her conjecture in any way.

In the post-task, Sam showed tremendous growth. He was able to accurately analyze the given information, monitor his progress and develop a solution plan, and choose a generative special case (MP1). One difficulty he had with the pre-task was choosing a special case that was too specific, and obscured some of the generality of the problem. In the post-task he chose a more appropriate special case (see Figure 2). Then he was able to generalize from the special case, something he had been unable to do in the pre-task. He also compared his special case argument to his more general argument and was able to evaluate them (MP3). Sam went from being unable to complete an argument to having a full, nearly complete proof. His work on the post-interview task was limited only by not attending to the meaning of his variables.

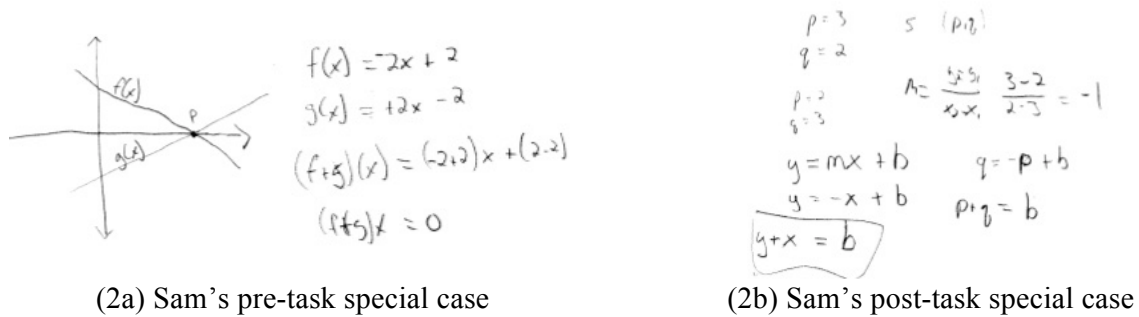


Figure 2: Sam's pre and post-task special cases

Laura too demonstrated substantial change through her work on the post-task (see Figure 3). In this case, she correctly selected a special case (MP1) and was able to construct a complete and correct argument for that special case (MP3). She did so through increased work connecting representations (MP1). She also engaged in communicating her conclusions and evaluating her arguments (MP3), two practices not visible in her pre-task interview due to her early incorrect conjecture. Though she did not attempt to generalize her special case result, Laura commented that she knew that was the next step. She showed important changes in the way in which she engaged in the practice of justification (MP3).

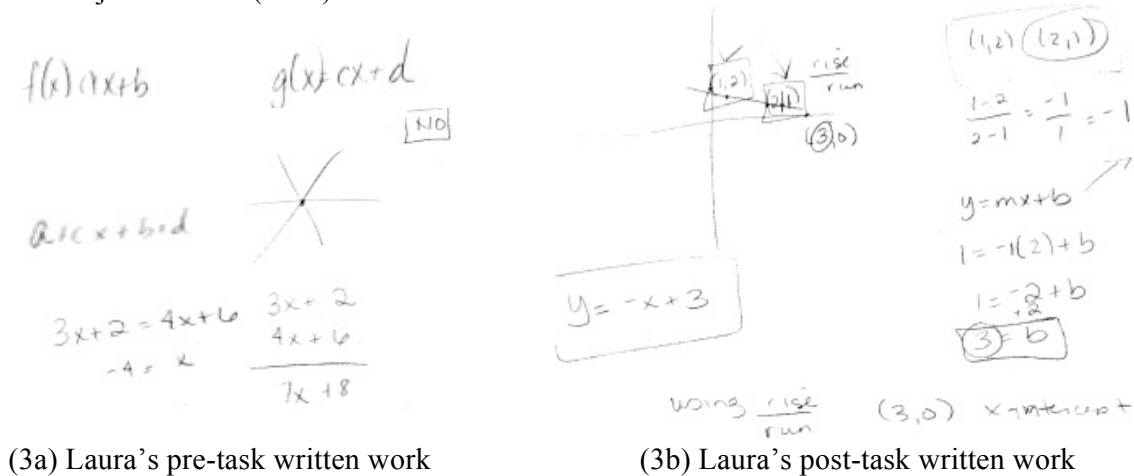


Figure 3: Laura's pre and post-task written work

Discussion: Connections to Opportunities to Learn

Overall, participants changed their engagement in mathematical practices, particularly making sense of problems (MP1), constructing arguments (MP3), and attending to precision (MP6). The results hint at the possibility of a learning trajectory around practice engagement. Laura and Sam improved in how they made sense of the problems, and this supported them to better construct reasonable arguments. Tim and Daniel demonstrated proficiency in making sense of the problems in the pre-task, but showed growth in communicating their conclusions and through their attention to precision. These particular practices were also a major focus of the abstract algebra class participants were taking (Baldinger, 2014).

Participants regularly had opportunities to make sense of problems during the abstract algebra course. For example, the professor emphasized how participants could use a special case to help them discover a more general solution. This is the approach Sam took on the post-task, and the approach Laura knew she should take. Participants also had opportunities to construct arguments and justifications. One such opportunity to learn occurred when the professor talked about

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communicating conclusions and the differences between scratch work and formal proof. This echoes the distinction Tim described in his work on the post-task. A third focus of the class was attention to precision. For example, the professor explicitly discussed the importance of defining the meaning of variables in a problem. Daniel and Tim both built on that in their work on the post-task. Participants' performance on the post-task reveals some important connections to the opportunities to learn in the abstract algebra course. This suggests the potential value of the content course as a site for learning about mathematical practices. Participants also reflected that they had learned how to engage in mathematical practices in the course (Baldinger, 2014), providing further evidence to support the idea that the improvements in performance on the post-task might be related to the learning opportunities in the abstract algebra course.

Implications and Future Directions

This case study illustrates what four pre-service secondary teachers learned around engagement in mathematical practices at the beginning of their teacher preparation. Though their learning was related to their experiences in the abstract algebra course, this is not necessarily a causal relationship; participants had numerous other learning experiences during this time. Additionally, these four participants are not representative of all pre-service teachers, and their learning trajectories are not necessarily "typical". However, each unique case provides insight into the variety of learning trajectories experienced by pre-service teachers.

There are several implications based on the results of this study. First, not surprisingly, these pre-service teachers exhibited distinctive learning trajectories for mathematical practices. In this case, despite differences in their starting places, all four participants showed changes in their engagement in mathematical practices. This suggests that teacher preparation programs can be sites for learning to engage in mathematical practices, just as the programs can be sites for developing other knowledge necessary for teaching. Additionally, such diverse learning trajectories need to be accounted for in the design of teacher preparation programs.

The abstract algebra course was clearly an important site for participants to learn to engage in mathematical practices. Incorporating more opportunities of this nature might support teacher engagement in a wider range of practices. Building on that, the practices proved portable across content levels. The participants learned to engage in these practices addressing college-level mathematics, but demonstrated their engagement on secondary-level tasks. This emphasizes the value of mathematics content courses beyond developing content knowledge.

Through engaging with multiple practices in an interconnected way, participants were more mathematically productive on the post-task than on the pre-task. Laura's post-task highlights the way engagement in communicating her conclusions (MP3) depended on her ability to connect her algebraic and graphical representations (MP1). Daniel and Tim's solutions show how attention to precision (MP6) can improve the quality of an argument. The interconnectedness suggests the value of learning about practices in conjunction with one another. The implication for teacher preparation is to provide multiple opportunities for pre-service teachers to engage in a variety of practices, rather than focusing on a single practice. Furthermore, this suggests the value of looking more holistically at practice engagement in research.

This study raises a question about how to measure engagement in mathematical practices in a way that accounts for varied learning trajectories. The interview tasks used in this study did this by focusing on accessible high school level content. However, the choice of tasks also limited the practices that might have been assessed. Additionally, the in-depth interviews conducted for this study were exceptionally illuminating but would be inefficient to implement across large teacher preparation programs. Given the importance of understanding teachers' engagement in mathematical practices, it is important to identify alternative measurement strategies.

The standards of mathematical practice in the Common Core (2010) provide a practical motivation for understanding how practices are learned and how they can be taught. It is reasonable to imagine that if teachers have not had opportunities to engage in mathematical practices themselves, it will be difficult for them to create opportunities for their students to engage in mathematical practices. Learning to engage in mathematical practices can be seen as a first step toward learning to teach others to do so, creating an imperative to more fully understand teacher learning around mathematical practices. Future research must consider other contexts for learning along with other content areas. That will help develop a more complete picture of teacher learning around practices and provide greater insight into how to structure relevant learning opportunities. Additionally, it will be valuable to consider the relationship between a teacher's ability to engage in mathematical practices and the strategies that teacher uses to support student engagement in mathematical practices. Teachers must be able to support students in all aspects of their mathematical learning, and should have opportunities to learn to do this as part of their preparation. Understanding teacher learning around mathematical practices is a crucial part of supporting teachers in their work with students.

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