

TEACHER NOTICING OF JUSTIFICATION: ATTENDING TO THE COMPLEXITY OF MATHEMATICAL CONTENT AND PRACTICE

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In this report, we will consider in-service elementary school teachers' noticing of the mathematical practice: justification. This study is part of a larger project evaluating the efficacy of a three year professional development built around attending to student thinking and promoting mathematical habits of justifying, generalizing and making sense. Noticing justification is a complex task requiring attention to both the (1) mathematical content and strategies and (2) the nature of the argument provided by a student. We have found that teachers' struggle to attend to both aspects simultaneously and offer a framework for considering teacher noticing of mathematical practices.

Keywords: Classroom Discourse; Teacher Education-Inservice; Reasoning and Proof; Standards

Reform curriculum and standards frequently treat mathematics as a dichotomous subject consisting of both content and practices (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Council of Teachers of Mathematics, 2000). Teachers are expected to foster classrooms not just based on mathematical content goals, but that also promote practices such as justification and generalization. However, practice goals often remain more mysterious. For example, many teachers lack an understanding of just what justification is and how it would look if a student was engaged in justification (Knuth, 2002; Simon & Blume, 1996). *Professional noticing* is a lens for making sense of what teachers attend to in their classrooms, how they interpret student strategies and build upon them. We consider teacher noticing of justification in the context of a professional development (PD) designed to transcend particular mathematical content and focus teachers on the mathematical practices that support and sustain students' development and learning of mathematics. We attend to teachers' noticing of justification, considering the interplay between noticing of justification practice and content-specific mathematical strategies.

Noticing

Noticing in a professional setting is both a lens for making sense of what teachers see in the complex setting of a classroom and a skill to be developed in PD for current and pre-service teachers. Much of the work on noticing stems from Mason's (2002) *intentional noticing*. Intentional or professional noticing differs from everyday noticing as to what is attended to and how it is interpreted is influenced and focused by the professional experience and knowledge of the individual. Jacobs, Lamb, and Philipp (2010) developed a framework for teachers' professional noticing of children's mathematical thinking consisting of: *attending to children's strategies*, *interpreting children's understandings*, and *deciding how to respond on the basis of children's understandings*.

We build on Jacobs, Lamb, and Philipp's work to consider not just children's mathematical thinking related to content and strategies, but also as it relates to general mathematical practices. We are considering practices to be mathematical activity that is not dependent on particular mathematical

content, but rather is embedded in all areas of mathematics such as the practices of justifying or generalizing. For this report, we will focus on the practice of justification.

Justification

Justification is an essential practice in mathematics classrooms listed in both the Common Core State Standards for Mathematics (CCSS) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000). Our usage of mathematical practice will reflect the usage found in the CCSS where mathematical practices describe ways in which students engage in the discipline of mathematics. Justification is of particular importance as it provides “a means by which students enhance their understanding of mathematics and their proficiency at doing mathematics...” (Staples, Bartlo, Thanheiser, p. 447). Justification provides a means to *both* deepen understanding of various mathematical content and develop mathematical practices.

Despite its importance, defining justification is a perilous task. Our definition will most closely follow Stylianides (2007) definition of a proof:

- It uses statements accepted by the classroom community (set of accepted statements) that are true and available without further justification;
- It employs forms of reasoning (modes of argumentation) that are valid and known to, or within the conceptual reach of, the classroom community; and
- It is communicated with forms of expression (modes of argument representation) that are appropriate or known to, or within the conceptual reach of the classroom community. (p. 291).

Within the PD, we define *justifying* as:

Reasons with meaning of ideas, definitions, math properties, established generalizations to:

- *show why an idea/solution is true*
- *refute the validity of an idea*
- *give mathematical defense of an idea that was challenged*

As in Stylianides’ definition of proof, justification derives its meaning from building on established facts in order to present a mathematical argument. However, in the PD, the emphasis switches from the product (a proof or justification) to the act of justifying. In this way, a student may be engaged in justifying even if his or her reasoning is incomplete or incorrect.

Previous work has shown that teachers and pre-service teachers may struggle to differentiate between justifications and non-justifications both when evaluating and creating their own justifications. Knuth (2002) found secondary teachers often evaluated non-proofs including empirical arguments as valid justifications for general cases. Pre-service teachers also often consider purely empirical arguments justifications for the general case (Stylianides & Stylianides, 2009; Martin & Harel, 1989). Simon & Blume (1996) illustrated that pre-service teachers provided alternate information when pressed to justify such as citing a rule for its efficiency or relying on procedures in place of a mathematical rationale. In light of these results, the noticing of student justification practices in classrooms is a challenging task. Lo and McCrory (2010), identified four factors necessary for teachers to promote justifying activity: (1) Knowing what counts as a valid justification for a given answer; (2) Familiarizing oneself with the struggles elementary school students may have; (3) Understanding how mathematical topics connect across operations and number systems; and (4) Knowing how to teach in a way that supports mathematical reasoning. (p. 150). These factors

highlight some of the complexity involved for teachers to promote justifying activity. While the PD aims to address all four factors, we will be focusing on the first factor. Knowing what counts as a justification (and recognizing justifying activity) serve as an important aim when promoting justification.

Attending to justification practices provide an extra layer of complexity to the already complex situation of making sense of students' mathematical thinking. Noticing justification requires both attention to content and practice. That is, there must be mathematical content to be justified. However, mathematical content alone is insufficient; justification also requires an argument to provide a *why* for mathematical claims and decisions. For example, consider the task in Figure 1.

[School Name Redacted]'s PTA provided 12 celery sticks, 17 carrot sticks, and 7 apple slices. If each student took 3 snack pieces, how many students would have a snack?

Figure 1: Task from Cycle 4

A non-justification for the solution of twelve students might be:

"I added 12, 17, and 7. $12+17$ is 29. $29+7$ is 36. Then 36 divided by 3 is 12. So 12 students would get snacks.

While the student is explaining *how* they arrived at the answer, they are not providing any rationale for *why* this procedure produces the correct answer. There is mathematical content in this example, but no evidence of the practice of justification.

In contrast, a justification for the solution of nine students might be:

"There are 12 of one snack, 17 of one snack and 7 of another. I could add all of the snacks together to find the total number of snacks since the types of snacks do not matter, which would be 36. Because division tells me how many groups of three are in 36, I could divide the 36 by 3. This would tell me how many groups of three snacks fit into 36 snacks. So 12 students could each have three snacks."

In this example, the student used their knowledge of addition and justified why totaling the different snacks would be appropriate. They then justified their division decision by using the known definition and connecting to the context. By connecting to a known meaning, they not only used an operation, but provided a justification for it. The mathematical content and argumentation were both aspects of the excerpt.

The Setting and PD

The PD takes place at an elementary school in a mid-sized urban district that is engaged in a three year PD for third to fifth grade teachers. The PD uses a *Studio Model* where one teacher (the studio teacher) works with a consultant to plan math lessons and then opens their classroom to the other teachers while teaching this lesson. The remainder of the third to fifth grade teachers (resident teachers) help plan the lesson, observe the enactment, and then debrief the lesson. At this elementary school, the third to fifth grade teachers engage in a yearly summer course (3 days) and five studio cycles throughout the year. These cycles include two days of PD sessions. Day one consists of leadership coaching with the principal and planning with the studio teacher. Day two involves all third to fifth teachers and consists of working together to do mathematics related to the lesson, planning and enacting the studio lesson and then debriefing the lesson. For this study, we are going to focus on the first year of the PD with attention to the lesson debriefs.

The central focus of the PD is to get all students to habitually justify and generalize in order to make sense of mathematical problems and ideas. To this end a set of *Habits of Mind* (such as working through stuck-points), *Habits of Interaction* (such as critique and debate), and *Mathematically Productive Teaching Routines* are the focus of the PD throughout the first year (Foreman, 2013).

Resident teachers' attention is focused on student discourse, in particular on discourse related to justifying and generalizing during the studio lesson. The teachers are provided with observation tools to help focus their attention (see Figure 2). The teachers are encouraged to write down discourse they observe during the lesson within the categories of: *procedures/facts*, *justifying*, and *generalizing*. After the lesson is enacted, the teachers engage in discussion around the various discourse they noticed and characterize them with respect to the three categories provided. In the discussion they are asked to justify their categorizations. Using this tool helps teachers focus/attend to children's discourse rather than other aspects of the classroom.

Teacher _____ Grade/Class _____ Date _____ Page _____

STUDENT DISCOURSE OBSERVATION TOOL		
PF USING PROCEDURES/FACTS	J JUSTIFYING	G GENERALIZING
<i>No evidence of reasoning</i> <ul style="list-style-type: none"> • Short answer to a direct question • Restating facts/statements/rules • Showing or asking for procedures <i>Uses meanings, definitions, properties, known math ideas to describe reasoning when:</i> <ul style="list-style-type: none"> • Explaining ideas & methods • Questioning to clarify • Noticing relationships/connections <i>But doesn't show why the ideas/methods work</i>	<i>Reasons with meanings of ideas, definitions, math properties, established generalizations to:</i> <ul style="list-style-type: none"> • Show why an idea/solution is true • Refute the validity of an idea • Give mathematical defense for an idea that was challenged 	<i>Reasons with math properties, definitions, meanings of ideas, established generalizations, and mathematical relationships as the basis for:</i> <ul style="list-style-type: none"> • Making conjectures about what might happen in the general or special cases or • Justifying a conjecture about what will happen in the general or special cases
Discourse Type	Student Mathematical Discourse * Indicates student thinking that I am especially curious about	Co-Inquiry Questions

Figure 2: Student Discourse Observation Tool. Reprinted from *Best Practices in Teaching Mathematics: How Math Teaching Matters* (p. 41), by L.C. Foreman, 213, West Linn, OR: Teachers Development Group. Copyright 2013 by Teachers Development Group. Reprinted with permission.

Methods

A member of the research team was present at each of the PD sessions and took detailed field notes. In addition, all PD sessions were video recorded. The field notes from year 1, provided the starting ground for identifying instances when teachers discussed student discourse and how it pertained to justifying, generalizing and procedures/facts. These episodes were then transcribed for analysis. Transcripts were analyzed for any instance of noticing during discussions of characterizing discourse. Each instance (measured as a turn in conversation) was then considered in light of whether the teacher noticed (1) the mathematical content, (2) the character of the discourse (justifying, generalizing, or procedures/facts), and (3) what evidence was provided for their interpretations and descriptions. The analysis focused on describing and interpreting aspects of noticing (rather than responding to student thinking), as the PD focused first on these aspects during year 1.

Results and Discussion

Each of the following excerpts comes from a teacher response when asked to share a piece of discourse from their discourse observation tool after the lesson. Teachers were prompted to

characterize the discourse in terms of justifying, generalizing and procedure/facts. We begin each section by sharing the teacher-selected piece of discourse and then analyzing it.

Teacher Noticing 1 (Cycle 1)

Teacher: They [a pair of students] were very much engaged with each other and happily exchanging their different ways of looking at the problem.

No mathematical content or practices. This quote exemplifies noticing that is general and does not provide evidence of student discourse or any interpretation of mathematical content or practices. This teacher was commenting on level of engagement without evidence. She did note that they had different ways of looking at the problem, but without evidence, this statement does not reflect noticing of the mathematics in terms of content or practice. Because of the PD's structure (and the observation tool's focus), teachers were nearly always focused on students rather than teachers. However, this focus often swayed to their noticing of student affect rather than mathematical discourse.

Teacher Noticing 2 (Cycle 1)

Teacher: I still think he's doing that justifying, defending the idea about that whole corner thing. I heard such awesome things, like, "How?," "I know because...," and then he's explaining, and then "I disagree, how did you get this number?" "Well, because you said this, this, this...did you know it was going to be this number" "Look. You said..." and he showed him on the paper!"

Noticing mathematical practice without content. The above noticing was of two students engaged in discussion as to whether the corners of the perimeters in Figure 3 should count once or twice in the total. The student used toothpicks to illustrate that a corner contributes two sides to the perimeter total. In this case, the teacher has noticed the practice of justification, but the evidence provided does not connect to the mathematical content. This noticing might be considered *keyword* justification noticing. While the teacher appeared to notice that a mathematical argument was being made, she did not report any student discourse that was mathematically focused. Noticing justification in this manner might reflect any number of justification conceptions and does not directly connect back to the definition of justifying being utilized in the PD. With the content of what follows, "I know because," an interpretation of justification lacks warrant. The "because" could be followed by a justification or it could be followed by a procedural explanation. Prior to attending to student mathematically thinking, it is unlikely a teacher could recognize justification beyond what might be a superficial interpretation.

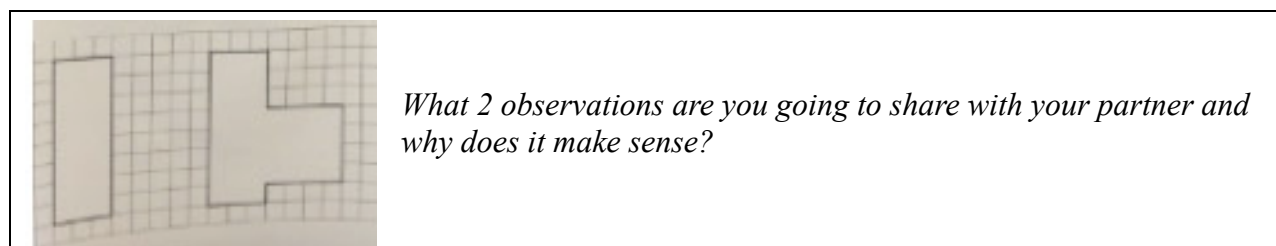


Figure 3: Task from Cycle 1

Teacher Noticing 3 (Cycle 2)

Teacher: But, then [student 1]— I really thought it was great when [student 1] on her own —cause we were just flipping it [a triangle] [motions upside down] you know what the kids would think

as right side up because we gave it to them “upside down” [air quotes] so he [student 2] thought if you flip it, it’s right side up so when she [student 1] turned it lengthwise, “well does that look like it to you?”

Noticing mathematical content without practice. In this excerpt, a teacher is noticing the mathematical content. Students were asked to identify if various shapes were triangles. Many students felt that whether or not a shape was a triangle related to orientation (see Figure 4). In this exchange, one student recognized and rotated a triangle to provide an argument that the top was in fact a triangle. The teacher noticed this exchange including interpreting that the students were focused on orientation. In this excerpt, some *evaluative* comments were made such as, “I really thought this was great.” Despite the prompt to characterize this discourse, this teacher did not consider whether there was justifying, generalizing or using facts/procedures. This was a fairly typical response type within a subset of teachers. It is unsurprising that a teacher’s focus might be solely on making sense of the students’ mathematical strategies, especially if attending to students’ mathematical thinking is a shift from their typical teaching. Interpreting the nature of the discourse and making sense of the mathematical understandings requires attending to two different (though interrelated) facets of a complex situation.

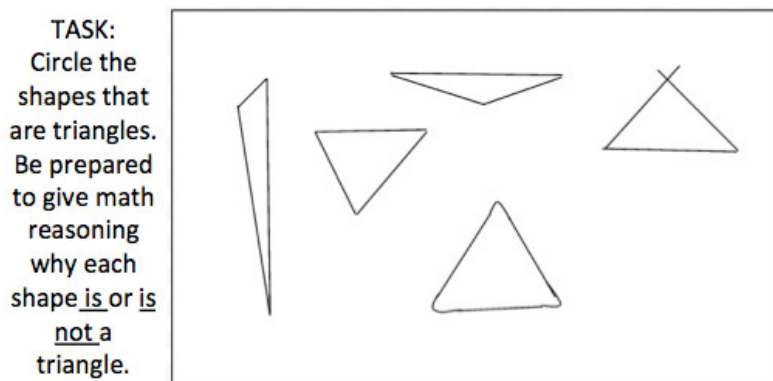


Figure 4: Task from Cycle 2

Teacher Noticing 4 (Cycle 4)

Teacher: [Student 1] was arguing with girl next to him [student 2] that she was wrong because she had the two and one remainders and the numbers added up to 11 even though [student 2] had 12 up there. They were focused on the remainders. He had 12 and knew he was right. When he talked to the whole group, it was when he said, “2 remainders plus 1 remainders equals 3” and he goes, “three divided by three equals one more group.” As soon as he said it out loud, you could see the light bulb flash and he was smiling and he told the girl next to her she’s right. The other girls said [inaudible]. He was having the whole conversation justifying it to himself.

Noticing mathematical content and practice. In this excerpt, the teacher is describing a debate between students based on the remainder from the prompt in Figure 1¹. The teacher in this case provided specific evidence of the mathematical discourse that included both content and interpreting the character of the discourse. While the connection between the mathematical content and characterizing of practice was tenuous, this excerpt represents one of the only cases of a teacher identifying a justification with concrete evidence. This might reflect both the complexity involved in attending to both aspects, but also the fact that this data is from the first year of the PD

Bartell, T. G., Bieda, K. N., Putnam, R. T., Bradfield, K., & Dominguez, H. (Eds.). (2015). *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. East Lansing, MI: Michigan State University.

implementation. At this point, the teachers are in the beginning phases of attending to student reasoning in a meaningful way.

A Framework for Noticing Mathematical Content and Practices

We summarize the four different ways teachers notice justification in Table 1.

Table 1: Framework for Noticing Mathematical Content and Practices

	Does not notice mathematical practices	Notices mathematical practices
Does not notice mathematical content and strategies	Statements are general in nature and contain no evidence of students' mathematics.	Statements may provide evidence of students engaged in a practice but no mathematics content is included.
Notices mathematical content and strategies	Statements include evidence of students' mathematical strategies and content-specific interpretations.	Statements include evidence and interpretation of both content and practices.

Attending to and promoting student justification may require a high level skill-set including both a knowledge of the nature of mathematical justification paired with being able to notice and make sense of students' mathematical thinking and strategies around content. The characterizing student discourse tool provides both a tool for writing down discourse (which focuses teachers on students), and provides a stable definition for the discourse types of: *generalizing*, *justifying* and *using procedures/facts*. As teachers learn to notice their justifications, their conceptions of justifications should continue to develop. At the same time, characterizing discourse (with evidence) necessarily requires understanding of students' mathematical thinking around given content. In this way, characterizing discourse is a way to promote attending to students' mathematical thinking and making sense of their reasoning.

Our analysis of Year 1 data, provided insight into the current state of teacher noticing of practices such as justification and generalizing. Noticing justification is an incredibly complex skill requiring both a deep understanding of elementary mathematics content and understanding of justification as a mathematical practice. The teachers in our study frequently attended to one or the other, but rarely created robust interpretations of student discourse that addressed both the character of the discourse and the mathematical content understandings. In fact, there were no examples of attending to both during the thirty minute debrief discussions in the first three cycles.

Based on this analysis, we were able to illustrate examples of what teachers notice when told to focus on characterizing discourse in terms of justifying, generalizing, and procedures/facts. Prior to extensive professional development, teachers were not able to characterize discourse using evidence. Further, we argue that noticing practices is a difficult and complex task due to the requirement to notice not just mathematical practices cues, but also notice mathematical content. Finally, we have introduced a framework that has helped to organize and make sense of the way teachers were noticing to student discourse. We conjecture that teachers will continue to shift from noticing only certain facets of student discourse to an integrated view of mathematical content and practice through Year 2 and Year 3 of the professional development.

Endnote

¹The division problem could be solved in one of two ways. The snacks could be first summed to arrive at the total number of snacks, and then divided by three. Alternately, each snack type could be divided by three first, leaving remainders of two carrot sticks and one apple slice.

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