USING WRITING TO ENCOURAGE PSMTS' REFLECTIONS ON AMBIGUITY IN MATHEMATICAL LANGUAGE

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Literature suggests that the mathematical language of teachers impacts a student's understanding of math concepts. When teachers unintentionally use ambiguous language, students' understanding of a subject can be negatively affected. We share background on specific instances in which teachers can create confusion with the language they use, and we investigate both pre-service teachers' and college algebra students' concepts of three common terms in mathematics: Solve, Evaluate, and Simplify by asking both groups to unpack their understanding of these terms through a writing prompt. We compare the language used by both groups in their definitions. Preservice teachers' reflections on their experience with the writing prompt are also examined to identify ways that such a task can help them identify gaps in their own understanding and in their thinking about student learning.

Keywords: Teacher Education-Preservice, Classroom Discourse, Mathematical Knowledge for Teaching

Introduction

By the time students have entered a college mathematics class, we might assume that they have developed a clear understanding of some basic terminology. Similarly, as teachers we might expect that we also have a clear definition in our minds for the words we use in our academic language everyday. However, when pressed to really examine our understanding of certain terms, it is possible that several gaps in understanding may come to light.

Teaching requires a sensitivity to the need for precision in mathematics (Ball & Bass, 2003). Mathematical terms are perhaps more precisely defined than those in many other disciplines (Barwell, Leung, Morgan, & Street, 2005), and ambiguities can only be accepted when there are shared experiences and assumptions across a community of learners (Jamison, 2007). However, we know that simply reading or hearing a precise definition of a mathematical term does not guarantee that a learner will attribute the same given meaning to the term. Meaning-making is dependent on an individual's construction experiences surrounding such mathematical expressions (Brown, 1997). It is important for teachers to consider ways in which they may unintentionally influence students' learning through their own ambiguous use of terminology and academic language in mathematics. Even simple vocabulary can have a large impact on students' understanding (Boulet, 2007; Gay, 2008).

In this study, we discuss the use of a tool for reflection called *writing to learn mathematics* (WTLM) to help preservice teachers not only examine their own understanding of certain vocabulary terms but also investigate how college algebra students understand them. Our study is housed within a unique college teaching seminar that supports the development of preservice secondary mathematics teachers' (PSMTs) knowledge about both mathematics and pedagogy as they work as the instructor of a college algebra course. We are interested in how both PSMTs and college algebra learners define words that they have been using in mathematics for many years. Specifically, we examine the following three questions in this study: (a) How do college students (specifically college algebra students and PSMTs) define the common terms, *solve, simplify, and evaluate* in mathematics? (b) What are PSMTs' reactions when required to interrogate their own understanding of these common mathematical terms? (c) What can PSMTs learn from reading students definitions of the terms?

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Background

Ambiguity in Mathematical Language

It is a common misconception in and out of the classroom that mathematics is a subject composed of "arcane rules for manipulating bizarre symbols something far removed from speech and writing" (Jamison, 2007, p. 45). This misconception places mathematics in a negative light. The meaning of mathematics almost literally gets lost in translation as vocabulary and terminology seem to take a backseat to the repetition of procedure and blindly following steps to solve a problem (Boulet, 2007). Jamison (2007) provides one suggestion for why students may struggle in mathematics classes:

> Ordinary speech is full of ambiguities, innuendoes, hidden agendas, and unspoken cultural assumptions. Paradoxically, the very clarity and lack of ambiguity in mathematics is actually a stumbling block for the neophyte. Being conditioned to resolving ambiguities in ordinary speech, many students are constantly searching for the hidden assumptions in mathematical assertions. But there are none, so inevitably they end up changing the stated meaning–and creating a misunderstanding. (p. 47)

The preciseness of math terminology juxtaposed with the implicit, sometimes vague definitions contained in natural language, may prevent students from developing adequate meanings; they are constantly switching between math and every day speaking. Problems may arise especially when there are shared meanings with everyday words, or when vocabulary used in natural language has a very different meaning in the mathematical context (Rubenstein, 2007). Table 1 shows some examples of potential vocabulary problems that teachers may overlook when trying to understand students' conceptual difficulties.

Table 1: Vocabulary Issues (adapted from Thompson & Rubenstein, 2000, p. 569)

Category of Potential Pitfall	Examples
Some words are shared by mathematics and everyday English, but they have distinct meanings.	*number*: prime, power, factor *algebra:* origin, function, domain, radical, imaginary *geometry*: volume, leg, right *statistics/probability*: mode, event, combination
Some mathematical words are related, but students confuse their distinct meanings.	*number:* factor and multiple, hundreds and hundredths *algebra:* equation and expression, solve and simplify *geometry:* theorem and theory *statistics/probability:* dependent and independent events

Students and teachers need to know the meaning of math vocabulary words and terminology in order to communicate in the classroom (Gay, 2008, NCTM, 2000, Thompson & Rubenstein, 2000). More specifically, it is important for teachers and their students to have the *same* (or at least similar) understanding of the words being used to convey ideas, objects, and actions. There is much literature devoted to the development of mathematical vocabulary, particularly in the context of students studying a field of mathematics (e.g., geometry, algebra, statistics) for the first time, or for students learning mathematics in a second language. However, by the time students are in college, we expect them already to have learned the basic vocabulary associated with a topic such as college algebra. In this study, the focus is not on teaching vocabulary, but on probing existing understanding of common vocabulary terms in mathematics.

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It is challenging for beginning teachers with little classroom experience to skillfully incorporate academic language for productive discourse. Gay (2008) shares examples of witnessing such challenges when preservice teachers ask students to do such things as “Graph this expression” and “Evaluate 6^3, 12^4, and n^4 if n=3” (p. 218). She suggests the importance of helping pre-service teachers be aware of the impact of their use of vocabulary on their students’ learning and understanding and shares strategies she has used with preservice teachers to help them build their understanding of mathematics terms, such as graphic organizers, concept circles, and the use of analogies. Below, we discuss the use of writing in mathematics as an effective tool for unpacking one’s understanding of the language used in mathematics.

Addressing the Problem

Nathan and Petrosino (2003) posit that, “discursive and reflective methods that are already commonplace in professional development and teacher education programs can serve as the basis for interventions aimed at aligning teachers’ views with accurate models of student reasoning and development” (p. 924). *Writing to Learn Mathematics* (WTLM) is one such method that incorporates writing prompts into content and methods courses to support the understanding and teaching of concepts and procedures. Through writing, learners (including PSMTs) can engage with mathematical content in new ways (Author and colleagues). In alignment with WTLM, careful reflection on written work can influence perspectives on teaching and learning in mathematics. Researchers suggest that giving learners opportunities to write in the content domain can play a major role in helping them to develop their voice in that domain (Kaplan, Fisher, Rogness, 2009; Thompson & Rubenstein, 2000). And can play a significant role in advancing and assessing learning (Inoue & Buczynski, 2011; Miller, 1992). Incorporating writing in the mathematics classroom offers multiple benefits to PSMTs committed to understanding the diverse ways in which students learn, discover, and create.

Research on WTLM focuses heavily on benefits it provides to the students doing the writing (Inoue & Buczynski, 2011). Investigations have also focused on the benefits that teachers can gain from reading their students’ writings in mathematics (Adu-Gyamfi, Bosse, Faulconer, 2010; Miller, 1992, Quinn & Wilson, 1997). In this study, we take this further to look at what teachers can learn from engaging in the writing prompts themselves before giving them to students.

Theoretical Perspectives

We believe that well-developed subject- matter knowledge is critical for effective teacher preparation. However, as PSMTs develop more expertise in their field, they can easily forget what novice students find easy and difficult to learn in mathematics (Nathan & Petrosino, 2003). One reason to explicitly teach WTLM stems from the existence of such *expert blind spots* (Nathan & Petrosino, 2003), where “teachers’ subject-matter expertise often overshadows their pedagogical knowledge about how their novice students learn and develop intellectually in the domain of interest” (p. 906). PSMTs need opportunities to engage with mathematics in ways that interrogate and reframe their current understandings and to perturb their basic idea of “knowing” mathematics. WTLM provides a tool for both expanding content knowledge and assessing students’ understandings of mathematics (Miller, 1992).

Reflective practice is one means of supporting WTLM’s incorporation in the classroom (Quinn & Wilson, 1997). It allows teachers to consider the implementation of novel practices in the classroom (Foss, 2010). When enacted as “a deliberate way of thinking leading to change in action,” (Shoffner, 2008) reflection allows PSMTs to develop and refine the knowledge needed to guide their teaching (Spalding & Wilson, 2002). Thus, reflection can support PSMTs’ understanding of teaching mathematics through consideration of prior understandings, past experiences, and current beliefs (Stockero, 2008).

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Working within a constructivist framework where learning is viewed as a process of transformation or modification of existing ideas, we see WTLM and reflection as powerful tools for perturbing PSMTs' existing ways of thinking in mathematics. They provide a way from them to interrogate understanding, enhance their thinking, and reflect on ways in which they and their students understand mathematics.

Methods

The participants in the study were senior PSMTs enrolled in a unique mathematics seminar at a university in the United States where they receive course credit to teach College Algebra. As course instructors, they were fully responsible for teaching the class to 20-30 students three days a week in 50-minute class sessions. They also had the added requirement of attending a seminar (taught by the first author) each class day to discuss pedagogical issues from the day and the mathematics that they would teach next. The College Algebra course was overseen by a course coordinator who designed the syllabus, pacing guide, and common exam and online homework sets. The added responsibilities for the undergraduate teachers were writing lesson plans, creating and grading their own quizzes twice a week, and proctoring exams.

Over the course of a semester, PSMTs were given WTLM prompts related to college algebra topics. In this study, we focus on the first prompt given, which dealt with mathematical vocabulary. We chose to address understanding of the common terms solve, simplify, and evaluate. The prompt given to the PSMTs was stated: **Explain the difference between the directions "Solve, Evaluate, and Simplify" in math problems. Then write an example using each with the expression 3(x+2)-x.** This question comes from a set of writing questions at the end of a chapter in Sullivan and Sullivan's (2004) algebra book. We chose it because of the target courses' focus on algebra and because confusion amongst these terms is mentioned in the literature (Thompson & Rubenstein, 2000).

After writing a response to the prompt in class, PSMTs engaged in collaborative reflection through asynchronous web discussions on a course wiki (c.f., Shoffner, 2008). They were asked to post their prompt response and post a reflection on the experience of answering the prompt, and were required to read and respond to other posts from their fellow PSMTs. They next created a quiz for their college algebra students (referenced below as "students") that included the same WTLM prompt, and posted another reflection on the WIKI to discuss that they had learned from reading their students' responses.

We collected prompt answers and reflections from 31 PSMTs over four semesters. These serve as our primary data source. In Fall 2014, we also collected prompt responses from 185 college algebra students. All data have been analyzed using content analysis to identify patterns in responses (Creswell, 2007) to address the research questions above. Our analysis included an examination of the following: (a) common language used to define the terms; (b) common misinterpretations of the terms; and (c) common themes in the PSMTs' reflections.

Findings

Definitions of Solve, Simplify and Evaluate

We examined the glossaries of several algebra textbooks to see examples of some formal definitions of the terms solve, simplify and evaluate. One McGraw Hill online algebra text gave the following three definitions: *evaluate* means to find the value of an expression; *solve* is the process of finding all values of the variable that make an equation a true statement; and *simplify* means to write an expression in simplest form. In comparison, we found that both the PSMTs and college algebra students used similar language in some of their definitions, but provided many more specific ideas from their experiences with these terms. Additionally, many of the college algebra students (and some PSMTs) struggled significantly with defining evaluate.

Table 2 shows a count of the number of acceptable vs. unacceptable/incorrect definitions.

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Table 2: Response Results

	Unacceptable Definition			Acceptable Definition			No Definition Given		
	Solve	Eval	Simp	Solve	Eval	Simp	Solve	Eval	Simp
Students (N=185)	16	133	15	169	30	168	0	22	3
PSMTs (N=31)	1	5	0	30	25	31	0	1	0

Solve was not a difficult term for the PSMTs or students to define. Over 90% of both groups provided an acceptable definition – some formal, some very informal. Table 3 shows the most common language used across both the PSMTs and students. For example, a majority of both groups mentioned that solve involves either finding a value of the given variable (similar to the textbook definition above) or finding an answer/solution to the problem (without specific mention of variables). A small percentage (3.9%) of the students felt that solve and simplify were actually the same thing, and another 7.8% defined solve as producing an "exact" answer – neither of these ideas was found with the PSMTs. Both groups made use of the language of "isolating" the variable or "getting the variable on one side" in their definitions.

Table 3: Results for Solve

SOLVE	Same as Simplify	Find value of variable	Find an answer/solution	Isolate variable	Exact
Students (N=185)	7 (3.9%)	65 (35.7%)	67 (36.8%)	9 (5%)	14 (7.8%)
PSMTs (N=31)	0	12 (38.7%)	8 (25.8%)	7 (22.6%)	0

Simplify was also relatively easy for the both groups to define, with 91% and 100% providing an acceptable response. Interestingly, in their reflections, several PSMTs mentioned this term as the one that gave them the most difficulty. This was due to the fact that they did not know how to define it without using the word "simplify" in their definition. As seen in Table 4, we did find that the most common language used by the college algebra students was to get something into the simplest terms or simplest form, which matches the textbook definition above. However, we also found that 14.6% of the students and 9.7% of PSMTs referred to getting an "equation" in simplest form, rather than an "expression." It is possible that this could be partially explained by how the responders may have been thinking about the purpose the simplification - some members of both groups of learners made reference to a need to simplify first before solving an equation. Thus, their reference to an equation makes sense for them in this context

The most common language in the PSMTs definitions for simplify involved "combining like terms." They were also more likely to mention specific strategies for simplifying such as "factor" or "cancel," while a larger percentage of students used the language of "breaking it down" to describe what was happening. Both groups had members who referred to the idea of "reducing" in their definitions in some way.

Table 4: Results for Simplify

SIMPLIFY	Combine like terms	Reduce	Break it down	Simplest terms/form	Involves an equation	Factor	Cancel

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Students (N=182)	13 (7%)	28 (15.1%)	28 (15.1%)	85 (46%)	27 (14.6%)	20 (10.8%)	1 (0.5%)
PSMTs (N=31)	18 (58.1%)	8 (25.8%)	1 (3.2%)	13 (41.9%)	3 (9.7%)	12 (38.7%)	9 (29%)

Evaluate was the most difficult for both the PSMTs and the college algebra students. Only 16.2% of students provided an acceptable definition of this term, and while 83% of the PSMTs were able to come up with a suitable definition, their reflections after answering indicated that most of them struggled to do so in the task. When members of both groups did have an acceptable definition, it was common to find the language of "plugging in" in their answer. The most common incorrect responses among the students was that evaluate meant the same thing as solve (28.8%) or simplify (10.4%) (see Table 5). Only a small percent (3.7%) used the language similar to the textbook definition above, though all of these students said that evaluate meant to "find a value of the equation" (while eight out of the nine PSMTS who used this language said "find a value of the expression). What we found most interesting where the definitions that attributed a non-mathematical definition of evaluate to this term. Some examples include: "To evaluate an equation is to determine how you are going to solve the given equation;" "evaluate means tell what kind of problem it is"; and "evaluate is to analyze a given expression in order to find further information" (the latter was from the one PSMT who answered with a non-math definition).

Table 5: Results for Evaluate

EVALUATE	Same as Solve	Same as Simplify	Find the value	Non-math definition	Plug in
Students (N=163)	47 (28.8%)	17 (10.4%)	6 (3.7%)	16 (9.8%)	12 (7.4%)
PSMTs (N=30)	2 (6.7%)	0	9 (30%)	1 (3.2%)	10

PSMTs' Reflections After Answering the Prompt and Reading Student's Responses

We next analyzed the PSMTs wiki reflection posts to identify common themes in the ideas that they posted and discussed asynchronously with each other. The left column in Table 6 includes general forms of the most common themes found across the PSMTs' reflections after they answered the writing prompt or after they read their students' answers. These themes are paraphrased from PSMTs' writings, while the quotes in the right column are one example of a direct quote from a teacher that fits the theme. Each one here appeared in some form in more than five different PSMTs' reflections and was therefore identified in our coding as an area of interest.

Table 6: Common Remarks from PSMTs' Reflections

	Theme	**Example Quote**
Reflections on mathematical	I had a hard time differentiating among the three words	"At first, I really had no idea what the differences were between the terms."
	I had a hard time putting my answer into written words	"Once I started writing my ideas down, I felt stupid. I couldn't find the words to describe what the process was for each one."

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understanding	I take things like this for granted	I think we are so conditioned to know what to look for we don't look at the directions unless we are confused... If [it says] 3(x+2)-x=0, we'd solve."
Reflections on pedagogy	This activity caused me to think about what I do or my own experiences	"I never realized how much informal language I use, and how I often find myself neglecting the more formal language of mathematics."
	I need to change what I'm doing in class	"I am going to be more strategic with the words I use in class to be sure that my students fully understand and that I am truly meaning what I am saying."
Reflections on student thinking	I can see why students struggle in mathematics	"Grading my quizzes today, I realized that a lot of students tried to factor when the problem was to expand or expanded when they were told to factor. I think if I had not done this prompt before grading, I might have been a little more judgmental. I probably would have thought: Can't these kids read directions?"
	If we change what we're doing, it will be easier for students to learn	"I think that by using the term evaluate, we can get our students to think about a problem more deeply."
	I learned a lot about the different ways that students think about mathematics	"When reading what the students write, I get to see an alternative way of thinking about a problem or a word, which helps me become a better educator."

As seen in the table, the PSMTs' reflections can be grouped into three main categories: reflections on their own mathematical understanding surrounding mathematical language/vocabulary; reflections on their own pedagogy; and reflections on their students' thinking and learning. The most common statement in the reflections (16 out of 31 PSMTs) was that struggling to define these terms made them feel like they were being placed in their students' shoes and that they could empathize with students' struggles. The second most common reflection focused on PSMTs' awareness that they have become too comfortable with terms in mathematics and the realization that they do not tend to pay attention to them in problem solving– instead they felt they usually let the mathematical symbols "tell" them what to do.

Conclusions

Teachers impact students in some obvious ways, but the focus for us is how they unintentionally influence students through ambiguous terminology and practices. Given the emphasis for a focus on developing student conceptual understanding and sense making in mathematics (NCTM, 2000), there is a critical need to understand and support effective practices that PSMTs can engage in to overcome their blind spots and enhance their content and pedagogical content knowledge in secondary mathematics. Our findings suggest that the use of writing prompts that unpack and encourage reflection on existing understandings of common mathematical terms can serve as one such effective practice to use in teacher preparation.

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