CONNECTING MULTIPLE MATHEMATICAL KNOWLEDGE BASES: PROSPECTIVE TEACHERS' CONCEPT MAPS OF ASSESSING CHILDREN'S UNDERSTANDING OF FRACTIONS

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This study investigates how 20 prospective elementary teachers make connections among children's multiple mathematical knowledge bases in their thinking about assessing children's understanding of fractions. The researcher facilitated concept-mapping tasks to examine the ways the prospective teachers linked concepts related to children's lives and experiences and children's mathematical thinking. This paper focuses on high-level tasks as a potential entry point to build stronger connections between assessing children's understanding of mathematics and children's multiple mathematical knowledge bases in teacher education. Additionally, I discuss implications for teacher educators and considerations for further research.

Keywords: Teacher Education-Preservice; Teacher Knowledge; Equity and Diversity

A role of teacher education programs is to provide support for prospective teachers to develop professional skills that are specific to and required for teaching. Mathematics teacher educators may provide opportunities for prospective teachers to recognize and validate children's many ways of knowing mathematics, which is especially powerful for addressing the needs of students who are traditionally marginalized in the mathematics classroom. Mathematics education researchers emphasize the importance of eliciting and building on children's mathematical thinking in teaching mathematics (e.g. Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Jacobs, Lamb, & Philipp, 2010). Supplementing this focus on children's thinking, more recent research calls for incorporating children's home and community-based mathematical *funds of knowledge* in mathematics teaching to support student learning of mathematics (e.g. Aguirre et al., 2012; Turner et al., 2012). When teachers consider children's funds of knowledge, they ultimately have more resources to draw upon and inform their teaching practice to design meaningful classroom experiences that incorporate their students' knowledge and experiences outside the mathematics classroom. There is limited research, however, on how prospective teachers connect the role of assessing children's understanding of mathematics to incorporating children's funds of knowledge in their mathematics teaching. The purpose of my study is to better understand in what ways prospective teachers make this connection in the context of an elementary mathematics methods course. In this paper, I address the following research question: In what ways do prospective elementary teachers link concepts related to children's multiple mathematical knowledge bases to assessing children's understanding of fractions?

Theoretical Framework

Aguirre and colleagues (2012) defined children's *multiple mathematical knowledge bases* as children's mathematical thinking and children's community, linguistic, and cultural funds of knowledge. By building stronger connections among children's multiple mathematical knowledge bases, teachers may have opportunities to learn from their own practice while making intentional instructional decisions that support their students' learning in meaningful ways by connecting students' funds of knowledge to their mathematics classroom experiences. In this study, I am interested in the connections that prospective teachers make between children's multiple mathematical knowledge bases and their thinking about assessing children's understandings as part of their professional practice.

Given the intricacies of teaching practice, I draw on concept mapping as a tool that provides prospective teachers with a learning opportunity to create a representation of their thinking about relationships between concepts related to teaching practice. The research literature on concept mapping in teacher education provides evidence that its use as a research tool is valid and robust. Scholars have used concept map artifacts to examine teacher knowledge about mathematics content (e.g. Williams, 1998; Hough, O'Rode, Terman, & Weissglass, 2007) and knowledge about teaching skills (e.g. Beyerbach & Smith, 1990; Koc, 2012).Other researchers studied the validity of using concept maps as a research tool in education. For example, Miller et al. (2009) examined the capability of concept maps as a research tool by studying pre- and post- concept maps by 251 prospective and practicing teachers. The authors used a concept map scoring method (Novak & Gowin, 1984) and found that participants' concept map scores distinguished expert to novice levels in conceptual understanding and growth over time. My research question focuses on how prospective teachers make connections among concepts related to children's multiple mathematical knowledge bases, and I am using concept maps to gather evidence of these connections by examining how concepts in their maps are linked together. I based my research design on work by Hough and colleagues (2007) who partially used qualitative content analysis to compare teachers' pre-maps and post-maps from beginning to end of a professional development program.

Method

I conducted this study within the context of a larger research project that produced modules designed to teach prospective elementary teachers to make stronger connections in mathematics lessons between children's mathematical thinking and children's lives and experiences. In my research, I collected data from20 prospective teachers enrolled in a15-week elementary mathematics methods course using activities from these modules. This paper reports data from and analysis of individual concept maps constructed in the last week of the course and individual reflections on the connections within the concept maps. All prospective teachers created these artifacts during their regular class meeting time and location; however, the methods course instructor was not present during these activities. All names in this paper are pseudonyms.

Data Collection

During the last class meeting of the semester, I explained concept maps and the concept map activity to the whole group. In this activity, I prompted prospective teachers to create a concept map to represent their knowledge about *assessing children's understanding of fractions*. I selected "fractions" as a specific mathematics topic to consider because it was a major topic in the mathematics methods course. I explicitly instructed prospective teachers to think about this concept map activity with a network structure, rather than a hierarchical structure with downward flow, to encourage making connections among any and all related concepts on their maps. I emphasized connections among concepts because I wanted prospective teachers to focus on indicating links between concepts to answer my research question.

First, prospective teachers individually created a concept map on a legal-sized sheet of paper with the option to draw the map entirely by hand or to write concepts on provided sticky notes to place on the sheet of paper. I did not provide prospective teachers with any initial concepts to consider, but I did provide black pens for this part of the activity. Next, I asked prospective teachers to have a focused discussion about evidence of connections among children's multiple mathematical knowledge bases in their concept maps. These discussions occurred in groups of four people. After a few minutes, I interrupted small group discussions to direct attention to specific concepts related to

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children's multiple mathematical knowledge bases that I provided on a sheet of paper: Children's mathematical thinking; Problem solving strategies; Making sense of students' mathematical ideas; Students' personal experiences; Students' interests and activities; Students' home and community knowledge bases (e.g. regular routines, places in community); and Funds of knowledge (e.g. cultural, community, and linguistic resources).

I asked each group to review the list and determine if any of these concepts connected to their concept maps. I also asked each group to discuss how they would change their concept maps to include any of the provided concepts but explicitly instructed them not to change their maps during the discussion. While prospective teachers had discussions in their groups, I collected all black pens and distributed red pens. I designed this activity in two parts with different colored pens to distinguish and collect data for both parts (before and after focused group discussion). After discussion, prospective teachers individually revisited their concept map to make any changes with red pens. Finally, prospective teachers individually wrote a brief reflection about evidence of children's multiple mathematical knowledge bases in their map.

Data Analysis

I used content analysis techniques to examine links and concepts within prospective teachers' concept maps by specifically looking for evidence of children's multiple mathematical knowledge bases in their maps. I first coded all concepts in the maps using three initial categories: Mathematical Concepts, Assessment, and Children's Multiple Mathematical Knowledge Bases. I created these a priori categories based on my research question and concept map prompt. I started analysis by coding five concept maps at a time with the initial coding scheme and identifying any concepts that did not seem to fit well in any of the categories. Through multiple rounds of this iterative process, I refined my coding scheme to include emergent categories based on patterns from my analysis. Table 1 contains my final coding scheme, definitions, and examples from concept maps.

Table 1: Final coding scheme with examples

Category	**Definition**	**Notes**	**Examples**
Mathematical Concepts	Includes all concepts related to fractions (types and parts of fractions, definitions) or number & operations more broadly	Same as a priori category	Numerator; Reciprocal; Common denominator
Representations and Tools	Includes representations such as number lines, manipulatives, and examples of fractions	Emerged from iterative coding process	Number lines; Pie charts; ½
Teaching Practices	Includes examples of and concepts related to assessment (design and types), activities, tasks, and instructional planning	Broader category of a priori *Assessment* category	High level tasks; Formal assessment; Differentiation
Children's Mathematical Thinking	Includes examples of and concepts related to students' prior knowledge, solution strategies, specific common understandings and misconceptions	*Children's Multiple Mathematical Knowledge Bases*	Problem solving strategies; Misconceptions; Seeing students' thinking
Children's Lives and Experiences	Includes examples and concepts related to children's funds of knowledge: linguistic, community, home, and cultural knowledge		Funds of knowledge; Relate to students' interests; Culturally relevant

Mathematical Concepts remained a category, Representations and Tools emerged as a new category, and I broadened the a priori Assessment category to include all concepts related to Teaching Practices. I decided to split the Children's Multiple Mathematical Knowledge Bases category to make a distinction between *Children's Mathematical Thinking* and *Children's Lives and*

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Experiences because I wanted to compare how prospective teachers represented both concepts in their maps. Finally, I used the prospective teachers' brief reflections as another source of data to examine how prospective teachers saw evidence of children's multiple mathematical knowledge bases in their concept maps.

Results

Across the group, nearly half (48.89%) of all links in the maps connected concepts related to children's multiple mathematical knowledge bases to concepts I coded as Teaching Practices. In particular, seven prospective teachers made a direct link between children's multiple mathematical knowledge bases and *high-level tasks*. A key finding from my analysis suggests that high-level tasks may be a possible entry point to strengthen connections between children's multiple mathematical knowledge bases and assessing children's understanding of mathematics. In this section, I will highlight three examples of how prospective teachers made these connections with particular attention to high-level tasks in their concept maps and reflections.

As shown in Figure 1, Avery has *high-level task* as a concept directly connected to *assessing children's understanding of fractions* with a cluster of concepts also connected to it. After editing the map, Avery added *problem-solving strategies* to this cluster of concepts, which I coded as a concept related to children's mathematical thinking. Avery also added *connect to students' experiences*, *use students' names*, and *connect appropriate funds of knowledge* to this cluster, which I coded as concepts related to children's lives and experiences.

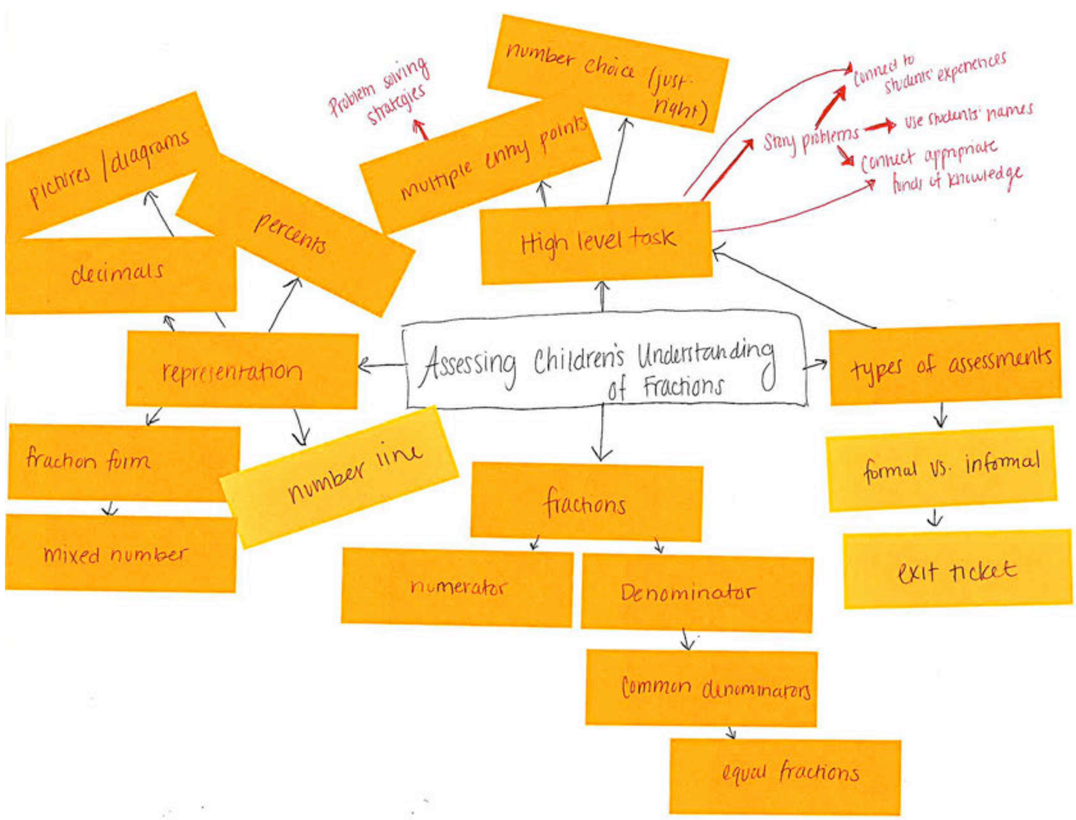

Figure 1. Avery's end of semester concept map.

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In the end of semester reflection, Avery explicitly wrote about seeing evidence of connecting children's multiple mathematical knowledge bases to assessing children's mathematical understanding by using high-level tasks:

> *Avery*: I made the biggest connection between developing *high-level tasks*[emphasis added]with relating students' experiences and funds of knowledge. When assessing students and how they think we need to make sure that all students relate to the problem and can understand the context or background of a problem. The material needs to be relevant to every child so that they can one day use their knowledge in the real world. Even something as small as changing the names in a story problem will increase student interest and motivation.

Avery points to a connection between creating high-level tasks to assess students' mathematical understanding and using relevant information about students' funds of knowledge. Avery also notes that it is important to acknowledge students' relationship to mathematical problems, including the problem context and background. Part of this relationship may be related to student motivation, but Avery focuses on the potential utility of mathematics in students' lives outside of the mathematics classroom.

Similarly, other prospective teachers indicated evidence of children's multiple mathematical knowledge bases by connecting these concepts to high-level tasks. Figure 2 shows Morgan's end of semester concept map with these connections. In this map, *utilize students' funds of knowledge* is the lead concept of an added cluster directly connected to *assessing children's understanding of fractions, fraction vocab*, *number talks*, *exit tickets*, *high-level tasks*, and *allow multiple representations*. Additionally, Morgan added a direct link between *high-level tasks* and *allow multiple representations* in the edited map.

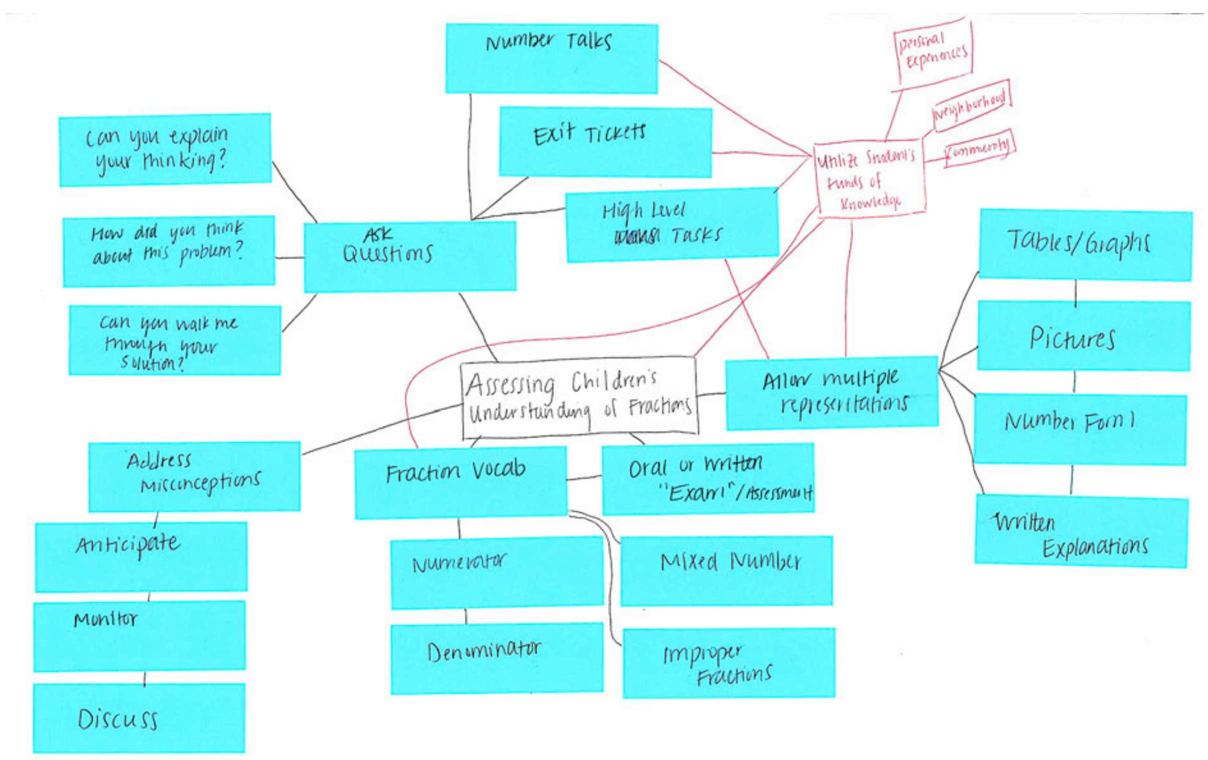

Figure 2. Morgan's end of semester concept map.

In the brief reflection, Morgan explained that *high-level tasks* and *multiple representations* connected to concepts related to children's multiple mathematical knowledge bases:

> *Morgan*: Creating *high-level tasks* [emphasis added] that provide students with multiple entry points into problems allowing them to think and use strategies that makes sense to them will tap into students' prior knowledge of what they already know and what strategies they are comfortable using. Allowing students to use *multiple representations* [emphasis added] also connects to their experiences of what types of strategies they have used in school before and what representations they prefer to use.

Morgan's brief reflection provides evidence of connections between high-level tasks and students' mathematical knowledge and strategies from in-school mathematics experiences. It is not clear, however, in what ways Morgan is making connections between high-level tasks and students' funds of knowledge. Similarly, it is not clear in what ways Morgan is making connections between multiple representations and students' funds of knowledge, even though there are links between these pairs of concepts on Morgan's map.

One prospective teacher, Harper, noted that there was evidence of concepts related to children's mathematical thinking before editing the map, such as *number talks*, *seeing students' thinking*, and *high-level tasks*. After editing the map, Harper added more concepts related to children's lives and experiences:

> *Harper*: At first, the only discussion I had about mathematical thinking was in describing how we can use number talks to see students thinking, and allowing them to explore different strategies with *high-level tasks* [emphasis added]. Once I edited the map, I added things about students' home life, community, personal experiences, etc.

Harper initially highlighted eliciting students' mathematical thinking through number talks, which is an activity in which students participate in 15-minute conversations about computation problems to communicate their mathematical thinking. Harper also describes how high-level tasks provide opportunities for students to explore different problem solving strategies, which is also closely connected to students' mathematical thinking. Harper does mention a shift to focusing on students' funds of knowledge only after the discussion and editing process.

An Interesting Case to Explore

Out of all the prospective teachers' brief reflections, Parker was the only participant who claimed to not see evidence of children's multiple mathematical knowledge bases in the end of semester concept map (see Figure 3). Parker explained that there was no evidence of concepts related to children's multiple mathematical knowledge bases in the map because of Parker's ways of thinking about assessing and about using funds of knowledge in mathematics teaching:

> *Parker*: I think there is no evidence because I thought of assessing in the pedantic sense. I thought that experiences of the children would go more along with the *actual teaching* [emphasis added] of concepts… I could add funds of knowledge to the concepts that I stated as being a part of a formal assessment.

I highlight Parker's reflection as an interesting case because I heard multiple prospective teachers voice similar thoughts during small group discussions about funds of knowledge being more directly related to the process of teaching mathematics rather than assessment, which could follow the act of teaching. From the methods course materials, I have evidence that prospective teachers adapted existing tasks and curriculum materials, but I have little evidence that prospective teachers have designed assessments at this point in their preparation program. One prospective teacher told me

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during the whole group discussion that they have experience adapting problems to align with students' needs although they have not created assessments in the course. Consequently, this evidence made me wonder about how prospective teachers' made sense of the phrase "*assessing children's understanding*" in the root of this concept map activity, and ultimately, how their understandings influenced the construction of their concept maps.

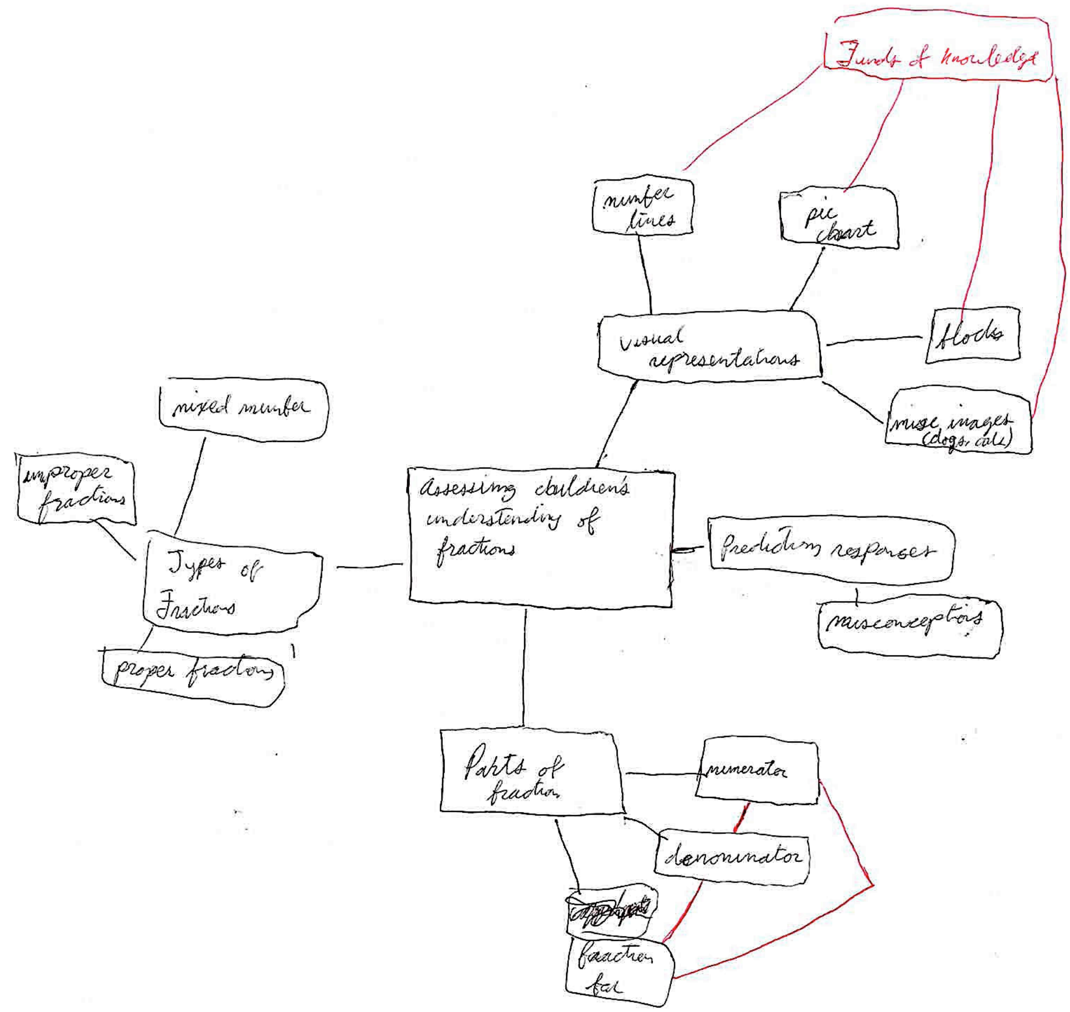

Figure 3. Parker's end of semester concept map.

Discussion

This study investigates in what ways prospective elementary teachers make connections among *children's multiple mathematical knowledge bases* in their thinking about assessing children's understanding of fractions. In the following, I describe implications for teacher educators and considerations for further research.

Teacher educators may support prospective elementary teachers to make explicit and stronger connections among concepts related to children's mathematical thinking, children's lives and experiences, and assessing children's understanding of mathematics. One entry point to better support these connections involves emphasizing *high-level tasks* as a concept related to teaching practices that connects to *both* children's mathematical thinking and children's lives and experiences. From my analysis of concept maps and brief reflections, I found strong evidence that the prospective teachers in my study made connections between high-level tasks and children's mathematical thinking. Based on an interesting case I found in the data, another implication for teacher educators is to be cognizant of a possible perception that funds of knowledge, including children's lives and experiences, are not used or useful in assessing children's understanding of mathematics. I would recommend a stronger focus on assessment as one of many teaching practices and the role of assessment for equitable teaching practices that serve the needs of all students, especially those who are traditionally marginalized in the mathematics classroom.

For further research, I would be interested in gathering more information about how prospective teachers make sense of the concepts related to *children's multiple mathematical knowledge bases*. Although prospective teachers made connections among these concepts, I noted that the concepts used more general language and did not include specific examples of how children's multiple mathematical knowledge bases connected to *children's understanding of fractions*. More specifically, I would like to examine *whose* multiple mathematical knowledge bases are represented in these concept maps. Are particular racial or ethnic groups of students in mind when we use the phrases *funds of knowledge* or *multiple mathematical knowledge bases*? Where do understandings of multiple mathematical knowledge bases come from (e.g. content in course readings/activities, prior experiences with children, etc.)?

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