AN EXPLORATORY ANALYSIS OF A VIRTUAL NETWORK OF MATHEMATICS EDUCATORS

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This study is part of a larger research design project that aims to cultivate an online community of mathematics educators. The purpose of this smaller study is to suggest interventions that will further support community cultivation efforts. Social network analytical methods are used to study project participants' interactions in virtual spaces. Investigation focused upon the connectivity of network structure, the most prominent members of the network, the presence of a core-periphery structure and the relationship between participant types and prominence. Analysis suggests that supporting interactions between newly involved and more long-standing participants will enhance community cultivation efforts. And, four participants emerged as most influential in the network, therefore being most effective in controlling and spreading novel information. These participants are suggested to receive additional professional development.

Keywords: Teacher Education-Inservice

Educational research and policy call for mathematics instruction to be student-centered, with a focus on argumentation and negotiation. While these calls were made over two decades ago, classroom instruction still does not reflection this "reform" oriented vision of classroom instruction (Stigler & Hiebert, 1999). One initiative that has shown promise in supporting teachers in shifting instructional orientation from teacher-centered to student-centered is engagement in community. Involvement in community provides opportunities for teachers to critically examine daily instructional issues, analyze student work, and plan mathematics tasks to support learning. While community engagement has been linked with improved instructional strategies (Vescio, Ross, & Adams, 2008), there is a lack in understanding of how to support emergence of teacher communities, particularly those that take place in online mediums.

This study is part of a larger research project that implements and innovative professional development (PD) model designed to cultivate an online community of mathematics educators. Due to extensive interactional data available from the larger study, and a particular focus on structural features of the network, we used social network analysis to investigate the patterns of interactions within the network. Social network analysis is a quantitative analytical method derived from graph theory used to study the structure of social networks (Scott & Carrington, 2011). Taken together, social network analysis can provide data about relationships in social networks that can be informative for future interventions that would not have been evident by other investigatory approaches. From an educational researcher's perspective, structural information about teacher networks is crucial for the implementation of interventions designed to foster community cultivation. For example, if teachers are identified as being sparsely connected, an intervention can be designed to support interaction between these teachers in order to improve the flow and access of information in the network as a whole. Interventions that serve the purpose of the one just mentioned may be critical in the process of cultivating a community of educators. Part of the purpose of this research is to make such suggestions for interventions.

The analytical methods briefly introduced here (and further elaborated in the methods section) serve as the backdrop against which research questions are formulated to focus investigation in this study. Consequently, this study seeks to answer the following research questions:

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- What are some of the structural properties of the social network?
- Who are the most central members of the participant network?
- To what extent is a core-peripheral structure present in the network?
- What is the relationship between participant type and centrality?

Literature Review and Conceptual Framework

This study is informed by literature on sociocultural learning theories (i.e. communities of practice and situated learning) and social network theory. These theories provide the conceptual lens to understand the structural relationships in the community of mathematics educators, and provide insights for interventions aimed at improving dissemination of instructional strategies and learning in the MathCom network.

Communities of practice

Communities of practice is a social perspective of learning that hinges learning upon individuals' enculturation into a community through increased participation within a social group. Three characteristics distinguish a group of individuals from a community of practice: mutual engagement, joint enterprise and shared repertoire (Wenger, 1998). Mutual engagement refers to the notion that communities develop and are maintained around engagement in shared practices. Engagement in meaningful activities of a community requires particular competencies that are valued by the community. Understanding of such competencies can be measured by the extent to which one interacts with community members. Therefore, identifying participation patterns both provides a sense of individuals' level of understanding of communal practices and the strength of community structure. Joint enterprise and shared repertoire refer to common goals, beliefs and a shared set of tools, respectively. While evidencing these aspects of community relies on more qualitative research, social network analysis is useful for investigation of mutual engagement to identify patterns of interactions (Wenger, Trayner, & de Laat, 2011).

Legitimate peripheral participation (LPP)

Communities of practice have a particular structure that is conducive to sustaining and maintaining engagement, which can be conceptualize through the lens of LPP. Communities consist of a core group of full participants, or "old-timers" who are experts in the community's practices (Lave & Wenger, 1991). However, if only experts are present, engagement typically becomes stagnant and interactions do not persist. Peripheral participants, or "newcomers", provide new perspectives and fresh outlooks to be considered by old-timers, which facilitates sustained engagement within a community (Lave & Wenger, 1991). Overtime, newcomers follow a trajectory of increased participation in the community, and eventually replace old-timers. The constant interchange of newcomers and old-timers is a core feature of a community of practice. Therefore, identifying places to support interaction between newcomers and old-timers may enhance community cultivation efforts.

Social network analysis and communities of practice

The discussion above underscores a perspective of learning that takes participation in a community as the unit of analysis. Researchers argue that communities of practice are comprised of social networks (Schenkel, Teigland, & Borgatti, 2001). Social network theory focuses upon the relational patterns embedded within a community and groups. In order for group norms to emerge and common beliefs to be present amongst the community, it is important for the social network structure to afford information flow. Recent studies have sought to identify central members—prominent members that have high levels of interaction within the group—in teacher communities to leverage their position in social networks to spread novel information (Daly, 2010). For example, through investigation of the importance of leadership for community success, Tsugawa, Ohsaki, and Imase (2010) conclude that in online communities, centrality can predict an individuals' ability to assume leadership responsibilities. In a similar study that sought to understand the relationship between individuals' centrality and successful implementation of reform, Atteberry and Bryk (2010) provide teachers professional development to act as "coaches" to spread information around school-based initiatives. In doing so, Atteberry and Bryk (2010) used social network analysis to study the relationship between teachers' centrality prior to receiving professional development and the success of the reform effort. They conclude that teachers with higher levels of centrality prior to implementation were more successful in dispersing novel information. In the current study, centrality measures are used to inform which participants, given additional developmental opportunities, may have the most impact on community cultivation.

Methodology

Social network analytical methods are used to investigate the "MathCom" network structure. In the following section these analytical methods are explained along with descriptions of the research setting, participants, data collection and processing, and data analysis plan.

Study Site and Participants

Data for this study is drawn from the MathCom project, which began in 2012. This project is designed to bring together and cultivate a virtual community of mathematics educators across the nation. A primary goal of the MathCom project is to foster mathematics educators' collective engagement in learning of mathematics and pedagogy. Accordingly, participants have been recruited in groups throughout the project as incremental efforts are being made to cultivate community. Over the past two years, multiple mediums of communication have been used as leverage points to support teacher interactions (i.e. face-to-face workshops, online classes and workshops, twitter discussion threads and email list serves) around different content, such as mathematics, instruction and assessment. Therefore, the research setting is in virtual spaces that foster online communication, such as twitter, online courses/workshops, and email listservs.

There are 82 participants (mathematics educators) involved in this study. Taking into consideration whether (and how) they participated in the two face-to-face summer institutes (at the end of Year 1 and Year 2 of the project) that focused on community and instructional development, we can distinguish between 6 types of participants:

- *2013 Fellows (n=12)* were part of the Year 1 institute
- *2014 Fellows* (*n=5*) began in Year 2 *and* were involved in the second summer institute.
- *2013 and 2014Fellows* (*n=12*) participated in both summer institutes.
- *Online participants (n=15)* were those who participated in the Year 2 institute solely via virtual video chat software.
- *Other(n=26)* participants participated strictly via online mediums
- *Staff(n=12)* are teacher educators responsible for virtual interactions with participants and developing project activities

However, given our interest in participants' interactions in the virtual spaces, we focused on the 67 who participated in those spaces.

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Data Collection and Processing:

Social network analysis is based on the premise that social life consists primarily of relations and the patterns formed by these relations. Social network analysts study these patterns of interactions between individuals in a network (Scott, 2013). In order to conduct the social network analysis, we examined the records of interactions between participants in each virtual space (twitter, online classes and workshops, email listservs) between May 2013 and September 2014. Participant (*i*) is said to have "talked to" Participant (*j*) if the latter initiated a communication tie to the former. In Twitter, a tie exists if Participant (*i*) included (*j's*) hashtag in the message (Ediger et al., 2010). On email and Blackboard learning systems, a tie is defined if Participant (j) responded to an initial or any other posts/emails from another participant (*i*). From this information, we created one communication matrix that records the interaction between study participants. Each cell (*Xij)* of the matrix takes on a value indicating how frequently (number of times) participant (*i*) directs or initiates a tie to participant (*j*). The resulting 67 x 67 binary asymmetric matrix became the basis/input for all of the structural analysis presented in this paper. The social network analysis software, *UCINET* version 6 (Borgatti, Everett, & Freeman, 2002) was used in all data preparation and analysis. Analysis results are reported as normalized indices. A normalized measurement is one in which the numerical value that results from analysis is standardized so one could compare the particular analytical result to other networks with different sizes.

Data Analysis Plan

This study uses four social network analysis measures (i.e. density, centralization, centrality and core-periphery) (Borgatti & Everett, 2013; Kadushin, 2011) to address the research questions and describe the communication structure of the MathCom network. First, density and network centralization measures allow us to address the first research question: *what are some of the structural properties of the MathCom Network? Network density* describes the general level of cohesion in a graph (Scott, 2000). The centralization score describes the extent to which the flow of information is organized around particular individuals or groups of individuals. Second, centrality analysis is employed to answer the second research question: *who are the most central members of the MathCom network?* Centrality is used in social network analysis to identify important nodes or those that occupy influential positions in a network. Three variants of centrality measures are used in this study: in-degree, out-degree and betweenness centrality (Freeman, 1979). *In-degree* centrality measures the ties an actor receives from others, while *out-degree* centrality measures the ties an actor directs to others. Betweenness centrality is a measure that indexes the number of times a participant falls on the shortest path between two participants in the network. Third, we employ a core-periphery model to examine the extent to which core-peripheral structures are present in the MathCom network (Question 3). This method bifurcates the network into a discrete model consisting of two classes. Fourth, distinguishing levels of centrality for participant types is also of interest (Question 4). We used specialized Analysis of Variance (ANOVA) tests for social network data to examine the differences in centrality by participant type (e.g. whether 2013 fellows are more central then 13 & 14 fellows).

Findings

Research Question 1: What are some of the structural properties of the MathCom network? To address this question, density and centralization measurements were used to give information about network connectivity. The MathCom network resulted in a density of 8.3%. Out of the 5,852 possible directed edges in the network, 488 of these edges are present. The network centralization analysis resulted in a measure of 0.222. This indicates that the MathCom network is more representative of a network that has centrality evenly distributed throughout the network than one that is controlled by a few central members. Density and centralization give an overall representation of the structure of the network. And in this case illustrate that over the 15 months in which this data was collected, close to 500 connections were made and certain central members are potentially emerging as a centralization score of .222 indicates there is some heterogeneity among centrality in the network.

Research Question 2: Who are the most central members of the MathCom network? As will be recalled, in-degree and out-degree centrality scores index the level of activity of participants in the communication network. On average, the top 10 participants have normalized out-degree scores of about 1.082, which is about 10 times larger than the average out-degree scores for the middle ten participants (0.111), and ninety times larger than the scores for the bottom ten participants (0.0117) respectively. On an individual level, participants 11018, 11006 and 11003 are the most active participants: actors 11006 and 11018 for instance have an outdegree score that is about 35% higher than the next highest out degree in the network. The indegree analysis yield similar results to those of outdegree: actors 11006, 11018 and 11003 have the highest normalized indegree centrality scores. Betweenness centrality analysis also yields actors 11018, 11006 and 11003 falling in the top ten, however actor 11016 also yielded a high betweenness centrality score. While actor 11016 did not have as high a valued in/out-degree score, her dichotomized indices where among the highest. Given the importance of betweenness centrality for indicating potential for leadership qualities and the control of information, actor 11016 is considered as one of the most central members.

The above analysis suggests that actors 11018, 11016, 11006 and 11003 are the most central members in the network. Each of the identified actors are 2013 and 2014 fellows, meaning they have been a part of the project since it began in 2013, a year longer than other members that began participation in 2014. Over a year of the project, the identified actors will have had many more chances to participate than those that began in 2014, therefore being a possible reason for their increased centrality. However, based on centrality alone, and drawing from previous research, this analysis suggests these are the most central members and should receive additional professional development opportunities.

In addition, in this analysis staff members are omitted in identification of the most central actors. For example, staff members 1 and 7 are present in the top ten of each centrality measure, however they are left out of this analysis. Part of the purpose of this research is to provide additional professional development opportunities for participants that are identified through social network analysis as most influential in the network. Therefore, identifying staff members that are most central does not support research goals; although, staff members are included in analysis as part of the 67 individuals because many interactions occur between staff (who act as role models) and participants, thereby disseminating best practices in mathematics pedagogy.

Research Question 3: To what extent is a core-peripheral structure present in the network? To address this question, we fitted a categorical core-periphery model to the MathCom data. The core-periphery analysis provided sub-group densities, which can be used to examine the connectivity, between/within, the core and periphery groups. The density profiles also help evaluate the extent to which the derived groups approach an ideal core-periphery structure where 100% of ties are expected between core members, 0% between peripheral members, and most of the connections expected between the core and periphery members. The core-to-core density of the MathCom network is 59%, the core to periphery is 16.7%, the periphery to core is 9.6% and the periphery to periphery is 3.2%. Although this network does not completely approach an ideal core-periphery structure (and few are expected to be), the analysis indicates the presence of a core periphery structure in the network (see Figure 1).

Of the 15 participants in the core of the network, 9 are 2013 & 2014 fellows, 2 are 2013 fellows, 3 are staff, and 1 is an "other" participant that was not part of either institute but was rather active in online classes. The high concentration of 13 & 14 fellows in the core of the network is expected, due to increased opportunities to participate throughout the life of the project. Thus, participation in the summer institute seems to be related to communication activity in the network. The staff members in the core of the network are also reasonable results due to their frequent activity in twitter and email. However, a rather interesting result is participant 13010 falling into the core, as she was not part of either institute. Further research will explore the position of participants such as 13010.

Research Question 4: What is the relationship between participant type and centrality? The analysis above suggests that there are variations in centrality of participants in MathCom. One determinant of this communication activity is participant type (whether they are 2013 Fellows, 2013-2014 Fellows, etc.). Table 1 displays the results of an ANOVA permutation model examining the differences in mean (betweenness, outdegree and indegree) centrality of participants by participant type. The ANOVA analysis shows that 2013 & 2014 fellows have the highest centrality means in each category. Their average betweenness, outdegree and indegree centralities are about 3 times larger than the next largest mean. For example, the mean out degree centrality for 2013 fellows is 0.2452, while that of 2013 & 2014 fellows is 0.7148, which is 2.9 times larger then the 2013 fellows. Second, 2014 fellows have substantially lower in/out-degree (valued and dichotomized) than 13 & 14 fellows. For example, the in/out degree of the 2014 fellows is 14.8% and 10.6% of the 13 & 14 fellows, respectively. These findings illustrates a clear divide in the amount of participation between the two groups: with the 2013 & 2014 fellows much more active than 2014 fellows.

Discussion

Network structure

The above analysis clearly shows the presence of a core and periphery in this network. While the core-periphery analysis will "force" members into the two categories regardless of the nature of the connections in the network, the density measures suggest a rather densely connected core (59% of the potential connections are present) and a rather sparsely connected periphery (3.2% of the potential connections are present). In the ideal core-periphery structure the core-to-core density is 100% and the periphery-to-periphery density is 0% (Borgatti & Everett, 2000). While the results shown here are not quite representative of the ideal core-periphery network, the divide suggested by Borgatti and Everett is a theoretical model rarely (if ever) observed in practice.

Lave and Wenger (1991) suggest a core-periphery structure is critical to sustaining productive interactions within a community while maintaining the structure of a community. In particular, they note that constant interaction between core and peripheral members is critical for knowledge creation and distribution in a community. In consideration of this guiding theory, results of this study are used to identify connections that should be fostered between core and peripheral members to ensure the suggested interactional patterns. Through intervention, members that have been most recently introduced to the MathCom network (e.g. 2014 fellows) and are in the periphery, will be connected with members in the core that have been part of the project for a longer period of time (e.g. 2013 and 2014 fellows). Furthermore, as the periphery begins to decrease in size due to peripheral members beginning to evolve into more centrally located members in the community; new participants should be gathered in order to maintain the core-periphery structure.

Centrality

Recent literature suggests that higher levels of centrality in networks increase individuals' likelihood of spreading novel information throughout the network (Atteberry & Bryk, 2010). In addition, Tsugawa, Ohsaki, and Imase (2010)mention that high levels of betweenness centrality are indicative of leadership qualities. Therefore, providing additional professional development opportunities to the most central actors may be an effective way of spreading novel information throughout the network in this study. The results of this study suggest that actors 11003, 11006, 11016 and 11018 should be given additional developmental opportunities in order to become "coaches" within the network that engage in practices with others to promote development of student-centered instructional strategies.

Conclusion

This study illustrates a quantitatively driven approach to enhancing professional development efforts. Social network analytical methods were used to identify opportunities for intervention that may have otherwise gone unnoticed. In particular, this investigation has evidenced the presence of a core-periphery structure and has identified 4 individuals that are most central in the MathCom network. These results provide justification for (1) providing professional development to specific individuals in the network, and (2) fostering relationships between specific groups of individuals in the network.

Table 1: Mean centrality measures and ANOVA tests of mean differences for each participant type

	2013 Fellows	*13 & 14 Fellows*	*2014 Fellows*	*Online*	*Staff*	*Other*	**One-Way ANOVA**		
Mean centrality							**F-Statistic**	**P-Level**	**R-Squared**
Betweenness	1.1381	0.2452	0.19	0.1125	0.0801	0.2357	3.966	0.003	0.108
Valued Outdegree	3.4667	0.7148	0.4265	0.2193	0.159	0.1331	4.44	0.015	0.127
Valued Indegree	0.5694	0.0762	0.0632	0.05	0.0498	0.0753	1.183	0.265	0.003
Dichotomized outdegree	0.679	0.211	0.079	0.066	0.066	0.0469	2.931	0.030	0.280
Dichotomized indegree	1.6592	0.2664	0.1535	0.0844	0.0712	0.0408	4.52	0.002	0.033
N	12	12	5	15	12	26	--	--	--

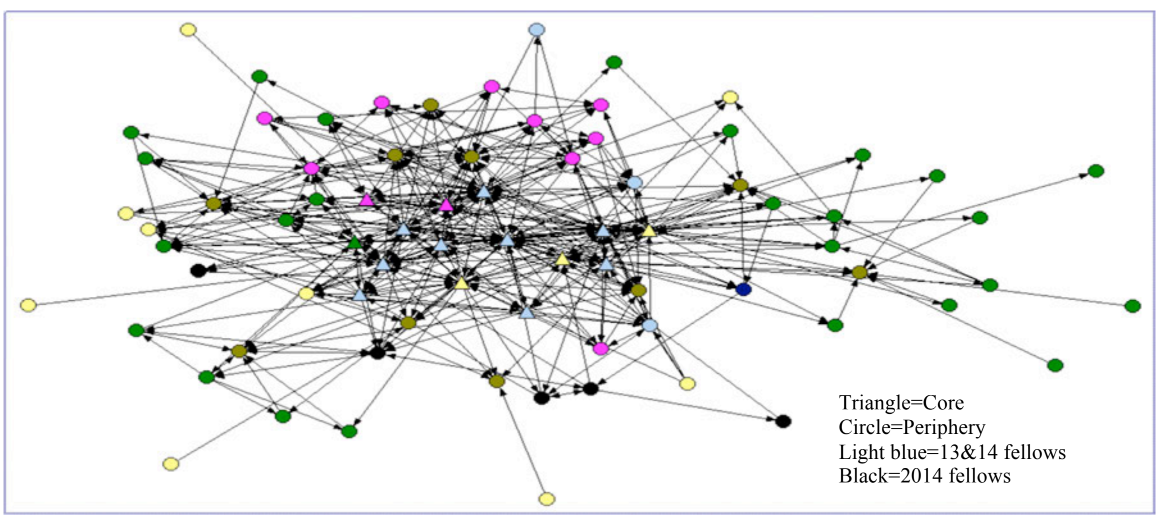

Figure 1: Communication interactions by participant type

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