ON FRAMING TEACHER MOVES FOR SUPPORTING STUDENT REASONING

Zekiye Ozgur
U. of Wisconsin-Madison
zozgur@wisc.edu

Lindsay Reiten
U. of Wisconsin-Madison
reiten@wisc.edu

Amy B. Ellis
U. of Wisconsin-Madison
amy.ellis@wisc.edu

Supporting students' mathematical reasoning is an important goal of mathematics instruction, but can be challenging for many teachers .We report the results of a study aimed at better understanding and identifying the ways in which teachers support student reasoning when provided with conceptually rich tasks. This study resulted in the Teacher Moves for Supporting Student Reasoning (TMSSR) framework, which organizes moves vis-à-vis their function and their potential for fostering student thinking. We describe the TMSSR framework, illustrate its affordances for studying teacher practices, and highlight its utility for teachers, teacher educators, and researchers.

Keywords: Instructional Activities and Practices; Algebra and Algebraic Thinking

Introduction

An important goal of mathematics instruction is to support meaningful and productive student reasoning; however, this is a goal that many teachers can find challenging (e.g., Rasmussen & Marrongelle, 2006). Two essential means for achieving this goal are the implementation of conceptually rich tasks and teachers' abilities to support and foster student engagement in such tasks. We report the results of a study aimed at better understanding and identifying the ways in which teachers can support student reasoning. This study is part of a larger project (http://tinyurl.com/badgerellis) that aims to a) help students develop deductive reasoning competencies in algebra through quantitative reasoning opportunities, and b) support teachers in achieving this goal. To scale up the findings from small-scale teaching experiments to whole-class settings, we partnered with practicing mathematics teachers to implement research-based curricular units in their classrooms. Analysis of a middle school classroom yielded the Teacher Moves for Supporting Student Reasoning (TMSSR) framework. Below we present the TMSSR framework, illustrate its affordances for studying teacher practices aimed at supporting student reasoning, and highlight its utility for teachers, teacher educators, and researchers.

Theoretical Background

Frameworks Investigating Teacher Moves

Various frameworks exist for investigating teacher moves during classroom instruction. While some frameworks focus on the questions teachers ask (e.g., Driscoll, 1999; Frey & Fisher, 2011), others focus on discursive moves (e.g., Herbel-Eisenmann, Steele, & Cirillo, 2013; Hufferd-Ackles, Fuson, & Sherin, 2004; Krussel, Edwards, & Springer, 2004). Yet others take a broader approach to include teacher questioning as well as other moves that teachers make in the course of instruction (e.g., Lampert et al., 2013; Staples, 2007). Taken together, these frameworks outline the general teacher moves and questions that occur in classrooms while teachers are eliciting, encouraging, and responding to students during instruction.

Additionally, there are frameworks that focus on student thinking. For example, Stockero et al.'s (2014) MOST framework for analyzing productive mathematical student thinking seeks to identify the most productive student thinking instances that warrant further teacher response. We build on these frameworks by investigating the moves teachers employ in quantitatively-rich contexts and the potential these moves have for supporting student reasoning. Like others have noted (e.g., Franke et al., 2009), teacher questioning is often used to help students make their thinking more explicit. However, in contrast to a focus exclusively on the questions teachers ask, a framework that includes additional practices that appear to support student thinking can provide a more complete picture of how teachers can foster student reasoning when engaged in conceptually rich tasks.

Quantitative Reasoning

Quantities are individuals' conceptions of measurable attributes of objects or events, such as length, area, volume, or speed. Relying on situations that involve quantities that students can make sense of, manipulate, and investigate can foster their abilities to reason flexibly about dynamically changing events (Carlson & Oehrtman, 2005). Reasoning with relationships between quantities has been found to support students' understanding of algebraic relationships and to encourage deductive argumentation (Ellis, 2007; Smith & Thompson, 2007). We therefore designed quantitatively rich tasks in a series of small teaching experiments, which we then provided to the teachers for implementation in their classrooms.

Methods

The study we report here occurred in an 8th grade mathematics classroom at a public middle school and consisted of ten days of instruction on linear relationships grounded in a context of gear ratios. We provided the teacher with a set of research-based tasks for exploring and identifying relationships between gears rotations; the teacher also had the liberty to make modifications to the tasks as she saw fit. All sessions were videotaped and transcribed. Additionally, field notes, student work, and an interview with the teacher provided supplementary data.

We began analyzing the transcripts (which also included the images of student work and written work on board) of the observed lessons via open coding, without any particular framework in mind. As we progressed into the data analysis using the constant comparison method (Glaser & Strauss, 1967), our attention focused on the teacher moves that supported student reasoning, and we eventually developed an emergent coding scheme through multiple passes of open coding. After the initial development of the coding scheme, we also analyzed the literature base to identify the ways in which our codes for teacher moves intersected with existing descriptions reported in the literature. After reaching a fairly stable coding scheme, we proceeded with focused coding (Saldaña, 2009) and two researchers independently re-coded the entire data set. Through constant comparison and discussion of each researcher's coding, the coding scheme was further refined by way of revising some definitions as well as delineating the functions the teacher moves served to support student reasoning.

Findings and Discussion

The TMSSR framework identifies and organizes teacher moves into four categories based on the function they serve in supporting student reasoning (i.e., eliciting, responding to, facilitating, and extending). In addition, teacher moves within the same category differ in their potential to support student reasoning. For instance, although *correcting a student error* and *prompting a student error correction* are both moves teachers make in response to student reasoning, prompting a student to correct her error has the potential to lead to a greater learning opportunity for the student than if the teacher had merely corrected the error (Speer, 2008). Drawing both from Speer's discussion about teacher moves offering different potential for supporting student learning and an analysis of how the teacher's moves affected student reasoning (as inferred from students' responses), we place teacher moves along a continuum for the potential each move has for supporting reasoning. More specifically, moves that offer greater potential are located towards the right hand side of each category. These moves occur during whole class discussions as well as when the teacher is working with students in small groups or individually. We begin by describing each category of the framework and then present an analysis of one teacher's classroom with the TMSSR framework, who we call Ms. L.

Focusing in on the TMSSR Framework

Tables 1-4 present the categories of the TMSSR framework. Although related teacher moves are organized along a continuum (signified by rows in the table), it is important to note that the continuum represents the *potential* each teacher move has for supporting student thinking. How the teacher enacts a move and the students' responses determine the *actual* affordance for supporting student reasoning. In some cases (e.g., re-voicing, encouraging student re-voicing, and re-representing) more than two teacher moves are organized within the same row to signify their related nature. Due to page constraints we are not able to demonstrate how all of the moves are placed along a continuum in the framework. We focus on a row from tables 1 and 4 since the moves in Tables 1 and 4 encourage students to take a more active role in the discussion (as opposed to Tables 2 and 3 where the teacher has a more prominent role).

Table 1: Teacher Moves for Eliciting Student Reasoning

Eliciting Answer: Teacher asks a question geared at eliciting the answer to a given task or problem.	***Eliciting Ideas***: Teacher asks a question geared at eliciting students' ideas for a solution strategy.
Eliciting Facts or Procedures*:* Teacher asks questions geared at eliciting students' recitation of facts or procedures.	***Eliciting Understanding****:* Teacher asks questions geared toward assessing what students understand and how they are reasoning.
Asking for Clarification*:* Teacher asks a question to clarify the student's meaning because teacher genuinely does not know what the student meant.	***Pressing for Explanation****:* Teacher asks student(s) to elaborate on their thinking, explain their reasoning, or reflect on and share their reasoning.
Figuring Out Student Reasoning: Teacher is trying to figure out a student's solution, or understand a student's explanation or reasoning.	
Checking for Understanding*:* Teacher asks a question to assess students' understanding of the mathematical ideas that are currently under discussion.	

The moves presented in Table 1 enabled Ms. L to elicit students' reasoning while implementing the quantities based tasks. These moves served to engage students in sharing their thinking and often occurred at the beginning of a discussion about a particular problem. These teacher moves commonly occurred when Ms. L worked with students in small groups as well as when she facilitated whole class discussions. The excerpts below demonstrate the potential difference that two related teacher moves, *Asking for Clarificatio*n and *Pressing for Explanation,* have for supporting student reasoning. As seen in the following excerpt, when Ms. L asks for clarification, the student's response (i.e., Leigh) is often minimal, with little or no elaboration:

Leigh: You could just plug in numbers so the middle gears teeth equals, the middle gears teeth is 12 and the big gears teeth are 16. So you need 3/4 times 12 and if it equals 16, if it equals 16, then…

Ms. L: So you are wondering this[*writes* $\frac{3}{4}(12)\overset{?}{=}16$ *on board*]?

Leigh: Yes.

Asking Leigh what she was wondering about may have clarified for Ms. L Leigh's current thinking about the task. However, if Ms. L had instead pressed Leigh for an explanation, as she did with Hope in the following episode, Leigh would have had more potential to think through and articulate her own strategy. For example, when Ms. L pressed Hope for an explanation (i.e., asking where the ratio 2/3 exists in Hope's work), this move encouraged Hope to think more about why her

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strategy made sense. Thus, Hope was able to start moving from a procedural explanation towards a more conceptual explanation:

> *Hope:* I made the table again. So like, so this is the first one, so Lewis' formula yesterday was to divide the smaller number which is the bigger gear by two. So I wrote, so it's 2.5 *(points to 5 on table)(writes 5: 2= 2.5)* and then instead of writing a whole new one I worked by like continuing, so I added 5 even though it isn't proper and it said 7.5 and then it equaled this one (*points to 7.5 on table*) so you know it works because of the 2/3.
> *Ms. L:* Okay, so can you maybe elaborate a little bit more? Which thing is the 2/3?
> *Hope:* Okay, so like this is like you're reducing it down a different way.What I did is I would say, like this, you're just making 'cause, I don't know how to say this, okay so like 7 and a half like you're trying to find 2/3 of 7.5.*(points to 5)*

Table 2: Teacher Moves for Responding to Student Reasoning

Validating a Correct Answer*:* Teacher actively confirms the student's idea by re-voicing, or re-wording in her own words, or adding a bit to the student's idea or response.	
Re-voicing*:* Teacher repeats student ideas (verbally or written) in order to make those ideas public.	***Re-representing****:* A form of re-voicing in which a teacher provides her own representation as a way to publicly share a student's idea, work, or strategy. The teacher may organize, re-frame, or formalize the student's statement or work.
Encouraging Student Re-voicing*:* Teacher asks students to re-voice other student ideas or solutions.	
Correcting Student Error*:* Teacher corrects a student error or supplies the correct answer more generally.	***Prompting Error Correction****:* Rather than correcting the student, the teacher prompts the student to address an error herself.

Due to space constraints, we primarily focus on the organizational structure of Tables 2 and 3, which present the teacher moves for responding to and facilitating student reasoning, respectively. The moves in Table 2 often occur after a teacher has already elicited student reasoning and s/he is trying to make students' reasoning more public or amend a student's statement. Teacher moves in Table 3 also generally occur after the teacher has elicited student reasoning and is now trying to assist students in developing their reasoning through various forms of guidance and explanations. These moves may help students engage with a task or summarize students' contributions before moving on to a new task. Although the teacher moves described in Table 3 are common, the *Topaze Effect* is worth noting. Stein, Grover, and Henningsen (1996) describe this move as reducing, however, we build from Brousseau's (1997) description of how a teacher breaks a task into smaller parts and thus significantly alters how a student conceptually engages in the task. In the following excerpt Ms. L asks Laura a question about the relationship between the two gears. However, before Laura has a chance to respond, she immediately asks an easier question. The second question reduces the original question (in which Laura would have to determine a fractional amount of a rotation) down to a yes/no response (whereby Laura merely has to identify whether the gear made an entire rotation).

> *Ms. L*: (*To Laura*) So if you turned the small gear once, how far around would the big gear go? (*Brief pause*) Would it make it all the way?
> *Laura*: No.

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Table 3: Teacher Moves for Facilitating Student Reasoning

<table>
<tr><td colspan="2">Cueing: Teacher cues students' attention by indicating that they should focus on a particular aspect of a problem, task, idea, solution, etc.</td><td colspan="2">Providing Guidance: Teacher provides hints, ideas, a potential strategy, or another type of conceptual scaffolding of the problem without outlining the solution structure or otherwise shutting down students' opportunities to reason on their own.</td></tr>
<tr><td colspan="2">Topaze Effect: Teacher breaks a task into smaller parts, reducing the complexity of the task by asking easier and easier questions, thereby reducing students' opportunity to engage in authentic problem solving.</td><td colspan="2" rowspan="2">Building: Teacher builds on students' earlier contributions to support new understanding, or encourages students to build on one another's contributions.</td></tr>
<tr><td colspan="2">Funneling: Teacher asks questions that move students down a specific path (e.g., through leading questions).</td></tr>
<tr><td rowspan="2">Providing Procedural Explanation: Teacher provides a procedural explanation for how to solve a problem. This move includes telling students a priori how to solve the problem by outlining the solution structure (or some other way).</td><td colspan="2">Providing Summary Explanation: Teacher summarizes for the class final thoughts about a task or problem, or a summary of information or discussion about the task.</td><td rowspan="2">Providing Conceptual Explanation: Teacher provides an explanation that has a conceptual basis, often focused on explaining why something works. This move can also be thought of as demonstrating logic.</td></tr>
<tr><td colspan="2">Providing Information: The teacher provides new information relevant to doing mathematics generally rather than information about a specific problem or task.</td></tr>
<tr><td></td><td colspan="2">Encouraging Multiple Solution Strategies: Teacher encourages a proliferation of solution strategies, including pressing students to come up with a different way to solve a problem.</td><td></td></tr>
<tr><td></td><td colspan="2">Providing Alternative Strategy: Teacher initiates a new or different way of solving a problem after students have shared their solution strategies or solutions.</td><td></td></tr>
</table>

Table 4 presents the teacher moves that were used to extend student reasoning. These moves usually occur after students have worked through a task for some time and have made some progress into the solution. To further extend students' initial reasoning, the teacher pushed students to provide complete answers rather than vague responses, to make connections to the context, to think about the underlying concepts involved in the task, and to justify their ideas. The excerpts below demonstrate the potential difference that two related teacher moves, *Topaze for Justification* and *Pressing for Justification,* have for supporting student reasoning. In both situations, Ms. L asked students to explain why an idea, solution, or strategy works; however, in the case of *Topaze for Justification*, Ms. L did not allow students enough time to grapple with the initial prompt to justify their ideas. Instead, she reduced the complexity of the question by following up with an easier question or by narrowing the question's focus. As an example, in the following excerpt Ms. L asked Gert to show why there was not a relationship between the gears, but she then immediately suggested testing with numbers. By doing so, Ms. L unwittingly prevented Gert from devising her own way to justify her claim:

Ms. L: What'd you come up with for a reason for part a?
Gert: I said because there's no relationship.
Ms. L: Good. Can you show why not? Can you show like with numbers?

Without the space to think of her own justification, Gert agreed to use Ms. L's suggestion, but the emphasis on numbers shifted the conversation to a calculational explanation. In contrast, Ms. L was able to better advance students' reasoning when she persistently pushed students to justify their ideas, allowing students enough time to think and solidify their reasoning. The following excerpt exemplifies the *Pressing for Justification* move:

Ms. L: Okay. So nine goes here is what we are saying (*writes 9 to "B" column*). So does that seem correct that if the medium gear spun twelve, the big gear would spin nine?
Students: Yes.
Ms. L: Okay. Yes, you are saying yes. (*spinning gears*) Anyone see a proof of why that works? Something you can use for evidence. Laura?
Laura: Okay. Well the ratio was three fourths
Ms. L: Uh-hum.
Laura: So then if you, um if you... Yeah, so the ratio is three fourths and now it's like, you could say it is nine twelfths. And then if you divide nine by three its three and if you divide twelve by three it is four.

Table 4: Teacher Moves for Extending Student Reasoning

***Pressing for Precision**:* Teacher encourages student(s) to provide an exact rather than vague answer, to check his or her work for accuracy, or to quantify a qualitative statement.	***Encouraging Reasoning :***Teacher encourages students to think about the task conceptually, for instance by thinking about why a strategy makes sense, by thinking about where the numbers connect to the quantitative situation, etc.
	Encouraging Reflection: Teacher asks students to reflect on provided answers or explanations (either from the teacher or from another student).
Topaze for Justification: Teacher initially pushes for justification, but then immediately downgrades her question to a less-sophisticated why question by heavily leading students into justification via easier questions.	***Pressing for Justification**:* Teacher asks students to explain why something works or to justify (logically, conceptually) their idea, strategy, or solution.
	Pushing for Generalization: Teacher encourages students to generalize their reasoning, either through formulating a rule, describing a process in general terms, or making connections across problems, numbers, cases, or events.

Analysis of Ms. L's Classroom Using the TMSSR Framework

Table 5 illustrates Ms. L's moves for supporting student reasoning while implementing the quantities-based unit. Frequency counts for each move are listed in parentheses. It is important to note that more than one teacher move may occur at the same time (e.g., a teacher often elicits facts or procedures while she is funneling). Shading corresponds to the proportion of a specific move compared to all moves in the table, with the darker cells representing the moves that occurred more frequently. When comparing the four functional categories in the TMSSR framework, the table suggests that Ms. L spent more instructional time eliciting student reasoning compared to any one other category. Eliciting moves occurred most frequently because the other three categories represent moves that generally occur after student reasoning had been elicited. Given that the data come from Ms. L's first implementation of the research-based unit, it is not surprising that her moves for supporting student reasoning were on the left hand side,with less potential for supporting student reasoning. As a teacher becomes more familiar with the tasks and the moves that have greater potential for supporting student reasoning, we would expect to see more moves located on the right hand side of the continuum. Although the TMSSR framework focuses on teacher moves, it is important to note that such moves are also related to the classroom environment (for instance, it had already been established that students were routinely encouraged to share their reasoning and were viewed as responsible learners by themselves and the teacher) and students' ability to engage with the tasks. Because the TMSSR framework focuses on the potential support teacher moves have for fostering student reasoning, two teachers could have similar illustrations (e.g., both could look similar to Table 1) but their students' development of reasoning could be different.

Table 5: Illustrating Ms. L's Moves with the TMSSR Framework

<table>
<tr><th colspan="4">Eliciting Student Reasoning</th><th colspan="2">Responding to Student Reasoning</th></tr>
<tr><td colspan="2">Eliciting Answer (53)</td><td colspan="2">Eliciting Ideas (13)</td><td>Validating a Correct Answer (85)</td><td></td></tr>
<tr><td colspan="2">Eliciting Facts or Procedures (144)</td><td colspan="2">Eliciting Understanding (37)</td><td>Re-voicing (27)</td><td rowspan="2">Re-representing (26)</td></tr>
<tr><td colspan="2">Asking for Clarification (25)</td><td colspan="2" rowspan="2">Pressing for Explanation (46)</td><td>Encouraging Student Re-voicing (4)</td></tr>
<tr><td colspan="2">Figuring Out Student Reasoning (46)</td><td>Correcting Student Error (26)</td><td>Prompting Error Correction (38)</td></tr>
<tr><td colspan="2">Checking for Understanding (10)</td><td colspan="2"></td><td></td><td></td></tr>
<tr><th colspan="4">Facilitating Student Reasoning</th><th colspan="2">Extending Student Reasoning</th></tr>
<tr><td colspan="2">Cueing (47)</td><td colspan="2">Providing Guidance (40)</td><td>Pressing for Precision (17)</td><td>Encouraging Reasoning (20)</td></tr>
<tr><td colspan="2">Topaze Effect (60)</td><td colspan="2" rowspan="2">Building (11)</td><td></td><td>Encouraging Reflection (25)</td></tr>
<tr><td colspan="2">Funneling (73)</td><td>Topaze for Justification (12)</td><td>Pressing for Justification (13)</td></tr>
<tr><td rowspan="2">Providing Procedural Explanation (74)</td><td colspan="2">Providing Summary Explanation (28)</td><td rowspan="2">Providing Conceptual Explanation (21)</td><td></td><td>Pushing for Generalization (5)</td></tr>
<tr><td colspan="2">Providing Information (14)</td><td rowspan="3"></td><td rowspan="3"></td></tr>
<tr><td rowspan="2"></td><td colspan="2">Encouraging Multiple Solution Strategies (26)</td><td rowspan="2"></td></tr>
<tr><td colspan="2">Providing Alternative Solution Strategy (9)</td></tr>
</table>

Conclusion

We propose a framework for teacher moves that support student reasoning by organizing moves vis-à-vis their function and their potential for fostering student thinking. Although the teacher moves were made while implementing quantities-based algebra units, many of the moves(e.g., building, eliciting, re-voicing) are similar to those that have been presented in other frameworks(e.g., Herbel-Eisenmann, Steele, & Cirillo, 2013; Lampert et al., 2013; and Staples, 2007). Therefore, we posit that these moves are not unique to these classrooms and can serve others working with teachers and investigating the moves that they make to support student reasoning.

By examining teacher moves holistically, we can better understand the various ways teachers support student reasoning. The TMSSR framework classifies teacher moves into functional categories and locates these moves along a continuum based on the potential support that a move has for supporting students' reasoning. Ideally, we would like to see teachers more frequently employ moves for extending student reasoning. However, we also caution that only attending to the frequency of teacher moves across categories may lead to incorrect conclusions about a teacher's practices. Our analysis revealed that some teacher moves have more potential for supporting student reasoning than others within the same category (e.g. *Topaze for Justification* and *Pressing for Justification)*;thus, placing moves along a continuum is helpful for better assessing teacher practices for their potential for supporting student reasoning. Teachers who are interested in informally assessing their teaching may benefit from thinking about both the four categories of moves and the ways in which teaching actions are organized within each category. Similarly, researchers and

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teacher educators studying teachers' practices could use the TMSSR framework to identify specific areas in which teachers may benefit from additional support.

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