BILINGUALS' NON-LINGUISTIC COMMUNICATION: GESTURES AND TOUCHSCREEN DRAGGING IN CALCULUS

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This paper discusses the importance of considering bilingual learners' non-linguistic forms of communication for understanding their mathematical thinking. In particular, I provide an analysis of communication involving a pair of high school calculus bilingual learners, who interacted with a touchscreen-based DGE (dynamic geometry environment). The paper focuses on the word-use, gestures, and touchscreen dragging actions in student-pair communication. Findings suggest that the students relied on gestures and dragging as multimodal features of the mathematical discourse to communicate dynamic aspects of calculus. This paper raises questions about new forms of communication mobilized in dynamic, touchscreen environments, particularly for bilingual learners.

Keywords: Classroom Discourse; Equity and Diversity; Technology

Introduction

In British Columbia, Canada, "In 2011–12, one in four (23.8 %) of public school students spoke a primary language at home other than English. Almost double the number of [English language learners] (135,651) live in families where the primary language spoken at home is other than English […]" (BCTF, 2012, p.11-12). Speaking from my own experience teaching mathematics in Canada, the home languages spoken in a typical mathematics classroom are very diverse, ranging from five to ten in any given classroom. This context is one result of globalisation and rapidly changing student demographics not only locally but worldwide.

Currently, research focusing on bilingual learners' mathematical communication has provided tremendous insight into the *complexities* of teaching and learning mathematics in multilingual contexts: the language dilemmas of teaching mathematics (Adler, 1999), the role of code switching in learning mathematics (Clarkson, 2007) as well as associating mathematics learning with socio-economic and epistemological access (Setati, 2005). These studies, however, had not critically examined bilingual learners' communication patterns and, in particular, addressed their competence in mathematical communication. As argued by Moschkovich (2010), future studies on bilingual learners must consider broader linguistic frameworks for understanding bilingual learning.

Research on multimodality can shed light on bilingual learners' communication as a multimodal activity that includes the use of language, gestures and interactions with diagrams (Arzarello, 2006; de Freitas & Sinclair, 2012; Radford, 2009). Aligned with the idea of multimodality in mathematical thinking, a small number of research studies have drawn on bilingual learners' non-linguistic forms of communication such as gestures and diagrams (Moschkovich, 2009). Moreover, although numerous studies have discussed the effect of the DGE-mediated learning of calculus concepts (Yerushalmy & Swidan, 2012), research on the effects of touchscreen-based DGE is limited. It is hypothesized that a touchscreen-based DGE may offer additional affordances by providing tactile and kinesthetic modes of interaction—hence, further facilitate bilingual learners' communication in calculus.

In a previous study working with bilingual learners, I showed that certain dragging actions on a touchscreen-based DGE constitute a form of communication (Ng, 2014). Using a Sfard's (2008, 2009) communicational approach, my analysis showed that some dragging actions were not merely dragging, but also instances of gestural communication—to communicate *dynamic* features and properties in the sketch as obtained by dragging. The touchscreen-dragging modality allows the

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dragging with one finger on the touchscreen and the gesturing with the index finger to blend together as one action. The importance here is that the *dragging-gesturing* action subsumes both dragging and gesturing characteristics, in that it allows the point to be moved on the screen (dragging), and it fulfills a communicational function (Sfard's definition of gesturing).

Building on this work, the current study examines the communication patterns that arise as bilingual learners interact with a touchscreen-based DGE. In particular, I investigate bilingual learners' language, gestures, and dragging in DGE as they communicate a given calculus concept in their non-native language. My study concerns the non-linguistic resources utilized by bilingual learners in communication as they work with a DGE. Further, it is the goal of the study that this analysis will identify bilingual learners' competence in mathematical communications.

Theoretical Framework

Sfard's communicational framework (2008) is based upon the social dimensions of learning, which suggests that learning is located neither in the head nor outside of the individual, but in the relationship between a person and a social world. It provides a suitable theoretical lens for highlighting the communicative aspects of thinking and learning. For Sfard, *thinking* is part and parcel of the process of *communicating*. This non-dualistic approach, which disobjectifies thinking as a purely cognitive phenomenon, is helpful for examining the relationships between talking, gesturing and mathematical thinking. Sfard (2009) defines *language* in unrestrictive terms, as any symbolic system used in communications, and *gestures* as bodily movements fulfilling communicational function: "Language is a tool for communication, whereas gesture… is an actual communicational action" (p.194). In this sense, a gesture can be performed to communicate with others (interpersonal) or with oneself (intrapersonal). Sfard's approach highlights the way in which thinking and communicating (for Sfard, this includes talking and gesturing) stop being "expressions" of thinking and become the process of thinking in itself.

Sfard (2008) proposes four features of the mathematical discourse, *word use, visual mediators, routines, and narratives*, which could be used to analyze mathematical thinking and changes in thinking. For the purpose of this paper, the first three features will be used for analyzing the use of language, gestures, and dragging in one's mathematical discourse. *Word use* is a main feature in mathematical discourse; it is "an-all important matter because […] it is what the user is able to say about (and thus to see in) the world" (p. 133). However, as a student engages in a mathematical problem, her mathematical discourse is not limited to the vocabulary she uses. For example, her hand-drawn diagrams and gestures can be taken as a form of *visual mediator* to complement word use. According to Sfard (2009), *utterances* and *gestures* inhabit different modalities that serve different functions in communication. Gestural communication ensures all interlocutors "speak about the same mathematical object" (p.197). Gestures are essential for effective mathematical communication: "Using gestures to make interlocutors' realizing procedures public is an effective way to help all the participants to interpret mathematical signifiers in the same way and thus to play with the same objects" (p.198). Gestures can be realized *actually* when the signifier is present, or *virtually* when the signifier is imagined. Sfard (2009) illustrates how a student uses "cutting", "splitting", and "slicing" gestures to realize the signifier "fraction". Since these gestures were performed in the air, where the signifier "fraction" is imagined, they provide an instance of virtual realization. Therefore, the same signifier "fraction" may be realized differently with different kinds of gesture or word use.

Routines are meta-rules defining a discursive pattern that repeats itself in certain types of situations. In learning situations, learners may use certain words or gestures repeatedly to model a discursive pattern, such as looking for patterns and what it means to be "the same". Drawing on Ng (2014), dragging is taken as a significant form of communication in this study; it is taken as both a routine for defining a discursive pattern that repeats itself in activities with touchscreen-based DGE, and a visual mediator as a multimodal feature of the students' discourse. A student may use dragging to explore (routine) and signify (visual mediator) the variation of the tangent slope. Using this notion, it is possible for students to incorporate *dragging-gesturing* to respond to each other in communications. Indeed, it was found that as one student suggested that the secant line will get "closer" to the tangent line, another student seemed to have responded by her *dragging-gesturing* to bring the lines "together". These gesture-utterance correspondences were also noted in the analysis of other pairs of bilingual learners' routine involving dynamic sketches (Ng, 2014).

Methodology

The participants of the study were three pairs of 12th grade students (aged 17 to 18) enrolled in a calculus class in a culturally diverse high school in Western Canada. All participants were bilingual learners and self-volunteered to participate from a class of 25, in which roughly half of the class were also bilingual learners. They were regular partners during assigned pair-work activities and were described by their teacher-researcher (also the author) as motivated and comfortable working with each other. The study took place at the end of the first trimester of the school year in the participants' regular calculus classroom, outside of school hours. At the time, the participants have just finished learning key concepts in differential calculus where the iPad-based DGE, *Sketchpad Explorer* (Jackiw, 2011) was consistently incorporated into the lessons. During these lessons, students were invited to explore dynamic sketches in pairs for roughly ten minutes followed by teacher-led classroom discussions about the activities. Therefore, the participants were experienced with exploring and discussing, in pairs, concepts in differential calculus with dynamic sketches.

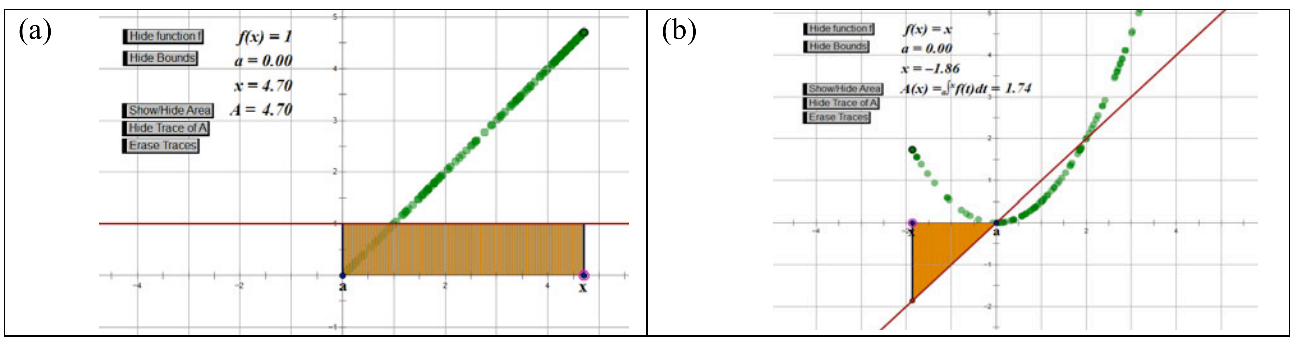

Figure 1(a-b): Screenshots of the DGE used in the study (with all buttons, "*Show function*", "*Show bounds*", "*Show Area under f*", and "*Show Trace of A*" activated). The bounds "a" and "x" are draggable; the green traces represent $A(x) = \int_a^x f(t)dt$.

The task used in this study invited the students to explore and discuss a sketch in *Sketchpad Explorer* that they had not previously seen. The sketch contains five pages all related to the concept of area-accumulating functions (Figure 1a, b). The participants were not told what concept the sketch was related to, but they were told that this concept was new to them (at the time of study, they had just spent one lesson related to integral calculus, that of indefinite integrals in their regular classroom). The participants were asked to "explore the pages, talk about what you see, what concepts may be involved" on all pages of the sketch before reporting back to the teacher as a pair. Before they began, they were ensured that their teacher would check in with them from time to time in order to make sure that they understood what they were expected to do and could ask questions that were related to the sketch. In total, 70 minutes of video data were collected in the study.

Analysis of Data

In this section, I provide detailed analyses of one participant pair, Sam and Mario's communications over the aforementioned task. Sam and Mario, whose home languages were Mandarin and Cantonese respectively, do not share a common home language. A 10-minute episode is chosen from the 30 minute discussion and analyzed in detail. The episode was chosen to highlight the different resources: language, dragging, and gesturing used by the student pair within the 10-minute discussion.

At the start, the students seemed unsure about what to do with the points *"a"*, *"x"*, and the green point in the sketch. They questioned the functions of the DGE with question markers "what" (eight times) and "how" (once) in the first two minutes of the episode. Most of these questions were formulated as Sam used the dragging modality to investigate the behavior of different points. For example, Sam asked "what" repeatedly as he tried to drag the green point which was not draggable. Although he had acknowledged that he had previously pressed a button which showed "an area", he had not realized that the green point had plotted the area in terms of *"x"*, evident in his questions "what's trace of A" and "how do we drag this trace". Then, upon dragging "*x*" around, he finally concluded "oh, oh, it's this one. Ok, make sense." His confusion seemed to have resolved perhaps because his dragging of *"x"* made the green point move; hence he realized the green point was a dependent and non-draggable object. However, it appeared that he remained unsure about what the green point meant, stating that he did not yet "understand this".

At 00:53, Sam and Mario took turns *dragging-gesturing* in a conversation-like manner, beginning with Sam's *dragging-gesturing*, which spanned 30 seconds (Figure 2a). During this occurrence, Sam dragged *"x"*, then *"a"*, and finally *"x"* again. Observing the students' word use and dragging actions, it seemed that both students made some progress in their learning of area-accumulating functions during this span. For example, as Sam was dragging *"x"*, he uttered, "*as* we drag this, the area *becomes*…" This utterance-dragging combination suggests that Sam was thinking about area as having dynamic qualities. It shows how dragging mediated the way Sam thought of the area as a becoming. The use of "become" implies something is happening, in particular, the area was changing *as "x"* was dragged. Furthermore, Sam's statement structure resembled an "if… then…" statement structure which calls upon a causal or functional relationship between *"x"* and the area. It was interesting to note that Sam never finished his sentence after uttering "become", perhaps because he had yet to *realize*, in a Sfardian sense, the simultaneous change in the variables despite noticing the area is changing. Similarly, Mario used a hedge word in his utterance, "it's *like* the area," suggesting a degree of uncertainty about whether or not the green traces meant the area.

Following Sam's prolonged dragging, different draggers and speakers were observed in the episode. For example, as Mario dragged *"x"* back and forth, Sam was responding verbally and simultaneously, "You see how this one moves? So it's like the area." A similar exchange was also noted earlier, where Sam was the dragger, as Mario spoke, "is that the area?" These two instances where the dragger and speaker were different people seemed effective for having the students communicate mutually and simultaneously. Although it may seem impolite and unconventional for one student to "talk over" another student, the presence of "talking over someone else's dragging" was not an issue here. Indeed, Sam's utterance did not interfere with Mario's dragging and vice versa; rather, from the way one talked about area while the other was dragging, they seemed to have made significant progress as a result of this concurrent communications.

Also observed in the first two minutes was the consistent use of gestures by the students in mathematical communications. Namely, Sam used three types of gestures, which in Sfard's terms, functioned quite differently in each usage. In the beginning, Sam used a pointing gesture as he talked about the bounds to make sure both interlocutors *spoke about the same mathematical object* (Figure 2b). Later, he used his hand to signify the linear pattern of the green traces, an instance of *actual* *realization* (Figure 2c). Finally, he flipped his right index finger left and right while uttering, "No you can't. You can only go like", which was another actual realization of the possible movement of the green point (Figure 2d). Moreover, this gesture was not accompanied by any speech, which suggests that Sam relied on gestures as a visual mediator, or a multimodal feature of his mathematical discourse, to communicate in the absence of word use.

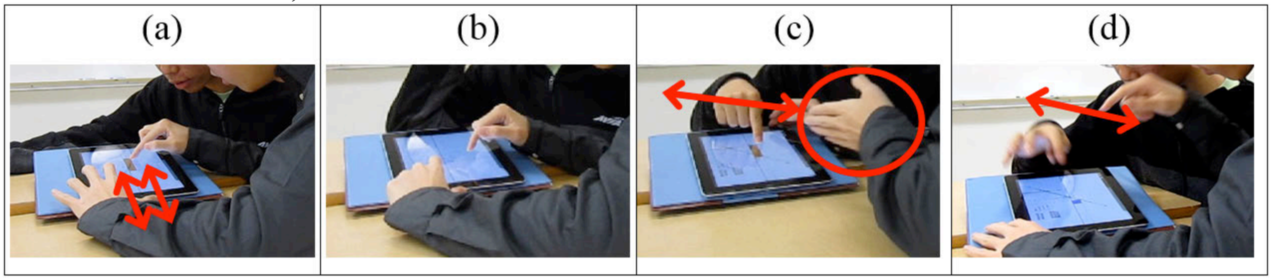

Figure 2: Screenshots of Sam's (a) dragging actions and (b-d) gesturing in the excerpt.

From 02:00 to 05:00, Sam and Mario consistently utilized dragging and gesturing to communicate mathematically. For example, as Sam took on the role of dragging and asked the question, "is that how the area is changing?" Unlike earlier, Sam was able to describe exactly that *the area is changing* with no hedge words at the 02:02 mark in the episode. He continued to drag for a span of 9 seconds without speech before letting Mario also tried dragging for another 6 seconds without speech. The switching of draggers suggest that both students were communicating mathematically while dragging. If speech was analyzed alone, some important analyses about the students' thinking in between speech would have been missed.

At 03:02, Sam and Mario performed a series of hand gestures. Initiated by his own dragging of *"x"*, Sam suggested that "oh, wait a sec. This is actually, the derivative of the graph, function," while he used a hand gesture to signify the shape of a linear function. To restate what he had said, he then used his index finger and traced a "U" shape in the air as he continued to conjecture that the line was "probably the derivative of x, x-squared". Mario responded with a similar "U" shape gesture as he asked "is this x-squared", suggesting that they verified Sam's conjecture. These gestures and word use pairings, which provide strong evidence that the students were engaging in *conjecturing* about the shape of the green traces, help identify Sam and Mario's competence in the mathematical activity.

It was noted that the deictic pointing word "this" was used extensively, appearing five times in this part of the episode. Using deictic words, the speakers no longer need to refer to the mathematical objects by describing them verbally, but they can use deictic words along with different gestures to replace the descriptions completely. This was found in Sam's "*this* is actually, the derivative", "no matter how you move, *this* one always", and "*this* is probably x, x-squared". As Sfard explains, gestures help ensure that the interlocutors speak about the same mathematical objects. Significantly for Sam and Mario, gestures served a complementary function to language in communication. The two students were able to use a combination of utterances and gestures to communicate about the mathematical objects effectively.

The students' realizations of the area-accumulating function could be observed in their discourse after the 05:00 mark of the episode. First, the questions posed hereafter were markedly different from before 05:00. Recall that previously, Sam had asked repeatedly, "what", at times without finishing his questions. It is possible that Sam's "what" questions reflected his uncertainty of what each point or button meant in the sketch. In contrast, having explored the sketch for some time, Sam asked three questions that began with "why". He asked, "why is there something to do with area" twice, and "why is it". By asking these "why" questions, it seemed that Sam was looking for the reason as to

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why the relationship of the two graphs were related to the area under a function. Considering Sam has only learned the topic of indefinite integral at the time of study, this was a valid question because Sam had yet to learn the idea of "definite integral as area" in his class. Regardless, asking "why" implies investigating the reasons of something that is clearly existential. In this case, Sam seemed to be investigating the reason why the *area* under a function had to do with its antiderivative.

At 05:17, a prolonged dragging action was performed by Sam, while the two students exchanged comments verbally back and forth. In particular, by far the longest spoken sentence was said by Sam within his own dragging action: "Ya. So the graph we have *here* is *specifically* the derivative of the… what we just graphed *here*, like the function *here* is *basically* the derivative of what we just graphed *here*." The sentence was very rich in a multimodal sense because it was spoken while the speaker was dragging, and gestures were used simultaneously as the speaker uttered "the function here". Some interesting word use was also observed. For instance, the word "here" was used four times, and the words "specifically" and "basically" each once. In line with a previous analysis of the use of deictic words, the use of locative noun "here" accompanied by gestures allowed the speakers to talk about the same mathematical object. Although Sam used the same word "here" four times, he actually meant to refer to two different mathematical objects, the function and its derivative. This could be why Sam used different gestures to specify which object he was talking about as he said "the function here". Secondly, the contrasting use between "specifically" and "basically" by Sam was also fascinating. Since Sam used the word "basically" quite frequently throughout the task, his word use "specifically" as opposed to "basically" in this sentence drew attention to the analysis. Consistent with his usage of "basically" in other parts of the transcript, it seemed that Sam used the word to suggest a generality or invariance that exists outside of the sketch. In contrast, it is speculated that he used "specifically" in the context of the specific page of the sketch to refer to the particular "graph" that was the derivative of another. According to this speculation, Sam was able to talk about area-accumulating functions both in its generality and particularity, which is a highly valued practice in the mathematics community.

Discussion

The analysis provides strong evidence that Sam and Mario, both bilingual learners, utilized a variety of resources in communication, with visual mediators in the form of gestures and dragging taking on a prevalent role. These included gestures accompanying deictic words and gestures for communicating geometrical notions of calculus. Moreover, the *dragging-gesturing* action emerged in the touchscreen dragging action and fulfilled the dual function of dragging and gesturing. These actions were repeatedly demonstrated by both students for communicating temporal relationships in calculus as well as in their routine of developing the mathematical discourse. In the presence of a dynamic visual mediator, the students' routine evolved from typical utterance-utterance sequences. Gestures-gestures and gestures-utterances sequences were observed in the conversation. Related to this, I observed one student *dragging-gesturing* simultaneously as the other spoke; this allowed two students to communicate simultaneously without interfering with each other. These observations support the claim that bilingual learners make use of gestures and dragging as important forms of communication. Using Sfard's communicational framework to define gestures as communicational acts is especially useful for understanding the mutual communications involved in these new communication routines.

The analysis shows that dragging and gestures transformed the students' word use. Initially, the students seemed unsure of what to make of the sketch; they used dragging to formulate their questions about the behavior of the sketch. Then, they began to explore and conjecture the relationship of the two functions in both geometrical and algebraic terms through dragging and gesturing. Sam and Mario made extensive use of verbs such as "become", "move" and "go," which

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imply change or motion while they used the dragging modality to change the area under a function. Moreover, gestures in the form of actual realizations were accompanied by the use of locative nouns "here" and deictic word "this". These gestures and word use pairings could potentially reduce the number of words to be spoken in a sentence.

The results of the study were encouraging not only in that the pair of bilingual learners was able to grasp quickly the functions offered by the touchscreen and DGE, but also in the way they communicated significant calculus ideas effectively incorporating linguistic and non-linguistic features in communications. These results have important implications towards the mathematics teaching and learning for all learners at large and bilingual learners in particular. For example, the study points to the use of DGE in pair-work activities for facilitating students' communications. At the beginning of their explorations, Sam and Mario did not use the functions of the DGE purposefully. As they began to learn to use the functions of the DGE, they began to engage in calculus ideas, and the DGE began to take on an important role in the communications. This implies that the role of the DGE is not a static one, but rather *dynamic* that is constantly evolving during the activity.

Also, it could be said that the design of the dynamic sketches played a significant role in facilitating the students' mathematical communications. The *Hide/Show* buttons allowed the students to talk about their ideas gradually, one button at a time, while the *dragging* affordance enabled them to attend to dynamic relationships and connect algebraic with geometric representations of calculus. In tune with previous studies on DGE-mediated student thinking (Falcade, Laborde & Mariotti, 2007), the students may have exploited these functionalities offered by the sketch and hence communicated about area and derivatives geometrically and dynamically as a result. Furthermore, the touchscreen-based DGE seemed to offer a haptic environment for learners to interact with dynamic relationships, where the nature of gestures and dragging is re-conceptualized (see Sinclair & de Freitas, to appear).

This study argues for an expanded view of bilingual learners' communication that includes utterances, gestures, dragging and diagrams. Due to their complementary functions, these elements must not be accounted for in isolation but as a full set of resources in mathematical communication. Although Sfard has not specifically addressed the distinction between dynamic and static visual mediators, the distinction is important for this study because of the potential for the dynamic visual mediators such as gestures and DGEs to evoke temporal and mathematical relations (Ng and Sinclair, 2013), particular for the study of calculus (Núñez, 2006). As shown in my analysis, *dragging-gesturing* emerged as a significant form of communication, and this was facilitated by the dynamic visual mediator presented on the touchscreen-based DGE. Future studies should consider extending the notion of visual mediators and routines to include gestures and dragging on touchscreen-based DGE. In particular, this paper calls for more studies in the area of DGE-mediated learning to investigate the role of *dragging-gesturing* in other types of mathematical activities and within other branches of mathematics.

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