MATHEMATICS, LANGUAGE, AND DEGREES OF CERTAINTY: BILINGUAL STUDENTS' MATHEMATICAL COMMUNICATION AND PROBABILITY

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While all mathematics is mediated by language, the role of language is especially complex in bi- and multilingual mathematics classrooms, and more so in bilingual education programs in which the explicit goals of both language and mathematics learning intersect. We explore bilingual French immersion students' linguistic and mathematical repertoires as they work through a series of probability problems. Focusing on the collaborative dialogue that occurred between students and researchers, our discourse analysis was informed by sociocultural theory and systemic functional linguistics. Findings indicate that students' linguistic and mathematical repertoires are intertwined, and that collaboration can offer opportunities for supporting bilingual learners' language and mathematics development. We conclude with implications and challenges for bilingual mathematics education.

Keywords: Classroom Discourse; Instructional Activities and Practices; Probability

One of the most novel characteristics of French immersion programs is that students whose first language is not French learn this target language not only through French language classes but also through content courses such as mathematics. Both mathematics learning and language learning are explicitly stated goals of the immersion program (Swain & Johnson, 1997). Around the world, students learn mathematics through languages other than their first or home language(s) in a variety of bi- and multilingual mathematics classroom contexts. Consequently, it is important for researchers and educators to examine how mathematics and language learning simultaneously take place in the classroom.

Theoretical framework

Work by Halliday (1978) and others has brought to the forefront issues related to language in mathematics and, more specifically, the mathematics classroom. Halliday described three aspects to consider with regard to any linguistic situation, including mathematical discussion: "first, what is actually taking place; secondly, who is taking part; and thirdly, what part the language is playing" (p. 31). Drawing on these ideas, we focus on mathematics as a meaning-making activity rooted in the social interactions of the learners. The notion of the mathematics register describes this in more detail: The mathematics register involves "the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes" (p. 195).

From a second language education standpoint, we adopt a theoretical framework that supports our view of mathematics and language. We draw on the work of Vygotsky (1962, 1978) and neo-Vygotskians (Cole, 1985; Donato, 1994; Lantolf, 2000; Lantolf & Appel, 1994; Swain, 2000; Swain, Kinnear, & Steinman, 2011; Wertsch, 1985, 1993), which has underscored the social element driving all individual cognitive functions. Moreover, this work has emphasized the key role language plays in all human interactions and learning. Language is viewed as a mediational means, in other words, language mediates thought and is not strictly a conveyor of thought. This approach, rooted in the exploration of language and learning through social interactions, guides our analysis of mathematical discourse.

Selected literature

Our review of selected literature has two main parts: first, a discussion of studies of mathematics and language from the mathematics education field, with particular focus on those based in bi- or multilingual contexts; and second, an exploration of some key sociocultural concepts related to second language education.

Mathematics and multilingual classrooms

Research based in a variety of multilingual mathematics classrooms has highlighted a number of important issues with regard to mathematics and language. In particular, scholars have pointed to a need to recognize that the mathematics register is enacted in unique ways within the mathematics classroom. The specialized language of the mathematics classroom is distinct from the specialized language of mathematicians (Barwell, 2005, 2007, 2009b; Barwell, Leung, Morgan, & Street, 2005; Morgan, Craig, Schüte, & Wagner, 2014; Moschkovich, 2003, 2007, 2010; Pimm, 2007; Setati & Adler, 2000). This work has underscored the importance of classroom context, and has viewed mathematics as a social, discursive activity. From this standpoint, "mathematical discourse includes not only ways of talking, acting, interacting, thinking, believing, reading, writing but also mathematical values, beliefs, and points of view" (Moschkovich, 2003, p. 326). Far from a homogeneous set of practices and norms, some general characteristics of mathematics classroom discourse can include "being precise and explicit, searching for certainty, abstracting, and generalizing,[… and] imagining" (Moschkovich, 2003, p. 327).

With regard to bilingual learners in particular, research has called for a refocusing on the resources these learners bring to the mathematics classroom rather than on their so-called problems or deficiencies (Barwell et al., 2005; Moschkovich, 2003, 2007). In this vein, studies have suggested that allowing for ambiguity, or the acceptance of multiple meanings, during mathematical collaboration can be a resource for mathematical understanding, particularly for bilingual students (Barwell, 2005). Moreover, hearing the mathematical in students' so-called everyday talk is also key to supporting bilingual learners, who may use this everyday talk to contextualize and understand the linguistic and mathematical aspects of problems (Barwell, 2009a; Moschkovich, 1999, 2003, 2005, 2009a, 2009b). This approach does not view mathematical and language learning as separate, but rather as intertwined and co-developing in a reflexive relationship (Barwell, 2005).

A further concept for consideration in bilingual mathematics classrooms is if, when, and how multiple languages are used and valued (or not). Often referred to as codeswitching, that is, the switching of languages "within the course of a single conversation, whether at word or sentence level or at the level of blocks of speech" (Baker, 2011, p. 107), this phenomenon has been explored in both mathematics and second language education contexts. As scholars in both fields have explained, historically codeswitching has been perceived as indicative of a deficiency with regard to bilingual students' mathematics and language proficiency. Recent work, however, has challenged this view. Researchers have argued for a positive, resource-oriented view of bilingual learners that recognizes the resources they bring to the mathematics classroom and, in line with sociocultural theory, this may include learners' first or home language(s). This approach challenges the monolingual norm and views codeswitching as socially and cognitively complex. However, codeswitching remains a contentious and controversial issue and the use of multiple languages in the mathematics classroom often conflicts with political agendas and language policy goals (Adler, 1999; Barwell, 2009b, 2014; Cummins, 2007; Moschkovich, 2005; Planas & Setati-Phakeng, 2014; Swain & Lapkin, 2000; Turnbull & Dailey-O'Cain, 2009; Setati & Adler, 2000).

Second language learning through content

With regard to second language learning in mathematics, two key sociocultural concepts emerge that are pertinent to our analysis. The first is the notion that as learners interact with a more capable

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other, who could be their teacher or their peers, they can achieve more than would have been possible on their own. In this scenario, the learner eventually gains control over the task, internalizes the skill, and is able to perform it independently. This movement from other- to self-regulation is described as what happens in the zone of proximal development (Vygotsky, 1978). It relates to a pedagogical notion called scaffolding, in which a temporary scaffold provided by an expert other is used to help learners with a particular learning task (Cole, 1985). The scaffolding is eventually dismantled as the learner becomes more capable and the responsibility for the task is gradually transferred from expert to learner.

The second pertinent concept is the notion that language learning occurs during collaborative dialogue. According to Swain (2000), collaborative dialogue "is where language use and language learning can co-occur. It is language use mediating language learning. It is cognitive activity and it is social activity" (p. 97). In this view, when language learners engage in problem-solving tasks they are able to notice and pay attention to linguistic elements and co-construct knowledge through producing output through collaborative dialogue. These language-related episodes mediate the learners' understanding of the problems and solutions (Donato, 1994; Swain, 2000; Swain & Lapkin, 1998).

The study

The current study is framed within a larger, 3-year longitudinal study entitled "Students' language repertoires for investigating mathematics" (supported by the Social Sciences and Humanities Research Council of Canada, Principal Investigator: David Wagner). In this paper, we focus on bilingual French immersion students' linguistic and mathematical repertoires during collaboration with an interviewer-researcher on probability-related problems and activities. (We have discussed different aspects of the larger study elsewhere. See, e.g., Culligan, Dicks, Kristmanson, & Wagner, 2014; Wagner, Dicks, & Kristmanson, 2015.)

Context and participants

The participants in the current study were Grade 3 French immersion mathematics students in their first year of the program. Students first engaged in a whole-class probability-based activity (Skunk die game, described in the next section) and then worked on related problem-solving tasks in small groups of two to three. As a follow-up, students interacted with an interviewer-researcher as an extension of the whole-class activity. During these interviews, students were introduced to a second probability-related activity (Skunk card game, described in the next section) and responded to questions related to the two games. The students were asked about their strategies for playing both games, about the differences between the two games, and about different words of interest (related to probability and degrees of certainty) they had used while responding to these questions and/or engaging in the problem solving.

The probability activities

In both the Skunk die game and the Skunk card game, students were introduced to the problem with a narrative: You are picking berries in the forest and trying to collect as many berries as possible before the skunk comes. Numbers 1 to 5 represent the berries you collect on each roll of the die. The number 6 represents the skunk and the end of the turn. If the skunk comes, you lose all of the berries you collected on that turn, unless you have "gone home" to avoid the skunk when you had enough berries. You do this for seven days (Monday to Sunday). The player with the most berries at the end wins the game.

In the Skunk card game, the interviewer-researcher laid playing cards (numbered 1 to 6, with 6 being the skunk) out on the table one by one, rather than rolling a die. The cards were not picked up once laid down. Rather, the interviewer-researcher continued laying down cards one by one as long

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as the student wished to continue. We moved on to the next day of the week once the students had decided to stop collecting berries for the current day, or once the skunk card was played. Cards were not picked up and reshuffled until all six cards had been laid down; this could happen in the middle of the current "day." Thus, the probability of getting the skunk on any given turn differs in the card game compared to the die game. In the card game, the events are mutually exclusive, and in the die game, the events are independent.

Data collection and analysis

Students were audio and video recorded during the whole-class activity and the follow-up interviews. Data were transcribed and written transcripts were the primary source for analysis. We analyzed the data using Swain and Lapkin's (1998) approach to discourse analysis, which entails describing and interpreting language-related episodes. Furthermore, we drew on the field of systemic functional linguistics (e.g., Halliday, 1994), which enabled us to describe and interpret specific instances of language use within our particular context.

Results

To discuss our results, we present selected excerpts of transcripts from the students' interviews with an interviewer-researcher and offer our interpretations.

Excerpt 1: Linguistic and mathematical uptake of "*absolument*"

The following is an excerpt from a Grade 3 interview. This is the first year of French medium learning for these children. English translations are provided on the right. The interviewer-researchers (R1 and R2) are asking the students (S1, S2, and S3) their predictions regarding the upcoming cards and their degree of certainty regarding these predictions. One researcher (R1) leads the interview and the other (R2) is behind the camera, taking note of students' language use and then participating in the interview later.

115	*R1:*	Est-ce que c'est absolument le quatre?	*Is it absolutely the four?*
116	*S3:*	Oui.	*Yes.*
117	*R1:*	Est-ce que tu es certain que c'est le quatre?	*Are you certain it's the four?*
118	*S3:*	Oui, non.	*Yes, no.*
119	*S3:*	Ça peut être une trois aussi.	*It could be a three too.*
120	*R2:*	Quelles sont les chances que ça soit un trois?	*What are the chances that it's a three?*
121	*S2:*	Beaucoup….	*A lot…*
…			
166	*R2:*	Alors ça doit être quoi ici?	*So it has to be what here?*
167	*S2:*	Il doit être, un, deux trois, quatre, cinq.	*It has to be one, two, three, four, five.*

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168	*R2:*	Ça doit être des fraises (et non pas la moufette)?	*It has to be berries (and not the skunk)?*
169	*S1, S2, S3:*	Oui.	*Yes.*
170	*R2:*	Absolument des fraises?	*Absolutely berries?*
171	*S2, S3:*	Oui.	*Yes.*
172	*S1:*	Absolument.	*Absolutely.*

In this exchange, we see linguistic uptake of *absolument* (used by R1 line 115, R2 line 170;taken up by S1 line 172). Moreover, there is mathematical uptake of *absolument*, a concept related to probability. The students go from being very certain (line 116), to questioning/hedging (line 118), to using a modal expressing a greater degree of uncertainty (line 119). Throughout the exchange, the researcher-interviewer acts as a more knowledgeable other, providing scaffolding and pushing students to go farther than they may have done on their own. Notably, however, the questioning of the student's response did not lead the student to change her answer ultimately—she worked through the task and decided she was *absolument certaine*. Here, mathematics and language work together to solidify the students' understanding of the probability concept.

Excerpt 2: Explaining the meaning of "*çadoit*"

In this Grade 3 excerpt, the interviewer-researcher (R1) is asking the students (S1, S2, and S3) the difference between "it has to be a 6" (card game) and "you have to brush your teeth" which present different senses of the modal verb "have to"—one indicating logic and the other obligation. The students relate this distinction to the English expressions "it is going to be a 6" and "you are going to brush your teeth." (In the translation at right, the underlined text is not translated because it is English in the original.)

388	*S1:*	En anglais « doit » dans la première phrase, ça doit être une moufette.	*In English "has to" in the first sentence, it has to be a skunk.*
389	*S1:*	Et, dans l'anglais, ça veut dire « it's going » and, dans l'autre phrase, ça dire « you have to. »	*And, in English, it means "<u>it's going</u>" <u>and</u>, in the other sentence, it means "<u>you have to</u>".*
390	*R1:*	« You have to », comme tu doit te brosser les dents et « it's going. »	*"You have to", like you have to brush your teeth and "<u>it's going</u>."*
391	*S2:*	Tu n'as pas une choix.	*You don't have a choice.*

Here, the students use their first language, English, to clarify their ideas. The first language seems to provide them with resources to strengthen and confirm their explanation that there is a difference between the two uses of "have (has) to". Students use English to clarify or confirm their interpretation of the French expression "*çadoit*". Comparing "it has to be a six" in the card game, in which students knew the next card "had to" be the skunk (it was the only card left to be played), to the sentence "you have to brush your teeth" was a cognitively challenging activity both mathematically and linguistically. In the last line, S2 raises the question of choice, which is inherent

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in the "you have to brush your teeth" example, but not in the "it has to be the skunk" example. Students use their first language as a tool for discussing the multiple meanings of "*çadoit*" and, in so doing, construct both mathematical and linguistic understanding.

Discussion

Our results highlight the mathematical and linguistic understanding that can occur during collaborative dialogue in the bilingual mathematics classroom. When viewed through a sociocultural theory lens, in the first excerpt, the learners, through the scaffolded guidance provided by the interviewer-researcher, are able to go farther, mathematically and linguistically, than they may have been able to individually. Through the interviewer-researcher's introduction of the term "*absolument*", students are able to pick up that language and use it to explore the mathematical concept of certainty. Similar to the reflexive relationship described by Barwell (2005), in this study students' mathematical understanding of the probability-related concept of certainty develops in an interwoven fashion with their linguistic understanding.

In the second interaction, students engage in a phenomenon that is of particular interest to many working in bilingual mathematics classrooms—codeswitching. Despite some traditional, deficit-oriented views of codeswitching, recent research in the field of second language education has argued that in the classroom, judicious use of students' first language can serve as a resource for second language learning (e.g., Cummins, 2007; Swain & Lapkin, 2000; Turnbull & Dailey-O'Cain, 2009). Moreover, research in mathematics education has argued that bi- and multilingual learners use their first language, home language(s), or shared language(s) as a resource for mathematical learning and that it plays an important social and political role (e.g., Adler, 1999; Barwell, 2014; Moschkovich, 2005; Planas & Setati-Phakeng, 2014). Barwell and Setati (2005), for example, have urged mathematics educators to find "ways of dealing with linguistic diversity that avoid reducing mathematics classroom interaction to a monolingual (English language) norm" (p. 23). Although the research contexts referred to here are varied and each is unique, codeswitching is a phenomenon that seems to occur throughout. A sociocultural theoretical framework that views language as a mediator of thought and as a cognitive tool, allows us to view students' codeswitching in this study as a resource for mathematical and language learning, rather than a problem or deficit to be overcome.

Implications and challenges

The two excerpts featured here point toward implications, and corresponding challenges, for mathematics educators working in bi- and multilingual contexts. First, we suggest that providing opportunities for students to engage in collaborative dialogue with each other and with their teacher is important mathematically, linguistically, and socially. Taking the time to allow these interactions to unfold is challenging when faced with the demands associated with covering curriculum outcomes and assessment, but can result in learning that is mathematically and linguistically valuable. It will be imperative for mathematics educators to recognize, value, and build upon the mathematics present in students' multiple meanings, and in their everyday talk. This is particularly true for bilingual learners. Viewing both mathematics and language as social, discursive activities may help foster collaborative exchanges.

Second, the ways in which multiple languages are used in any context, including the mathematics classroom, are complex. Interpretations of codeswitching practices must take into account contextual, political, and language policy factors. Nonetheless, researchers across contexts are increasingly viewing student codeswitching as a potentially resourceful way of understanding complex mathematical and linguistic content. In spite of this, local policy often dictates that one language only, the target language, be used as the language of teaching and learning in the classroom (and this is certainly the case in our study's context, French immersion). The challenge will be for researchers and educators to continue to explore in more detail if and how students' multiple languages can be used in the mathematics classroom, and how to do this in a way that results in effective and efficient language and content learning.

Conclusion

In sum, our results suggest that collaborative dialogue can be a meaningful activity in the bilingual mathematics classroom. In particular, interaction may provide opportunities for bi- and multilingual learners to learn not only mathematical content but also language. Learners can build on the scaffolding provided by teachers and even their peers to extend their understanding of linguistic and mathematical concepts.

We argue for a need to move beyond viewing strictly academic mathematics vocabulary as the only acceptable or valuable mathematical communication. While gaining control over mathematics terminology is without a doubt important, students also need to acquire the language necessary to talk about mathematics. Moreover, language use, language learning, and mathematics learning are largely, if not entirely, inseparable.

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